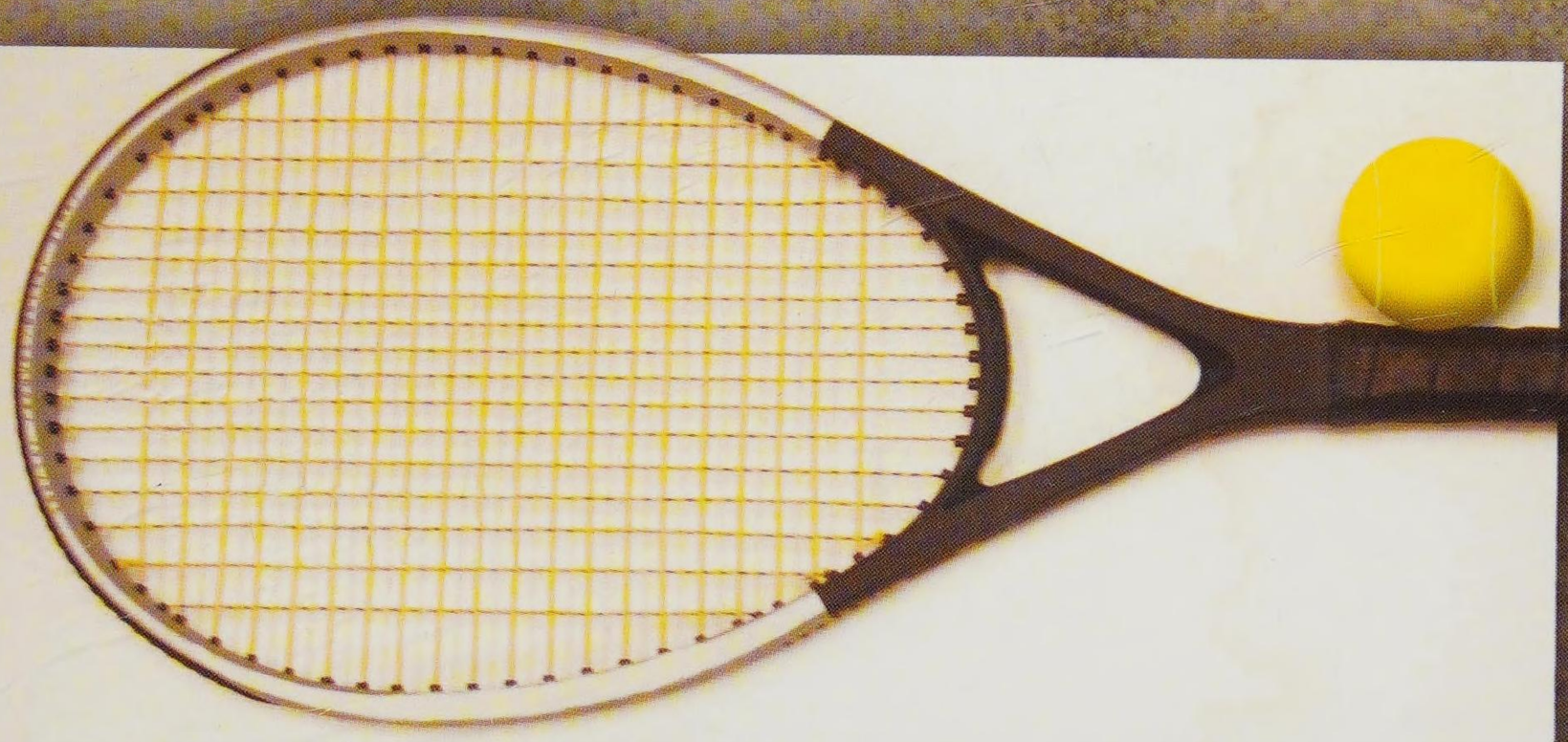
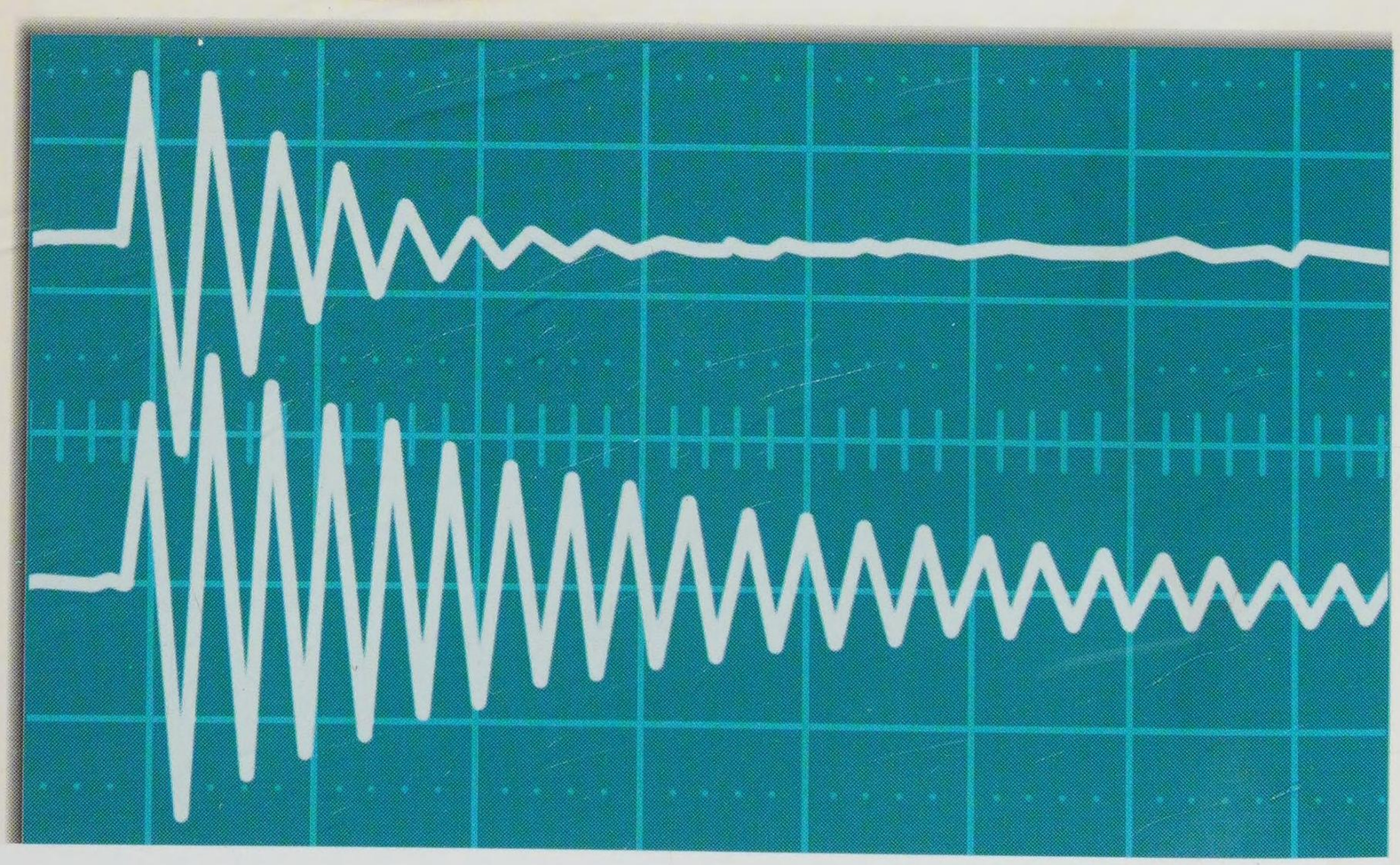
THE PHYSICS AND TECHNOLOGY OF TENNIS





HOWARD BRODY, ROD CROSS
& CRAWFORD LINDSEY

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The Physics and Technology of Tennis

Howard Brody
Rod Cross
Crawford Lindsey

USRSA
United States Racquet Stringers Association

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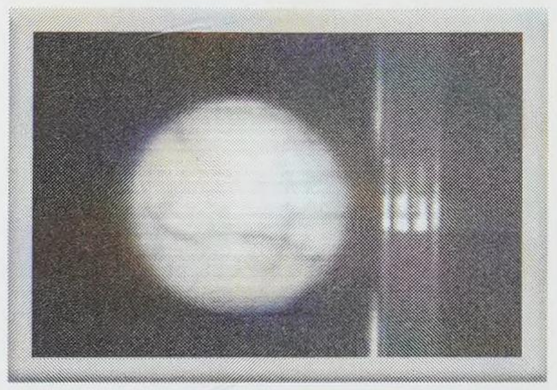

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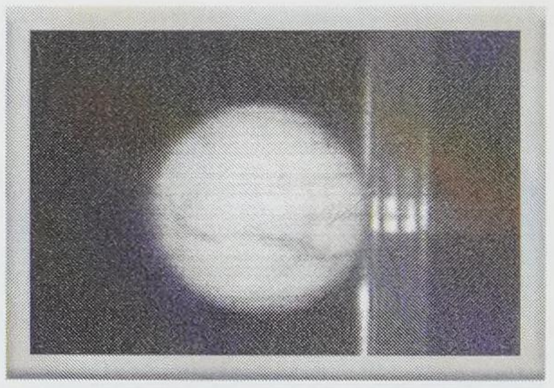

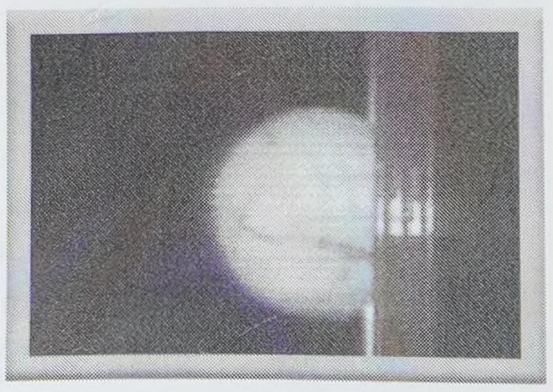

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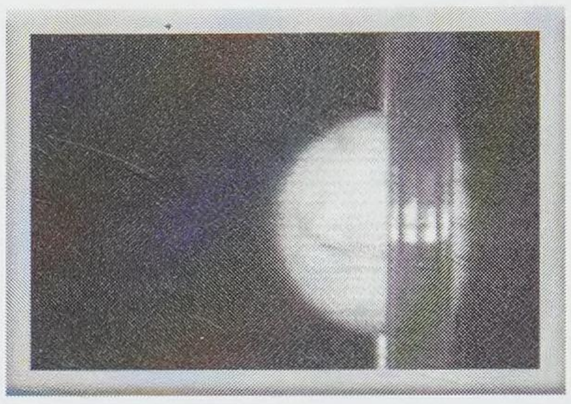

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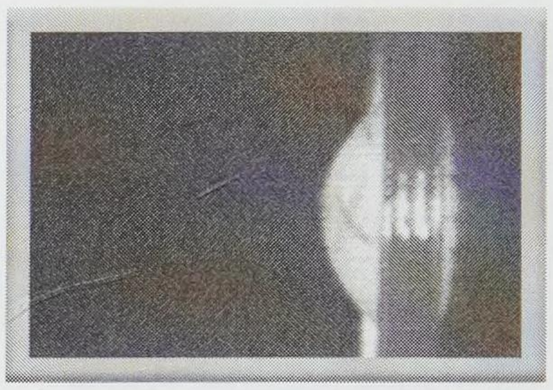

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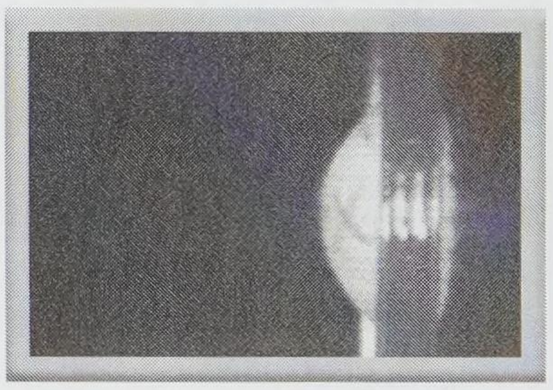

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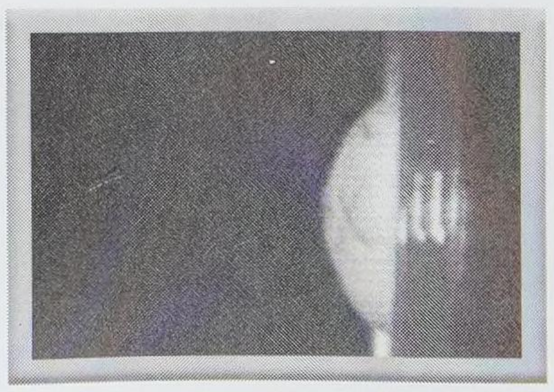

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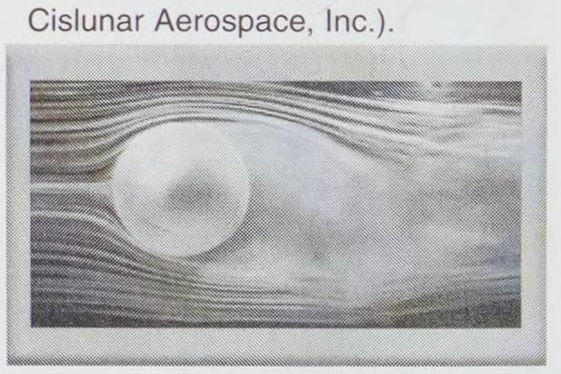

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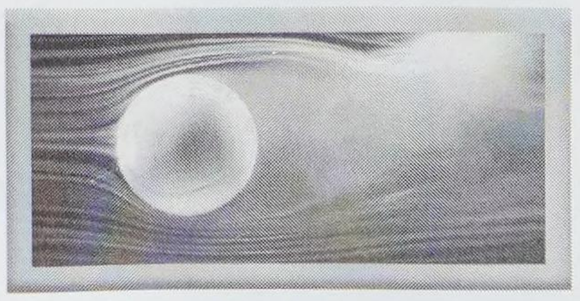

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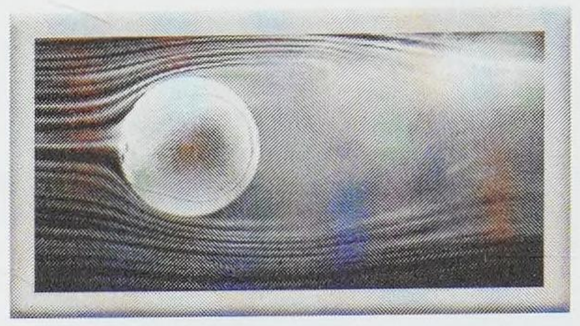

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Preface

This book fills several niches. It was conceived to be the all-in-one source of enlightenment for all things scientific and technological about tennis equipment and its affect on a player's performance. This task was not undertaken simply for the pure joy of intellectual pursuit. There is a great need and want in the marketplace and on the tennis courts for information about equipment. Tennis equipment has become so technologically advanced that buying a racquet or choosing a string requires either a Ph.D., in order to make an intelligent choice, or total blissful ignorance, to avoid the stress of making the wrong decision. Other sports have become the same way — golf in particular.

For years, players have pondered many tennis mysteries:

- Is a stiff frame good or bad?
- Where is the sweetspot and what does it do for me?
- What causes shock and vibration? Is it injurious, and does it affect my shots?
- Does string elasticity make a difference, and how can I influence that elasticity?
- What difference does string tension make?
- Where should I impact the ball on my racquet?
- Should I use a lighter or heavier racquet?
- Can I and should I alter the weight and balance of my racquet?
- How much difference do different string materials, constructions and gauges make?
- How can I create ball spin, react to spin, and read spin?
- How can I hit with more power?
- And many more ...

All of these questions can be answered with a little understanding of the physical fundamentals underlying the universe — tennis, of course, being its most important microcosm. In the past, these questions have been answered with one part science and two parts folklore, and a pinch of myth. Sometimes the answers are correct, but the explanations are wrong. Sometimes both are correct, but if you measure the physical quantities involved, they are not enough to make a difference anyway. And sometimes, players' perceptions of what is happening at ball impact with the strings, racquet and court are at odds with the measured physical reality. And we all know that you can't talk players out of what they feel.

So this book is here to set the record straight. This book is a hybrid of sorts. It is part textbook, part reference, and part "good read." There is tennis science here for every level of expertise and curiosity. It is "popular science," but without leaving out the hard explanations that make for true understanding. Concepts are fully explained in words. But, where necessary, words are supplemented with formulas, and the formulas, if very important, interesting, or intrinsically fascinating, are derived. In all cases, the math can be skipped without loss of the conceptual train of thought, and most of the chapters are formula free anyway. Where present, the formulas help to better understand the relationships that are involved in any equipment associated event on the tennis court. And, of course, if you want the answer to a specific question like, "How fast will the ball go, or where will the ball land if I hit it in such and such a way with such and such a racquet," then you will need to

use the equations. The book is also heavily illustrated with diagrams and graphs to complement the text and to add visual understanding and eye appeal.

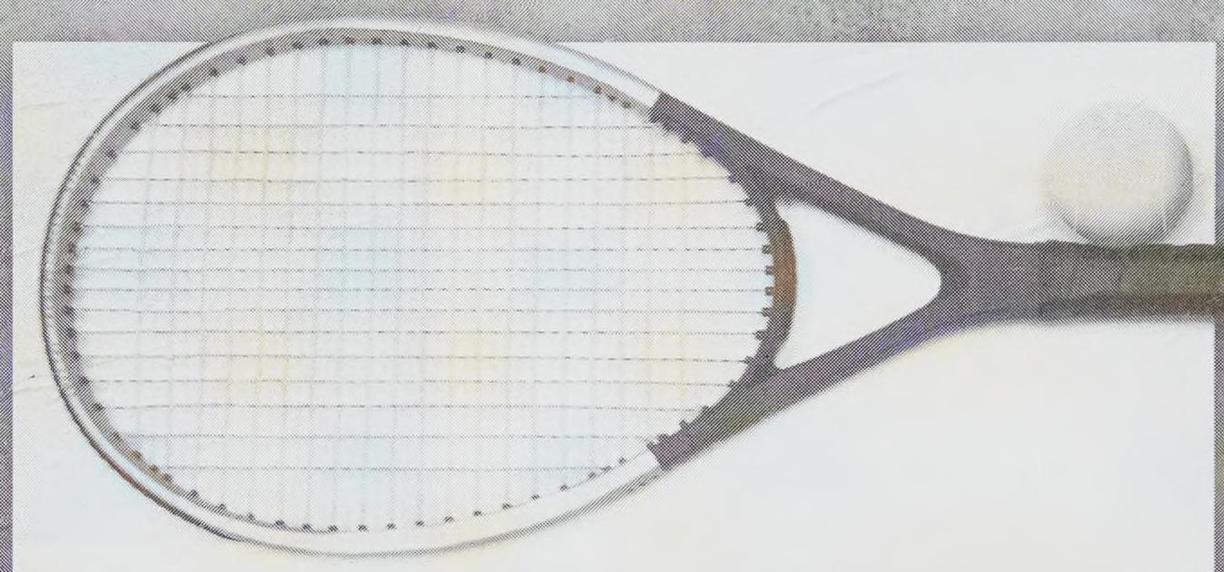
There is some duplication of discussions and explanations in the book. This is on purpose. Not every reader is going to read the book from front to back. Nor is it our intention that they do so. Rather, our intention is to have each chapter stand by itself as much as possible, so the reader can delve in anywhere one's need or whim dictates. To do this, some chapters require concepts and explanations from other chapters. We make no apology for that. Quite the contrary. The end result is that each chapter's discussion can be understood in a larger context, without reading any prerequisite chapters.

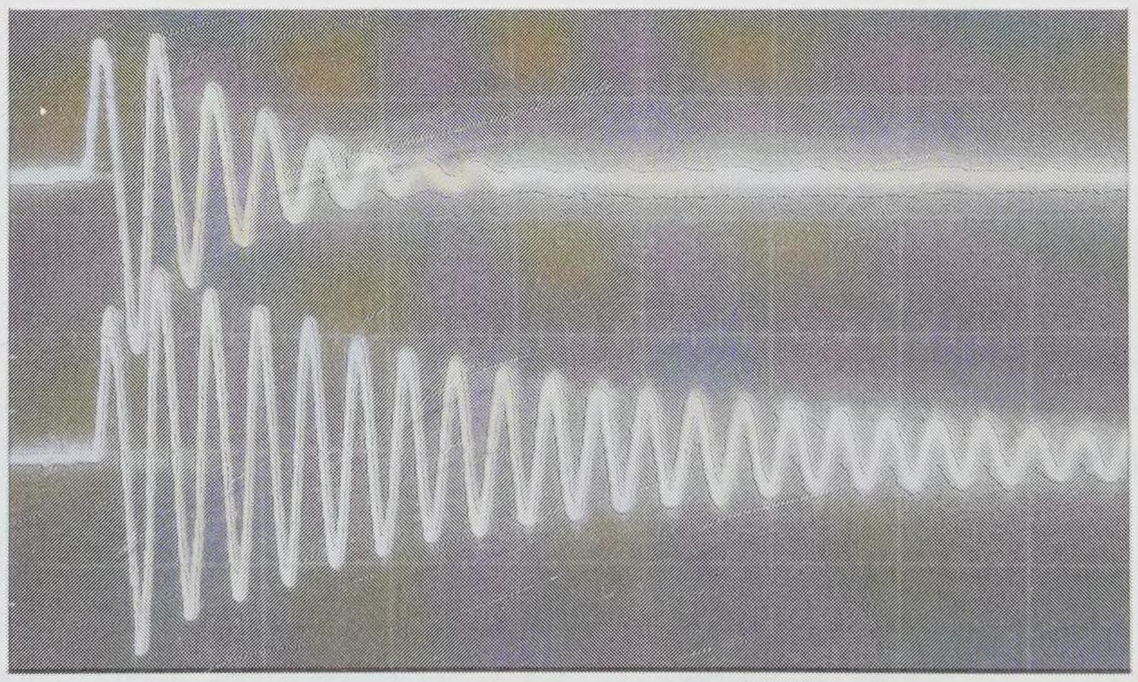
Appendices complete the presentation. Here you will find a glossary, handy units and conversions, equipment rules, court dimensions, references and other more esoteric information.

The authors would like to thank the following for photographs: Ron Waite/Photosportacular for the player photographs at the beginning of each chapter; The University of Sheffield and the ITF (International Tennis Federation) for the ball bounce photos in the Contents; and NASA Ames Research Center Fluid Mechanics Laboratory and Cislunar Aerospace, Inc. for the wind tunnel photos in the Contents. Thanks also to R. Christian Anderson for his beautiful cover design.

Recognition is due also to Racquet Tech Publishing, which is a division of the United States Racquet Stringers Association (USRSA). The USRSA is a worldwide organization of over 7000 racquet technicians, teaching professionals, racquet sports retailers, manufacturers, sales organizations and tennis (racquet sports) enthusiasts. Its mission is to educate all its constituencies to better understand, service, perform with, and enjoy the technological wonders known as racquets, strings, balls, courts, shoes, and stringing machines. For information about membership, products, or services, go to its website at www.racquettech.com, call 858-481-3545, or write to USRSA, 337 S. Cedros, Suite D, Solana Beach, CA 92075.

PART ONE
RACQUETS





Pete Sampras, United States

1

Choosing A Racquet

Why Some Racquets Are Better Than Others

By Rod Cross

A difficult question to answer is why some racquets are better than others even though they all look much the same and they are all made from the same basic ingredients. If you pick ten racquets at random and hit a few balls with each, one or two are likely to feel a lot better than the others. It is as if some racquets have a magic ingredient that makes them perform better — at least better than others for some people. Because of differences in abilities and styles of play, there is no universally acknowledged "best racquet." The perfect racquet has not yet been invented. If it had, then everyone would be using it. There definitely are, however, some underlying basic principles or ingredients that make some racquets very good and others just ordinary.

No one uses wood or aluminum racquets any more. They have been replaced by lighter, stiffer and stronger graphite versions with larger heads. These factors, then, must be part of the formula that makes one racquet better than another. But what is the optimum combination of weight, stiffness, weight distribution and head size that best suits the individual player? And what of string type, string gauge, string tension, sweetspot size and the amount of vibration damping built into the racquet? If one mixes all these ingredients in the right proportions, then it should be possible to come up with a really good racquet.

In theory, it should be possible to come up with a formula for the ideal racquet based on the physical properties of a racquet and a player's individual style of play. If we knew what the formula was, it would be relative-

ly easy to design the ideal racquet for each different type of player. In fact, this has already happened to some extent. Racquets these days are commonly classified as either "player's" racquets or "recreational" racquets. A "player's" racquet is one designed for professional or elite players, and it typically has a medium to small size head and is relatively heavy. Recreational racquets are for everyone else and are generally lighter and have a larger head. Racquets used by the professionals all look much the same and the lengths are all between 27.0 and 27.5 inches. Professionals don't use any of the high-tech latest models and they don't choose oversize or widebody or extra long frames. The differences are mainly in the weight and the way that the racquets are customized with lead tape in the head and the handle to suit each player.

There are also intangibles in good racquets. For example, many players (including professionals) cannot play without a string dampener inserted in the strings. But why? The string dampener stops string vibrations after the ball has left the strings, but it has no effect on ball speed or ball control or frame vibrations. It just deadens the sound of the strings. So a good racquet must be one that not only feels good, it also has to sound good.

Choosing a Racquet

The best way to choose a good racquet is to take a dozen or so different racquets onto a court and hit a few balls with each racquet. Some tennis shops allow you to do this but they can be hard to find. In fact, twelve racquets is a bare mimimum. Ideally, each of the twelve racquets you try should be multiplied by three so that you can try each model strung at low, medium and high tension. Then multiply that by four so you can try gut, nylon, polyester and kevlar. You will never actually find your ultimate weapon, since no one will lend you three identical racquets strung at three different tensions, and new models keep appearing faster than you can try them. You will never reach your ideal playing standard either. But it is fun trying to get there and a good racquet will make a big difference.

The first racquet I ever bought felt perfect in the shop, but it was also the worst racquet I have ever owned. I made the mistake of assuming that a racquet that feels good when you swing it will also feel good when you hit the ball. Nothing could be further from the truth. The truth is, hitting a ball is vastly different from hitting air. There is no jarring and no vibration and no sensation of power when you hit the air. It is absolutely essential to hit a ball when choosing a racquet to buy.

How Does It Feel?

The main thing to look for is whether the racquet feels comfortable to swing and feels good when you hit the ball — not too much shock and vibration, not too heavy and not too light. Every racquet you try will feel different from the next. However, if you hit the ball in the middle of the strings then there will be no vibrations at all in any racquet. Racquets vibrate only when you hit a few inches away from the center, especially if you hit the ball near the tip or the throat. So, if one racquet vibrates more than another, it might be because you didn't hit the ball in the middle of the strings. Don't assume that it was the racquet's fault. Keep hitting until you can decide if it is you or the racquet that is causing the vibrations.

Some racquets DO vibrate more than others. Stiff racquets always vibrate less than flexible racquets since they are harder to bend. A racquet that vibrates does so because it bends rapidly one way and then the other. Consequently, flexible racquets vibrate more than stiff racquets. Also, racquets strung at high tension vibrate more than racquets strung at low tension. It is perhaps surprising that a racquet vibrates less when the frame is stiffer but it vibrates more when the strings are stiffer. The reason is that the ball sits on soft strings for a longer time, which helps to dampen both the string and the frame vibrations. The least amount of vibration therefore occurs in stiff racquets strung at low tension. Try hitting with an old wood racquet if you can find one. They are very flexible compared with a modern graphite frame, and they vibrate MUCH more than a modern racquet.

Grip Size

An important factor in choosing a racquet is to select the correct grip size. The best racquet in the world will feel terrible if the grip is too large or too small for your hand. Good racquet shops will have a selection of handle sizes that you can try and they should also have a chart based on hand size to select the correct grip. Grips are cheap and easy to change. Some people prefer a tacky feel, and others like a soft feel since it helps to cushion the blow when you mishit a ball. If you decide that the handle size is too small, the problem can be fixed by an overgrip or by building up the handle size with plastic heat-shrink tubing under the grip.

Racquet Weight and Power

Should you choose a light or a heavy racquet? This is largely a matter of personal taste, but light racquets are easier to swing and you can swing them faster. Light racquets are more maneuverable so you can move them from point A to point B quicker and you can rotate them faster around the long axis. However, the ball can also rotate a light racquet faster and further when the ball pushes on the strings. If the ball is hit in the middle of the strings, the impact normally causes the wrist to rotate backwards. With a "wristy" shot, the impact slows the rate of wrist rotation even if it does not result in a backwards rotation of the wrist. If the ball impacts towards the top or bottom of the frame, the racquet will rotate about its long axis. Light racquets rotate further, resulting in a larger error in the rebound angle off the strings. Ball control suffers if you mishit the ball. Light racquets are less forgiving in this respect.

If you want raw power, then the most powerful racquet possible would weigh about 1000 gm — about the same as a baseball bat. However, baseball players swing their bat only a few times in a whole game; they don't have to run with it to get to the ball; they don't have to serve the ball; and they can play only one half-decent shot — the double-handed forehand. Obviously, raw power is not the only factor in choosing a racquet. In fact, power tends to be overrated as a selling point. Ball speed off the strings is easy to increase simply by hitting the ball harder. Players will do so when it is appropriate, but maximum possible ball speed is not necessary or desirable on every shot.

Tennis players can get enough power (and much better maneuverability) with a racquet weight between 250 and 400 gm. The speed of the ball off the strings depends on three factors. The first is the speed of the incoming ball. Apart from the serve, the incoming ball speed is determined by your opponent, so there

are really only two factors to consider. The most important of these is the speed of the racquet. If you swing the racquet faster, then the speed of the ball off the strings will also be faster. The other factor is how well the ball bounces off the strings. The main factor here is the weight of the racquet. String tension also plays a small role since it affects the softness of the impact and that influences the energy loss in the ball and the energy lost in vibrations of the frame. There are no other factors involved, unless you want to include tiny effects such as wind speed or the type of string used. Many players and string manufacturers will tell you that certain strings give more power than others, but no one has ever measured this, and I am fairly certain that it is psychological, either something to do with the sound made by the strings or the way the impact duration influences the feel of the racquet.

The speed of the ball for a serve is given by the simple formula, $v = (1 + e_A)V$, where v is the serve speed, V is the speed of the racquet at the impact point and e_A is a number known as the apparent coefficient of restitution (ACOR) typically about 0.4 that describes how well the ball bounces off the strings. For example, if V = 72 mph and e_A = 0.4 then v = 1.4 x 72 = 100 mph. Consequently, 72% of the ball speed is determined by the speed of the racquet. The value of e_A is determined by (a) the relative weight of the ball and the racquet, (b) the energy loss in the ball and the racquet, and (c) the actual impact point on the strings. For an impact in the middle of the strings, e_A can be as high as 0.45 for a heavy racquet, but heavy racquets cannot be swung as fast as light racquets. Even if they could, and if e_A increases from 0.4 to 0.45, the serve speed increases by only 4% to 1.45 x 72 = 104 mph. But if V decreases by 4% when e_A increases to 0.45, there is no change in serve speed at all. Tests have shown that players can serve almost as fast with an old wood racquet as with a modern graphite racquet.

A tennis ball bounces faster off the strings than a baseball since a tennis ball is lighter, but it bounces slower than a superball or a golf ball. A tennis ball will bounce better off a heavy racquet since a heavy racquet is not pushed backwards by the ball as far as a light racquet. For example, if you throw a tennis ball at a brick wall it will bounce back a lot faster than off the strings of a racquet held in the hand. But you can't swing a brick wall as fast as a tennis racquet. When these two factors (swing speed and bounce) are considered together, it turns out that there is very little difference between a light racquet swung fast and a heavy racquet swung a bit slower. The ball speed will be noticeably smaller only if the racquet weight is less than 250 gm, or more than 1000 gm. That's why it is largely a matter of preference. Similarly, string tension is largely a matter of preference. Lower string tension will give you more power, but it's only about 1% more. You can get 1% more power by swinging the racquet 1% faster.

There is an interesting lesson here. Whenever you read an article or whenever someone tells you that a certain racquet or technique gives "more" power or "more" control or "more" of anything else, then make sure you find out exactly how big "more" is. If it is only 1% more, then it's usually not worth worrying about. If "more" is not backed up by a number that has actually been measured or calculated, then the person giving you the information is giving you an opinion rather than a statement of fact. In fact, very few performance measurements of racquets have ever been made, and those made by the manufacturers are closely guarded trade secrets.

Sweetspots

A common selling point for a racquet is that it has a large sweetspot. This needs to be taken with a grain of salt. There are two sweetspots on the strings of a racquet, but they are so close together that it is impossible to distinguish between them with anything other than sensitive vibration equipment. The two spots are near the center of the strings, only a few centimeters apart, so the ball can land on both of them at once.

If a racquet hits a ball, then three things happen to the racquet. It starts to vibrate, it suddenly slows down, and it suddenly starts to rotate about a different axis to the one you used to swing the racquet. This all happens in a split second, and you would have to concentrate hard or take high speed film to see that the racquet actually slows down. It is easier to see this if you hold the racquet at rest in your hand and someone throws a ball at the strings, or if you drop a ball onto the strings. The ball bounces off in one direction, the racquet recoils in the other direction, and the racquet forces your wrist to rotate backwards since the whole racquet rotates about an axis through your wrist. The same things happen when you swing the racquet towards the ball, but you would need a small video camera strapped to your arm to capture the action.

1. The node. When a racquet vibrates, most of the vibration occurs at the tip, the throat and at the butt end of the handle. There is a spot near the middle of the strings that doesn't vibrate at all (Figure 1.1). It is actually not a spot but a curved line extending from the middle of the strings to points at about 2 and 10 o'clock on the frame. The spot that most people talk about is the intersection of this curved line with the long axis of the racquet. It turns out that if a ball hits this spot (or anywhere else along the curved line) then the racquet will not vibrate at all, even at the tip or the butt end. This spot is therefore a sweetspot, technically known as a vibration node. If the ball hits the strings say one inch away from this spot, then the racquet will vibrate a small amount. The further away from the spot, the more the racquet vibrates. If the ball impacts at a point on the node line near the 2 or 10 o'clock position, then the frame will not vibrate, but it will twist in your hand. Consequently, the sweetest point on the cuved node line is the one that intersects the long axis.

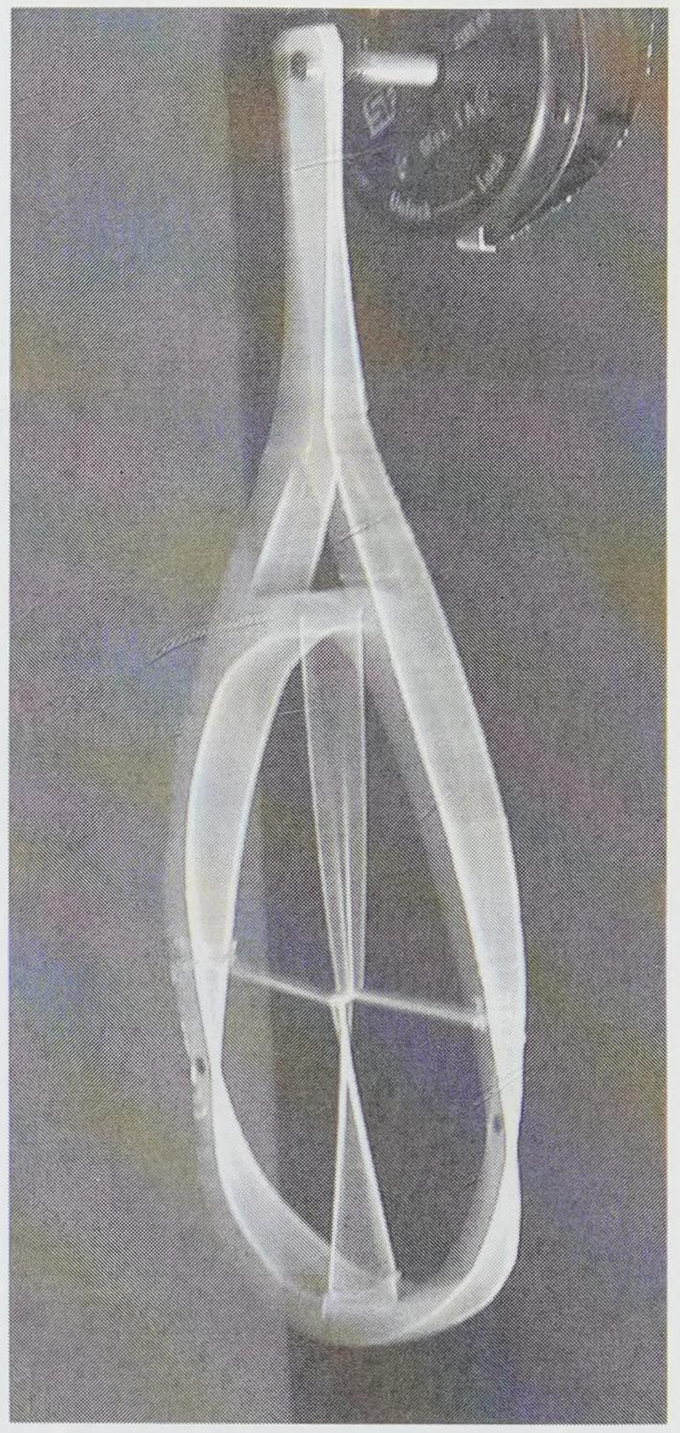


Figure 1.1

Time exposure of a vibrating racquet – actually a stiff cardboard model with one elastic main and one cross string. A real racquet would not vibrate as much as this. The node points in the frame (at 2 and 10 o'clock) and the strings (near but not at the intersection of the two strings) are clearly visible. The handle is connected to a vibration generator (a metal rod connected to a speaker cone) in order to vibrate the handle back and forth 1 mm at 25 Hz. Real racquets are stiffer and vibrate at frequencies from about 120 Hz up to about 200 Hz.

Hence, we can describe different parts of the string bed as feeling perfect, pretty good, or terrible in terms of the amount of vibration transmitted to the handle. A stiff racquet will have a bigger "pretty good" area than a flexible racquet since the overall level of vibrations is reduced in a stiff racquet.

2. The COP. The second sweetspot is a few centimeters closer to the throat and it is known technically as the center of percussion or COP. A ball impacting at the COP will not cause any sudden motion of the handle either forward (towards the net) or backward, but the whole racquet and the wrist will rotate regardless of where the ball impacts on the strings. Sudden motion of the handle towards the net occurs if the ball impacts near the tip of the racquet, in which case the handle jerks forward and tugs on the hand and arm. Sudden motion backward results when the racquet strikes the ball in the throat area. An impact at the COP

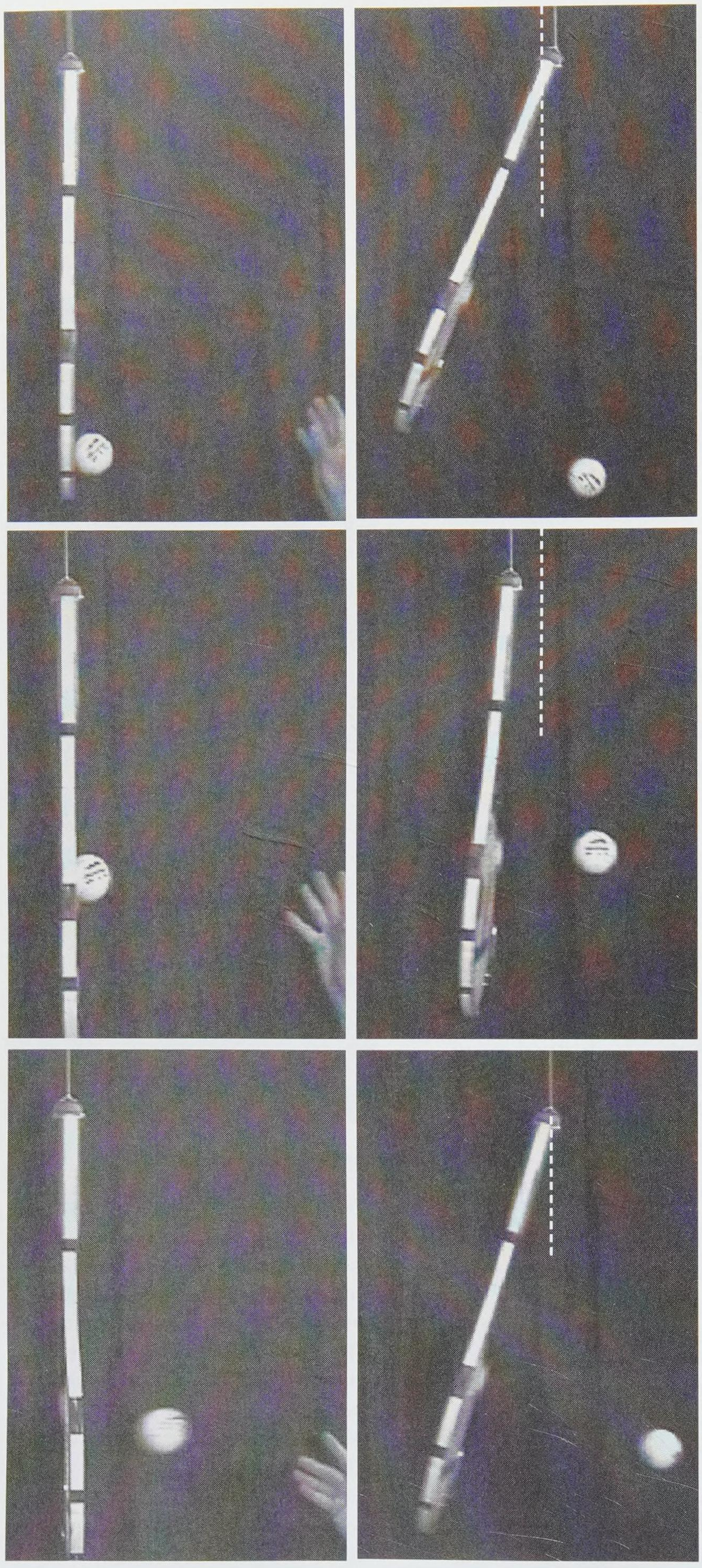


Figure 1.2

Series of photos taken from video film showing the location of the COP. A racquet was suspended by its handle using a long length of string, and a ball was thrown by hand at low speed to impact at either the tip, throat or COP on the strings. Dotted white line is starting position of string and racquet.

Impact at the tip: The handle swings to the right. The ball bounces at low speed and drops vertically downwards. Racquet tip moves rapidly away from the ball at about the same speed as the incident ball. Time between photos = 0.04 s.

Impact at throat: The handle swings slightly to the left. Ball bounces best here. Racquet tip moves slowly away from the ball. Time between photos = 0.04 s.

Impact at COP: The COP for this racquet was closer to the throat than to the middle of the strings. Racquet rotates, but string and butt end of handle don't move. Ball bounces at slightly lower speed than off throat. Time between photos = 0.08 s.

generates a small amount of vibration but no shock, while an impact at the first sweetspot generates a small shock but no vibration. An impact at the tip or throat results in both shock and vibration (Figure 1.2).

3. Maximum power. Many people (especially the manufacturers) like to include a third sweetspot — the point at which the ball comes off the strings with maximum speed. However, I don't like the label "sweet" for this third spot since it doesn't necessarily feel sweet. If you hold a racquet in your hand and drop a ball on the strings, the ball bounces best off the throat and worst off the tip. In other words, the ball comes off the strings with maximum speed when the ball impacts in the throat region. This is an impact region that causes the whole racquet to vibrate and should not be identified as a sweetspot.

The ball comes off the strings with maximum speed in the throat area only when the racquet is initially at rest or moving at relatively low speed. If the racquet is swung at a higher speed, then the point at which the ball comes off the strings with maximum speed shifts towards the center of the strings since the center is moving faster than the throat. At the opposite extreme where the ball is initially at rest (as in a serve), then the ball comes off the strings fastest at a point roughly half way between the center and the tip. The maximum power point can be shifted even closer to the tip by

adding small lead weights to the tip. Even though the racquet travels fastest right at the tip, the maximum power point is not right at the tip. As described above, the speed off the strings is given by $(1 + e_A)V$, where V is the speed of the racquet at the impact point. Since e_A is smallest right at the tip, it turns out that the serve speed from the tip is about 10% lower than the serve speed from the middle of the strings.

Manufacturers like to specify the size of the sweetspot in terms of the area of the string bed that generates about 50% or more of maximum power. This is a larger area than the size of the comfortable hitting zone, and therefore appears more impressive. This is another example where the advertising brochures can be misleading and where the actual figures are usually left off the graphs.

Balance Point and Swingweight

Racquets are classified as either head-heavy, head-light or evenly balanced depending on whether the center of mass is respectively nearer the tip, nearer the handle, or in the middle of the racquet. The center of mass is also called the balance point since a racquet can be balanced on one finger, or on the edge of some other object, if the point of support is directly underneath the center of mass. Most players have no idea where the balance point of their racquet is located, or why it might be important.

The balance point is closely related to swingweight but it is not the same thing. Swingweight depends on the balance point, but it also depends on the actual weight of the racquet. Both the balance point and the swingweight depend on how the weight is distributed along the racquet, but the balance point doesn't depend on how much actual weight there is, whereas the swingweight does. The situation is complicated by the fact that two racquets of the same length and the same weight and having the same balance point can have different values of the swingweight, depending on how the weight is distributed. The important thing to know is that the swingweight determines the resistance felt in the hand and arm when you swing a racquet through the air. It is greater for heavy racquets, it is greater when the racquet is head-heavy and it also increases if the racquet is longer. For that reason, extra long racquets are generally very light so that the swingweight will be about the same as shorter racquets.

You can get a good feel for balance point and swingweight by screwing a nut onto a bolt through the strings at the tip so that it is attached firmly to the strings. A couple of washers will be needed. A small 10 or 20 gram bolt will make a big difference to the position of the balance point and to the swingweight, even though the total weight of the racquet is increased by only 10 or 20 gm. If you tape the same bolt to the handle, there is no change in the total weight, but the balance point will shift about 30 mm closer to the handle, and the swingweight is a lot less than when the bolt is located at the tip. Extra weight at the tip will slow down the head speed and it increases racquet power a small amount, but it doesn't slow down your arm speed by much. Extra weight in the handle has no effect on racquet head speed relative to the wrist and no effect on racquet power, but it will slow down your arm a tiny bit. However, the handle and the hand is already relatively heavy, so an extra 10 gm in the handle is unlikely to achieve much. Some players say it helps, probably because extra weight in the handle increases the moment of inertia about the racquet's center of mass, and hence, it will reduce shock caused by sud-

den rotation of the handle. The same result could be achieved by adding mass at the tip, but that has a much bigger effect on the swingweight about an axis through the wrist. Depending on your style of play, and the amount of wrist action, an extra 10 gm of lead tape at the tip could make a significant difference to the way you hit the ball. It is easy to add lead tape to a racquet to see if it helps. If it doesn't, peel it off and throw it away or try another spot.

Headsize

In theory, the head size of a racquet needs to be only slightly larger than the ball. The head size of a golf club or a baseball bat is not much bigger than the ball, so what is different about tennis? In golf, the ball sits on the ground waiting to be hit. The idea is to hit the ball in the middle of the club head, so the head can't be a lot bigger than the ball, otherwise it might hit the ground. Nevertheless, the head size of a modern golf club has grown to about twice the size of older club heads. In baseball, it is the batter who remains at rest waiting to hit the ball. Even so, the batter often misses the ball completely. That would be embarrasing in tennis, which is why the head is considerably larger than the ball.

Professional or elite players hit the ball in the middle of the strings most of the time, so they don't need to use a racquet with a really large head. If they did, it would be too cumbersome and not very maneuverable at the speeds at which they play. On the other hand, recreational players benefit from a large head since they don't always hit the ball in the middle of the strings, and they don't swing the racquet as fast. A larger head has a larger polar moment of inertia since the mass of the frame is further from the rotation axis. As a result, a hit off-center (towards the top or bottom of the frame in a groundstroke) does not cause the racquet to twist in the hand as much.

In addition, a racquet with a larger head has a larger sweetspot area over which the shot still feels pretty good, partly because the sweetspot is closer to the center of the strings. If the head size is reduced, the sweetspot tends to shift towards the throat. The sweetspot remains at the same distance from the tip, but if the head size is reduced and the length of the racquet is kept the same, then the center of the strings shifts towards the tip. This can be countered by locating extra mass in the tip in racquets with a small head, which also helps to increase racquet power.

Frame Stiffness

Modern graphite racquets are much stiffer than older wood or aluminum racquets. They are also stronger, which means that they can be made lighter and still not break. Part of the reason is that graphite itself is stiffer and stronger than wood or aluminum, but an equally important factor is that graphite racquets are hollow whereas old wood racquets were solid. For any given mass of material, a hollow section is stiffer than a solid section. For example, a solid cylindrical aluminum rod weighing about 300 gm is much easier to bend than a 300 gm aluminum tube of the same length, provided it has a reasonable wall thickness, since it will have a larger overall diameter. That is why the metal legs of a chair are made from hollow tubing rather than solid material.

If you take a wood ruler and bend it, it is much easier to bend across the narrow part than edgewise. In other words, a length of material that is only 1 mm

thick and 20 mm wide is much easier to bend than material that is 20 mm thick and 1 mm wide. For this reason, tennis racquet frames have a hollow cross section that is longer in the bending direction (at right angles to the string plane) and narrower in the other direction. One cannot make the frame very wide and narrow like a ruler since it would buckle as soon as it was strung. The cross-section of the frame is therefore elliptical rather than circular or flat. Widebody racquets are quite a bit longer in the bending direction, which makes them very stiff.

An interesting question is whether a stiff frame will feel stiffer than a more flexible frame. The answer is that stiff racquets reduce the impact force on the hand. The opposite situation prevails with the strings. A stiff string plane acts to increase the impact force on the hand and therefore feels stiffer than a soft string plane. If you hit the ball properly in the middle of the strings, then the racquet will not vibrate at all and it will not slam into your hand, regardless of the frame stiffness. Frame stiffness is a factor only if you mishit the ball or if you deliberately hit the ball at a point nearer the tip or throat of the racquet.

Stiff racquets bend less when they vibrate compared to flexible racquets, but they vibrate faster. The vibration frequency of most racquets is around 120 or 130 Hz, but very stiff racquets vibrate at frequencies up to about 200 Hz. By contrast, old wood racquets vibrate at about 90 Hz. If you hold the handle near your ear and tap the frame, you can hear how stiff the racquet is. It's the same as hitting the strings so that you can hear how tight they are. However, in order to hear the frame vibrate, you need to hold the handle lightly between two fingers at a point about half way along the handle, otherwise your hand will damp the vibrations.

String Type and Diameter

There are four basic string types you can choose from. In increasing order of stiffness they are natural gut, nylon, polyester and kevlar. Each type is available in a range of different diameters from about 17G (1.20 mm) to 15G (1.40 mm). Natural gut is the most expensive and is the choice of many professional players. Thicker strings are more durable since they take longer to break. Most commonly it is one of the central main strings that breaks first since they move along the cross strings every time you hit the ball. This cuts a groove in the main strings and weakens them.

Further details on the general behaviour and properties of particular strings are discussed in the chapters devoted specifically to this topic.

String Tension

The best tension at which to string a racquet is a matter of personal choice. Most people prefer around 60 lb (27 kg), but some people like 70 lb (32 kg), and others like 50 lb (23 kg). Lower tensions give slightly more power and higher tensions give slightly better control. Recent tests show that most people cannot pick a 10 lb difference in tension either in terms of the feel or performance of the racquet. This result is surprising since many players say they can definitely pick a difference, even though when tested blind, many of them can't. However, just as many players admit that they can't pick a difference and they don't have any preferred tension. If you can notice a difference then

maybe you are in the minority who can, in which case you should get your racquet strung at whatever tension you prefer.

Even though a 10 lb difference in string tension seems a lot, the actual effect on racquet feel and performance is unlikely to be large. At 60 lb tension the string plane is moderately stiff and the string plane will stretch about 20 mm in a fast serve. At a tension of 50 lb the string plane is softer so it will stretch further, with the result that the increase in tension during impact is larger, and the maximum tension during impact is about the same as that for strings at 60 lb. The peak force acting on the ball will be about the same and the speed of the ball off the strings will also be about the same.

Kim Clijsters, Belgium

2

Racquet Materials

Graphite, Titanium and Piezoelectric

By Crawford Lindsey

GRAPHITE

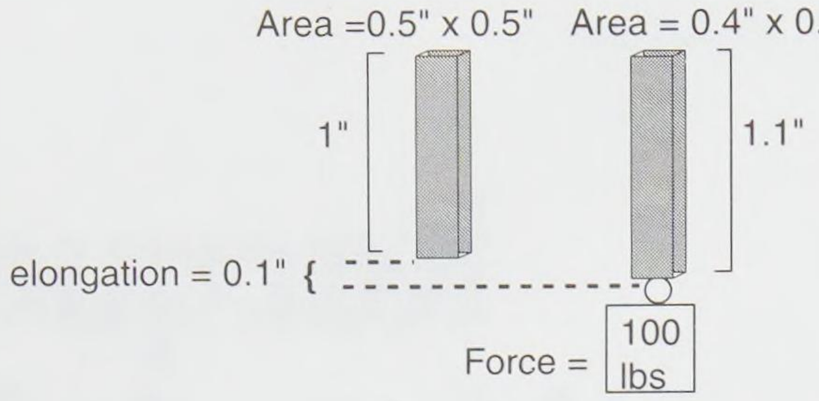
Marketing campaigns brag a lot about the materials with which racquets are made. In days of yore, it was wood. Then came aluminum and fiberglass. But all the recent advancements in racquet technology are due to one thing — graphite composite construction. Without graphite we wouldn't have oversize, superoversize, widebody, or superlight racquets. And graphite itself comes in many flavors. Manufacturers also brag about racquets being made of "high," "ultra-high" or "hyper-high" modulus graphite. What is graphite anyway? What is modulus? How is graphite used in the construction of racquets? Why does higher modulus graphite cost so much? How do you achieve stiffness and strength with these materials?

The basic racquet graphite recipe is to combine carbon fibers with a polymer resin to form a composite material. The composite material is then layered, shaped, and molded into a racquet with various properties includ-

ing weight, strength and stiffness. But all along the way, manufacturers are able to alter different variables to change a material's final contribution to the properties of the product. Each point of difference is eligible to end up as a marketing point of difference somewhere, someday.

Modulus and Strength

Modulus Defined



Stress = force/area = 100 lb/0.16 in.^{2} = 625 lb/in.^{2}
Strain = elongation/original length = 0.1 in./1 in. = 0.1
Modulus = Stress/strain = 625 lb/in.^{2}/0.1 = 6250 lb/in.^{2}

Figure 2.1

Although racquets are not stretched, the material modulus most often quoted is the tensile modulus, not the bending modulus.

Modulus, as it is most commonly used, describes the relationship of stress to strain on a object when a pull of a given force is applied along its length (Figure 2.1). More specifically,

$$\textbf{Modulus = stress/strain} = \frac{\textbf{force/cross section area}}{\textbf{elongation/original length}}.$$

Under normal conditions, stress and strain always change in the same proportion, so the modulus (stress divided by the strain) is always the same, and that is why "modulus" is considered a property of a material. If you double the stress, the strain will also double.

Manufacturers have typically advertised the graphite in their racquets as either intermediate or high modulus graphite. Then, "ultra-high" modulus started to creep into promotional literature. And then, Wilson recently came out with a graphite called Hyper Carbon. This caused quite a stir because their aggressive marketing campaign really forced tennis players to become educated about the different grades of graphite, if they were to make heads or tails of all the competing claims.

The first thing to understand is that "hyper" is not a new category above ultra-high. The materials industry simply uses four modulus categories:

Standard = 30-39 Msi (million pounds per square inch)
Intermediate = 40-49 Msi
High = 50-58 Msi
Ultra-high = > 58 Msi.

"Hyper" is a brand name for a particular ultra-high product from a specific racquet manufacturer. The same is true for Prince's GraphitExtreme or any of the other brand's naming of the material. Hyper Carbon has a modulus of 63.3 million pounds per square inch (Msi) and a breaking strength of 612 thousand pounds per square inch (ksi). So, "Hyper" is the marketing tag, "ultra-high" is the modulus classification. Figure 2.2 shows a plotting of some generic carbon fiber products on today's market that are used in racquets.

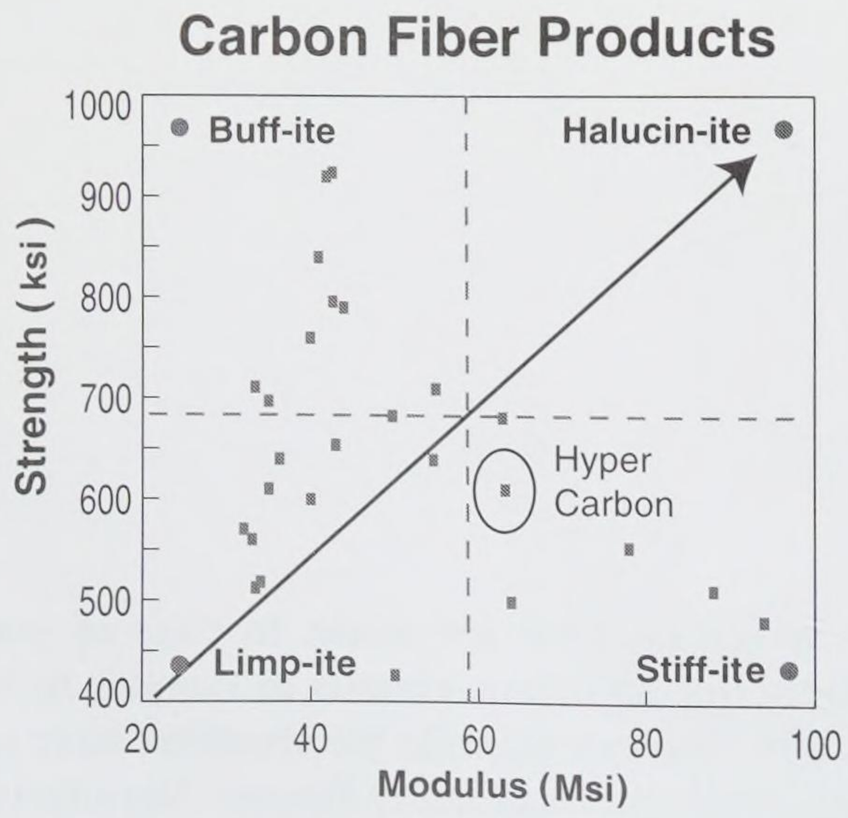


Figure 2.2

Is higher modulus good or bad? It depends on how you use it. As is discussed later in the text, as modulus goes beyond 40 Msi, strength tends to go down. Moving along the diagonal line gives equal increases in strength and stiffness. As you can see, that is difficult to achieve. That is why we have playfully named that quadrant "Halucin-ite."

You will notice that each category is a range. So, if you really wanted to know which racquets have the highest modulus graphite, you would need to know the actual specs of the raw material. Numbering of the modulus, of course, gives consumers a better comparative guideline

than category names do, especially if marketing brand names are then added to each category. But even if you know the exact modulus specs of the material in a racquet, how do you know whether that is good or bad for the desired product and whether it makes a difference to the product's performance? That is more complicated. To answer that, we must first look at how graphite is made and how it is used in racquets.

Making High Modulus Carbon

Phase 1 — The Precursor Material: Carbon fibers are made from one of three carbon-rich starting materials — rayon and polyacrylonitrile (PAN), both polymer processed fibers, and pitch, a residue of coal and oil refining. All three are heated into liquids and extruded through a disk about the size of a nickel with thousands of microscopic holes that shape the liquid into filaments. The filaments are cooled, congealed and bundled into fiber-yarns containing thousands of filaments. The fibers are then rolled onto large bobbins. Pitch-based fiber is the precursor for very high modulus fibers over 75 million pounds per square inch (Msi). Rayon and PAN are used in clothing or further processed into carbon fiber. PAN is the precursor for most racquet carbon.

Phase 2 — The Carbon Fiber: The PAN fibers (literally, "acrylic sweater yarn") are pulled through two ovens of varying high temperature and oxygen-free atmosphere to burn off all non-carbon elements (carbonization). The fibers are tensioned throughout the process to align the carbon molecules in hexagonal crystal sheets along the fiber. At this stage, modulus is affected by temperature, time at temperature, and the amount of orientation achieved in pulling. Temperature exposure varies from 1-3 hours. Higher temperatures produce purer carbon fiber. The grades of carbon are typically 90-99 percent carbon. Though the terms "carbon" and "graphite" have been used interchangeably over the years, graphite is defined as greater than 95% pure carbon or as having a modulus greater than 50 Msi. The higher the carbon content, the higher the modulus.

Higher moduli are created by higher temperatures. Temperatures for good carbon fiber might be 1,200-1,400 degrees centigrade and for good graphite between 2,000 and 3,000 degrees centigrade. The energy costs and quality control for attaining higher modulus are considerable. Also, higher modulus fibers are more brittle and difficult to handle, so they have to be processed slower, and that also costs more.

Emerging from the ovens, the fibers go through a cleaning bath and are coated with a one-percent epoxy solution known as a "sizing." This facilitates handling. Without sizing, the fibers are like wispy microscopic hairs floating in the air. The fibers are gathered in either 1K, 3K, 6K or 12K bundles (tows) composing a carbon fiber yarn that is then rolled onto spools. Racquet manufacturers typically

What Is Resin?

Resin binds the fibers together. In fact, a composite is often referred to as "fiber reinforced matrix," which implies the matrix has top billing in this relationship. Behind every fiber there's a matrix. Resins are polymers (long chains of molecules) and are of two kinds — thermoplastic and thermosetting. Resins, too, have their own strength and elasticity properties.

Thermoplastic resins soften with temperature. These resins are elastic because the individual molecules are not bonded to their neighbors and can slip past each other when being stretched. Consequently, thermoplastic resins are good candidates for string.

Thermosetting resins are used in racquets. These are two types — polyester and epoxy — of which the epoxy predominates in racquets. Thermosets harden, or are cured, under high temperature. The distinguishing feature of thermosets is that molecules link to each other, forming a cross-linked supporting structure. This results in additional strength and stiffness. Because the molecules in the matrix are not aligned in any one direction, the strength and stiffness properties are the same in all directions — they are isotropic. Fibers, on the other hand, are anisotropic, meaning the properties change depending on the direction of the stress to the long axis of the fiber. In many stress situations, the matrix does more than the fiber. But it is not strong enough to do it all, so fibers are placed at many angles to handle all possible load conditions.

use 12K bundles because they cost less and are easier to handle in the next step — prepregging. Fiber strength and modulus specs refer to these fibers before they are combined into a composite.This is important to note, because, as we will see, the properties of the composite will depend on both the strength and modulus of the composing graphite and resin, and also on the relative proportion of each material in the composite.

Resin: The Weakest Link

If the matrix is the weakest link, why isn't that the strength of the composite? Epoxy resin has a strength index up to about 20,000 lb/in^2 (20 ksi) and a modulus of 600,000 lb/in^2 (0.6 Msi). In the case of Hyper Carbon, the strength is 612 ksi and modulus is 63.3 Msi. So why doesn't the composite break when the stress exceeds that of the matrix? The answer is that the resin doesn't share the load equally with the fibers because it is more elastic. Elasticity is a measure of how far you can stretch a material before the bonds between atoms and molecules build up enough force to balance and counteract the stress — as when tensioning a string at 50 pounds and it stretches until it works up 50 pounds of resistance.

The relationship between the matrix and the fiber is much like gluing a Kevlar and nylon string together. If you tension the combination in a stringing machine, each fiber will elongate the same amount (because they are glued together), but that amount of stretch will not build up very much stress in the nylon. Instead, most of the stress will be transferred to the Kevlar, which will bear most of the load. If the Kevlar is 10 times as stiff as the nylon, it will bear 10 times the load. That's exactly how fiber and matrix work together.

Phase 3 — Prepreg: This is where a "composite" is created. Hundreds, or even thousands, of spools of bundled fiber are placed on one end of the prepreg line, and the ends are pulled over a comb to spread and align all the fibers into "sheets" 12" to 48" wide. The sheet is pulled through an epoxy resin bath. The excess resin is scraped off with a squeegee device and the sheet is rolled onto a drum with a removable paper backing. Because all the graphite fibers are arranged parallel to each other, this is known as unidirectional graphite prepreg. The term "prepreg" refers to the fibers being impregnated with resin before the actual molding process. The combination of the resin and the fiber is referred to as a composite. The resin also contains a curing or hardening agent (catalyst) and a coupling agent that together causes the resin molecules to link together in a supporting cross-linked network and to adhere to the carbon fiber. This occurs under heat and pressure in the racquet molding process.

The epoxy resin is also known as the matrix. Neither the fiber nor the matrix is much good without the other. The loose fibers by themselves can't transmit loads to each other, and the matrix is not nearly as stiff or strong as the fibers. So the matrix serves three purposes: to bind fibers together, transmit loads to each fiber, and to protect the fibers. The strength and stiffness of the composite falls somewhere in-between those of the fiber and the matrix. In other words, the strength and modulus of the composite is less than that of the fiber specs by themselves.

The strength and modulus properties of the composite depend on the volume ratio of fiber to matrix. In fact, the modulus of the composite as a whole equals the modulus of the fiber times it's volume percentage in the composite plus the same for the matrix. The fiber/matrix ratio typically varies between 50/50 to 60/40. But there is a theoretical limit to how high the ratio can be and still allow the epoxy resin to serve its purpose. That limit is around 90%, but it's not approachable in any practical manufacturing application. Middle 70s might be doable in some high-end space applications, but the generally accepted practical limit is around 60/40 fiber-to-matrix. The theoretical limit is determined by the geometry of the fibers and by how much space is left over when they are packed as close together as possible. If the packing is too tight, the heated resin can't flow into the spaces between fibers during molding. You end up with weakness-causing voids as a result.

The sheets of prepreg are then either stored or first cut into shapes for later lay-up, catalogued by shape or product, and then stored. Prepreg is usually

Thin Prepreg

Strength and stiffness can be increased by using thin prepregs — also known as low FAW (fiber-average-weight) prepregs. This involves an additional step in prepreg manufacture. Before the fibers are resin-coated and rolled onto the paperbacked drums, they go through a series of rollers that spread the fibers as much as possible. The goal is to minimize the number of fibers per inch and yet not have any space between fibers. In other words, as little overlapping as possible. Thinner prepregs have lower weight per square meter of material. The thinnest FAW available is between 60-70 grams per square meter. Just ten years ago, the lightest prepreg was 115 grams per square meter.

Thin prepreg offers a couple advantages. Thinner layers allow a manufacturer to use more layers while maintaining the same weight and thickness. More layers allow more combinations of fiber angles, allowing greater customization and optimization of strength and stiffness properties. This facilitates using less material overall, resulting in thinner frame walls and a lower weight racquet.

Low FAW prepreg costs more because it requires more machine time and equipment. Many technologies have been developed, but there is a cost associated with each one. If you want a lightweight racquet, you start with more expensive material, processing takes more time, and quality standards are higher. All these things build in cost.

refrigerated to stabilize any room temperature curing (hardening), though it can last several months without refrigeration. One prepreg company even claims one year.

Making a Racquet

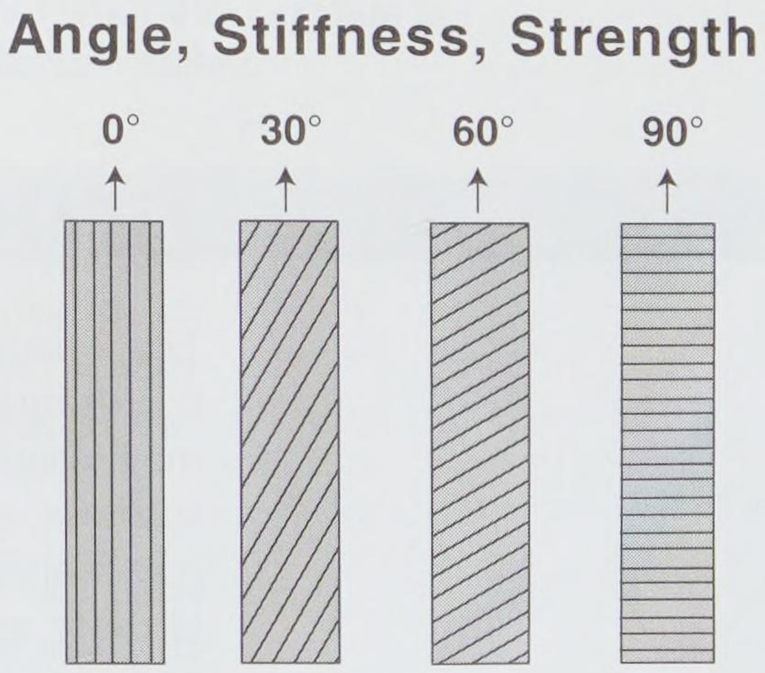
Now we want to make a racquet. This is where it becomes fun. There are a number of considerations:

1. Fibers are very stiff along the length-wise axis, but not so at any fiber angle off that axis. If the load is applied just 15 degrees off axis, the modulus may decrease to 60% of maximum. At 45 degrees it could be as low as 20% and so on down to 0 at 90 degrees (Figure 2.3).

2. As modulus goes up, strength goes down (see Figure 2.2). Why? It has to do with the number of imperfections on the fiber surface per unit of length. Imperfections are both uncovered and created during the heating process. It has been proven that microscopically short fiber lengths of different moduli have approximately the same strength. However, a one-inch piece of high modulus fiber will have more imperfections than one inch of a lower modulus, high strength fiber.

3. As modulus goes up, fiber becomes more brittle. When the atoms and molecules of a material are so completely oriented that there is no room left for movement, there is no elongation or deformation prior to breaking. There is just instantaneous failure — especially if impacted at an angle off axis to the strongest direction of the fiber. Brittleness is an impact phenomenon.

4. As modulus goes up, the fiber's density goes up because the lighter elements have all been burned off. Higher mod-



Stiffness (modulus) drops off very steeply when the load (direction of arrows) is applied just a little off the fiber direction. As the angle increases, the pull is less on the fibers and more on the matrix.

Figure 2.3

ulus material is heavier per foot or volume.

5. Higher modulus costs more. There are higher energy, quality control, and handling costs.

6. The lighter the racquet, the more strength and stiffness is needed to make up for less material. But stiffness and strength move in opposite directions.

These factors limit how much of any material you can use and what you can use it for. Every racquet must combine a blend of different modulus material to balance strength, durability, stiffness, and economics. This is accomplished in a racquet's lay-up.

Racquet Lay-up

Most racquets are made of multiple layers (laminations or plies) of unidirectional prepreg. One layer won't do it. So no matter what the fiber's or composites' specs, racquet stiffness and strength depend on prepreg. The art of lay-up lies in determining the angles at which the different layers of unidirectional prepreg — each perhaps with different modulus and strength properties — will be oriented to each other.

Five to ten layers are common for tennis racquets, so several angle combinations are possible. If we consider the length of the racquet as the reference axis, a zero degree angle is parallel to that. Zero degree high modulus fiber makes the racquet stiff to bending. Ninety degree prepreg is used to stiffen the hoop in the string pull-through direction, and every angle in-between will increase torsional stiffness. Manufacturers typically think in terms of 7.5- or 15-degree increments. The trick is to stack all the layers so the strength and stiffness are optimized to match the racquet's target player (see thin prepreg sidebar).

Modulus classifications are not good or bad in themselves. High modulus is not better than intermediate or low modulus carbon fiber. It depends on how they are used and to what end purpose. The key is to use the material in a design that will optimize the material's properties. Without the design, the material's properties may be wasted. Without the material, the design may be impossible.

TITANIUM

Titanium is the ninth most abundant element in the earth's surface and is the fourth most used metal in industrial applications — behind aluminum, iron and magnesium. It is the strongest and lightest metal. It has very good torsional stiffness, and like most metals, is isotropic, meaning it is equally strong in all directions. Graphite, on the other hand, is anisotropic. Its strength and stiffness is more oriented in the direction of the fibers and not, for example, at right angles to them. However, titanium is about two and one-half times heavier and three times less stiff than high modulus graphite (see Table 2.1). As the stiffness/weight column shows, it theoretically takes 3.15 times (5.235/1.66) the titanium to equal the graphite's stiffness.

The question then arises, why are racquets containing a small amount of titanium so light? Manufacturers tell us that it is how titanium and graphite are combined that makes the difference. Though its specific properties don't equal

graphite in weight and stiffness, their use together allows their properties to complement each other — sort of the whole is greater than the sum of the parts argument. The theory is that in a graphite racquet, separate layers of graphite fibers must be oriented in different directions to gain strength. But by adding isotropic titanium fibers to a single layer of unidirectional graphite, less right-angle graphite has to be added. It then becomes a complicated mathematical/mechanical balancing act between titanium's isotropic strength benefits and torsional stability, but heftier weight, and the weight savings of eliminating some of the high modulus graphite.

Table 2.1

Mechanical Properties Comparison

Material	Tensile Strength (lb/in^2)	Density (lb/in^3)	Modulus (lb/in^2)	Modulus/Wt (in^2)
Aluminium	48,000	0.09548	10,100,000	1.057
Titanium	130,000	0.16257	27,000,000	1.660
Kevlar	203,052	0.052745	11,020,000	2.089
Graphite	217,000	0.057803	26,251,830	4.541
HM Graphite	638,166	0.06647	34,800,000	5.235

Source: Mark's Standard Handbook for Mechanical Engineers and Composite Materials Handbook.

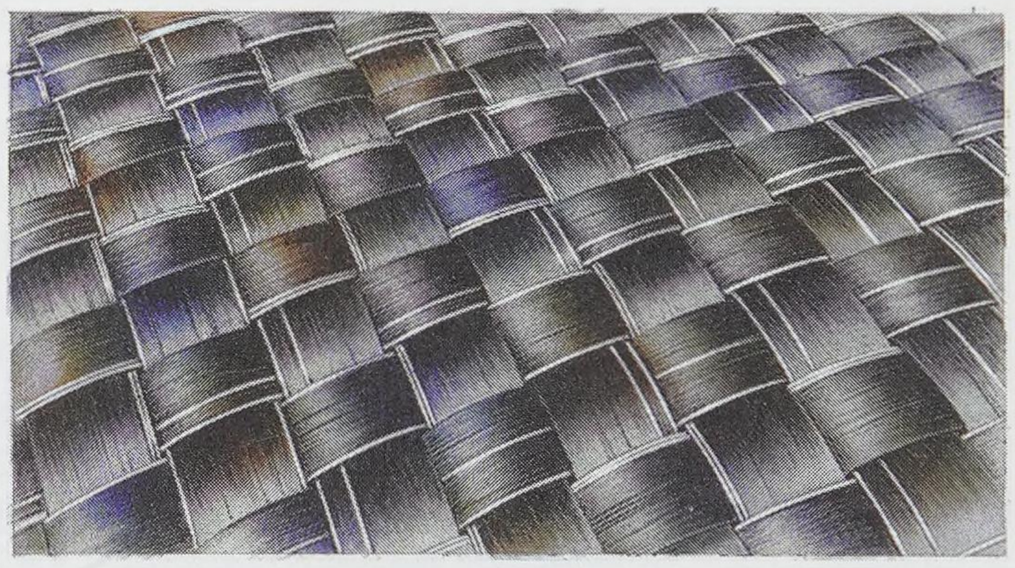


Figure 2.4

Titanium fibers interwoven with graphite in Head's "graphite/titanium matrix."

Furthermore, the titanium is only used in limited strategic strength areas like the throat and shoulder area where there are increased torsional and bending stresses. It is also typically used in small percentages. Does it work? It does in golf.

Titanium is used differently in golf and tennis, however. In golf heads, metals are used as the predominant structural element. Titanium, rather than graphite, is used here for three main reasons. It is much more abrasion and impact resistant than graphite; it is more resilient upon impact, thus adding to ball velocity; and its isotropic properties are ideal for bearing the golf ball's impact loads. Arranging enough light-weight graphite fibers in the directions necessary to resist golf ball impact might make the graphite head almost as heavy as the titanium head, without the other advantages of titanium. So sometimes light weight can be heavy (golf) and sometimes heavy can be light weight (tennis).

For these reasons, titanium is used as a predominant material in golf but plays more of a supporting role in tennis. Racquets became superlight just as titanium was being marketed. Titanium is not the reason racquets are light. So, in spite of the marketing hype, titanium plays only a supporting role in racquet construction. Graphite is the workhorse material that makes everything possible.

But manufacturers are constantly pushing the materials envelope. Head was the first racquet company to put "smart" materials into their racquets.

PIEZOELECTRIC MATERIALS

Head Sports recently developed a science-fiction-come-true racquet technology — a computerized self-powered auto-controlled piezoelectric dampening and stiffening system. What the heck is that?

In a nutshell, piezoelectric material generates an electric charge when a force deforms it. The charge produced is proportional to the force — more force, more charge. The opposite is also true. Piezo material is deformed when it is subjected to an electrical charge. In this case, deformation is proportional to the amount of charge. They are reverse processes to each other. One will exactly undo the other.

Mechanical energy is turned into electrical energy and electrical energy is turned into mechanical. This all happens by the miracle of positive and negative charged particles pushing each other around as materials are pressed, bent and electrified. If you get the right pushing and pulling, you get an electric current as your reward (Figure 2.5).

This creates two benefits. First, energy turned into electrical energy does not go into vibrating the racquet. That's the damping effect. Second, there's a self-induced restorative force that acts to stiffen the racquet the more it is bent. Racquets with piezo material are intelligent to the extent that they react variably and adaptively to their environment. They sense the force and dissipate it in electrical energy in proportion to the size of the force. At the same time, they produce some internal stiffening that is also proportional to the force. But as smart as these racquets are, they are still passive.

Head also developed a racquet that is an active participant in its universe. It possesses a higher intelligence in the form of an electronic brain that is resident in the handle and connected to its motor system by an electronic nervous system. The brain takes the electric signal, measures it, times it, processes it, and redirects it back to whence it came to do work. The return signal tells the piezo material ("intellifibers") how much to contract to counter the impact force and when to do it so as to create a canceling vibration to the impact vibration.

So we have stiffness and damping as calculated by the brain. Upon impact, the piezo material acts as a sensor ("generator" in piezo lingo) device, sends the data to a control device, and then actively performs a motor ("actuator") function. In this case, that would be the "active counter-force" that acts within one millisecond to limit the amplitude of the impact-induced bend. Then this "counter-bend," if you will, creates vibrations that will help cancel the residual impact vibrations because it is timed to be "out of phase" with the latter. This residual cancellation is just that – residual – because mechanical vibrations travel through the racquet at the speed of sound, whereas the electrical signals travel at the speed of light.

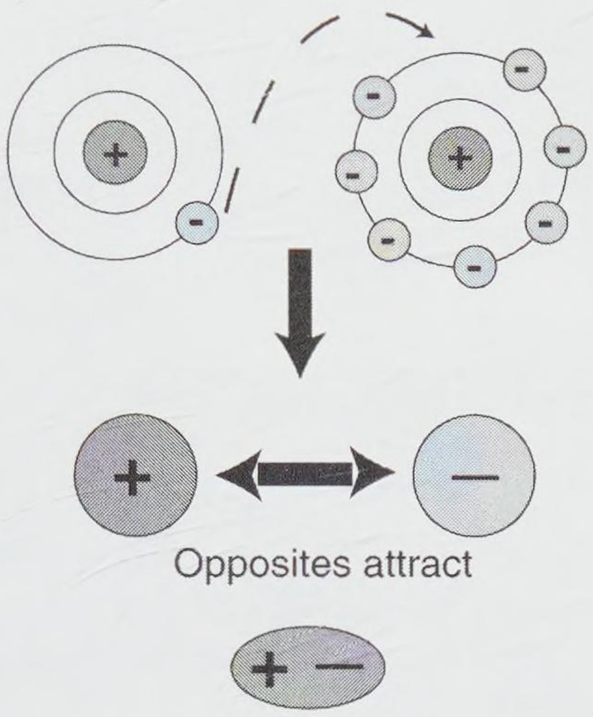
Piezo materials are one of a class of materials called "smart materials." Besides piezoelectric materials, there are shape memory alloys, magnetostrictive materials and polyelectrolytic gels. Basically these all match a change in shape with a change in one of the following variables: electric charge, magnetic field, or temperature. And it goes both ways—a change in shape (like bending) due to a force will result in an electric charge, magnetic field, or change in temperature. And within certain ranges of behavior, these changes are very precise,

consistent, proportional, and can be calculated in nanometers. (So exactly, in fact, that piezo applications are copiously abundant in such high tech areas as positioning scanner heads, precision tooling, and extremely accurate measuring devices.) As such, a change in shape can be used to measure a change in charge or temperature, just as a change in charge can be used to measure a change in shape. Or, thinking less passively, a change in shape can be used to produce an electric charge, which in turn can produce a change in shape. In other words, you can use these changes to do things, as the Intelligence racquets do.

Figure 2.5

How piezoelectricity and computer chips work in a racquet

1. Make ceramic molecules with charged ends



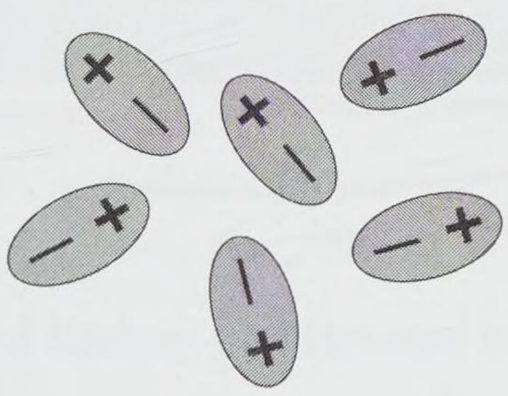
Atoms are electrically neutral. They have an equal number of positive protons in the nucleus and negative electrons flying about the nucleus.

Atoms combine into ceramics (the material used in the Intelligence Series is lead zirconium titantate [PZT]) by giving and receiving electrons. Atoms try to get 8 electrons in their outer shell by combining with other atoms that have the number needed to complete the shell.

When this happens, one atom becomes negatively charged and the other positive. They then electrically attract each other (opposites attract) and form a bond uniting them into a molecule of the new material.

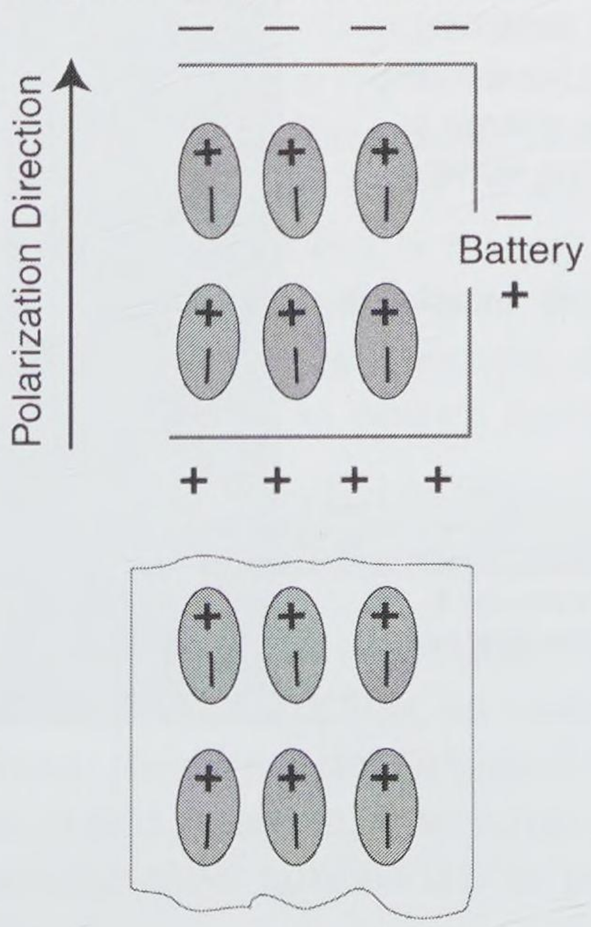
One end of the combination is negative and the other positive. This is called a **dipole.**

2. Mix up molecules into ceramic material



Billions and billions of molecules of the material come together to form the material. (Actually a powder is mixed, milled, shaped and heated.) At this stage, the dipoles are facing every which way, and the material as a whole is neutral as dipoles arrange themselves so they are all happy in electrical equilibrium.

3. Polarize the material--make it piezoelectric



The key to the unique piezoelectric effect is the ability to orient and freeze the dipoles in an aligned manner.

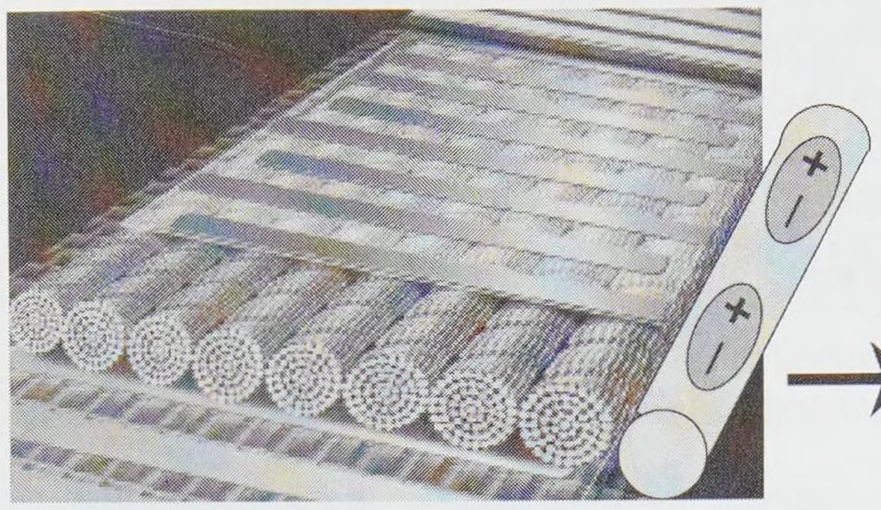
To make the material piezoelectric, it is polarized by exposing it to an electric field. The charged ends of each dipole align themselves toward the opposite charge electrodes that are exerting the electric field. The field and heat are then removed and the dipoles "freeze" permanently into alignment.

The result is an electrically neutral material on the inside (all the charges balance each other out), but the top surface has a positive charge and the bottom surface a negative charge. These charged surfaces attract opposite charges from the atmosphere around them to balance the charge so the surface is neutral also, but the stage is set for intellifibers to do their thing.

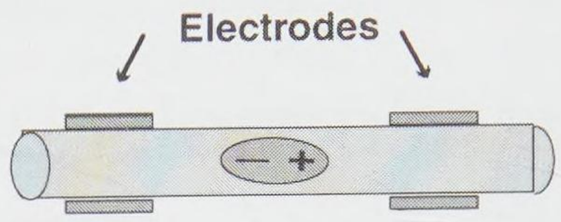
Figure 2.5 continued on next page

Figure 2.5 (continued)

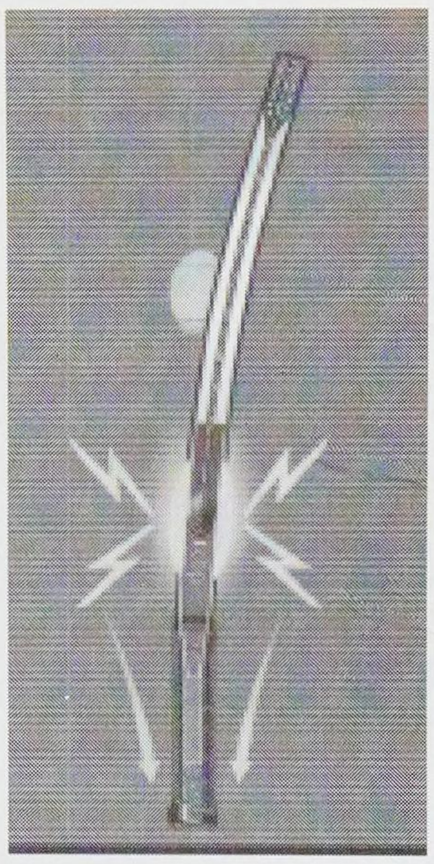
4. Create fibers and attach electrodes



The dipoles within each fiber are aligned from top to bottom. Before impact, neither the electrodes or the fibers have a charge. Everything is neutral.



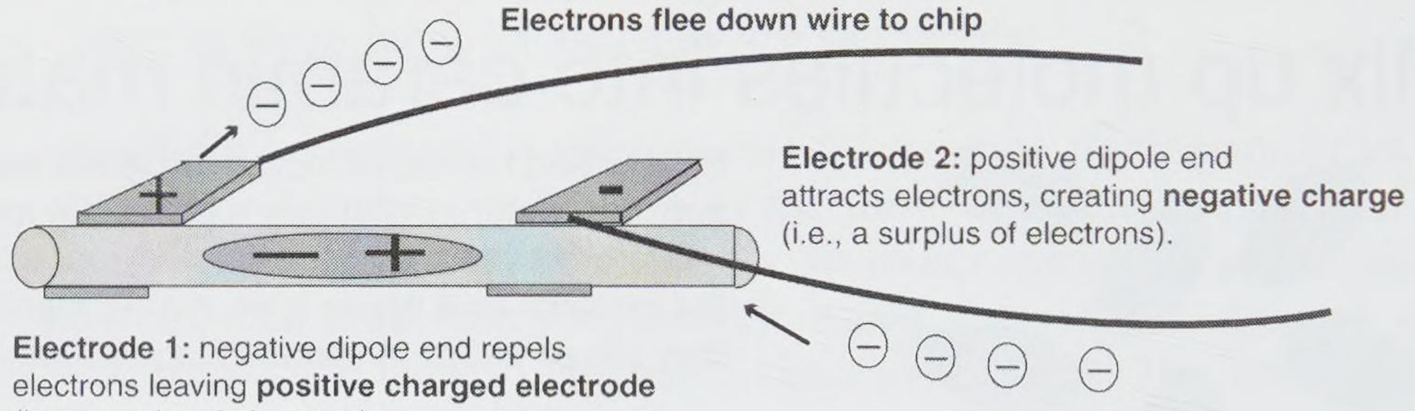
Electrodes are metal conductors with "loose" electrons that jump from atom to atom away from an electric field of negative charge... or towards a field of postive charge. As they do so, they repel other electrons in their path creating a domino effect called electric current.



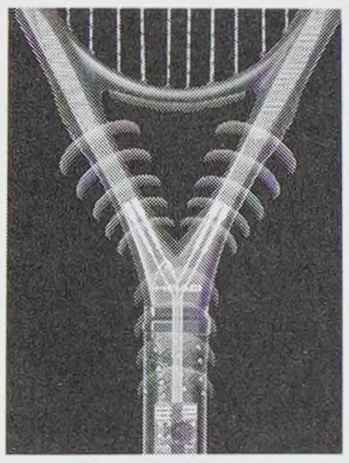
5. Stretch fibers to create charge

Racquet bends at impact, stretching intellifibers

When piezo material is stretched or compressed, the polarized dipoles are also stretched (in this case) and displaced from their neutral positions and the dipoles become even more polarized because the charges separate. This chases away like charges on the surface electrodes, and attracts opposites, thus creating a current.

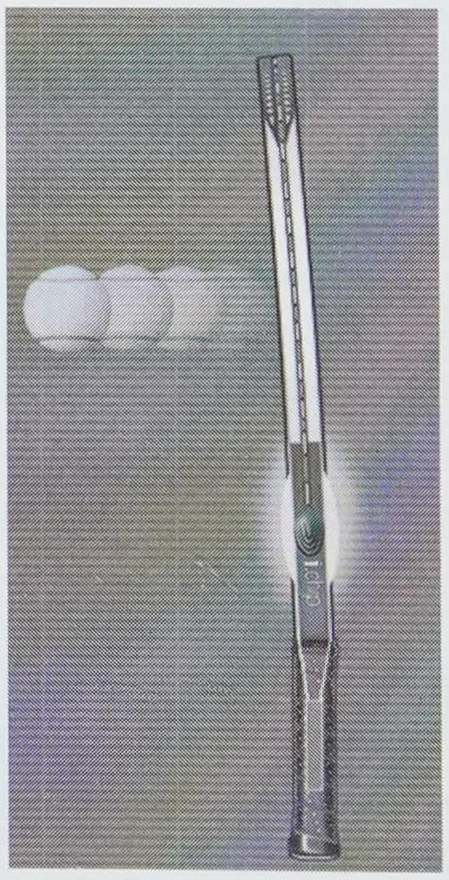


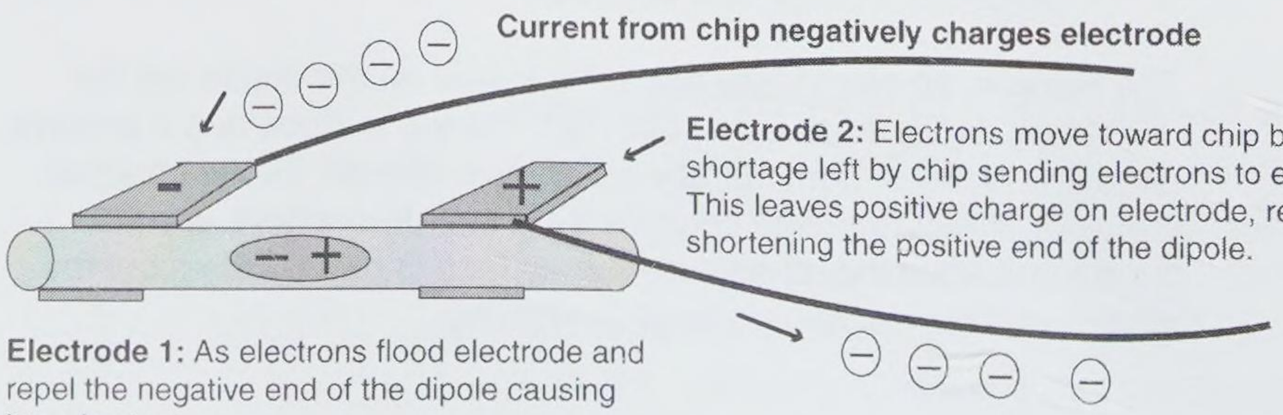
6. Reverse charge to reverse stretch



The current produced in step 5 travels to the chip, turns it on, and then is sent back to the fiber electrodes at a timed interval that will contract the fibers in the opposite frequency at which they were stretched, thus minimizing vibration by cancellation. The current from the chip sends electrons into the positive electrode, reversing its charge to repel the negative end of the dipole. Meanwhile, electrons leave electrode 2 toward the now positive chip.

Repeat the entire process back to step 5, and so on until damping is complete.





Bjorn Borg, Sweden

3

Weight, Balance & Swingweight

Distribution of Weight and a Player's Swing

By Crawford Lindsey and Howard Brody

Why do I feel one kind of weight when I pick up a racquet, and another kind when I swing it? It seems that weight takes on different guises. First, there is weight, which is a force – the force of gravity on an object. Then there is mass – an object's resistance to changes in its motion – its inertia. Mass and weight are different physical quantities, but we will neglect that (see Note 1). The racquet weighs ten and a half ounces and has a mass of 300 grams. That's easy. But why does it feel like it weighs more depending on where I pick it up — at the handle, head, or somewhere in the middle? This is where distribution of mass and balance point come into play. And last, a racquet may have the same weight and balance point as another, yet still be more difficult to swing. This is where distribution of mass combines with racquet acceleration in swingweight. This chapter will explain the concepts of mass, weight, balance, and swingweight. Other chapters will deal specifically with their role in power, maneuverability, feel, shock, and vibration.

Force, Mass and Inertia

Nothing changes the motion of an object without a force — tennis balls included. Without force, all objects tend to maintain their speed, direction, and rotation. All objects have mass or inertia, which is the resistance to change in motion. Force, therefore, doesn't make the universe go (it goes by itself), but it does change how it goes. And to change an object's motion, you must change its speed or direction. In other words, you must accelerate it. Thus, we have the most famous law in physics and tennis for making things happen: F=ma (force equals mass

times acceleration). This says that you need force applied to a mass to get the object moving to the desired velocity. So, mass is the primary restraint in whatever you want to do. And the effect of mass will depend on its amount and also its location.

The implications for racquet sports are obvious. You have to get your racquet in motion, but once you do, it takes little force to keep it going, just enough to overcome the force of air resistance. But then you have to stop it, change its direction, and start all over — all of which require additional force. Sometimes inertia helps you, sometimes it hurts you. Your racquet resists everything you want to do with it, but once it is doing it, it doesn't want to stop. So every atom in your racquet is a little mass of resistance. Sounds somewhat daunting, doesn't it? But before we try to swing these rascals, let's see what mass has to do with the feel of the racquet.

Weight

You know very well what the weight of a racquet means. You put your racquet on a scale and read how much it weighs — 10.5 ounces, 300 grams, etc. If you hold the racquet at the balance point, the weight is the amount of force your hand must apply to keep the racquet at rest and the force your hand is applying is equal and opposite to the force of gravity on the racquet. Most racquets feel remarkably light when held at the balance point instead of the grip, but you don't hold the racquet at the balance point when you play tennis. For the first half of the twentieth century, most racquets were made of wood and weighed 14 ounces (400 grams or 0.4 kg), or more. With the introduction of graphite composite frames, engineers were able to produce racquets with weights as low as 7 or 8 ounces (200 – 230 grams). This is a great technological feat, but may not be the best racquet weight to play tennis with. The Rules of Tennis do not specify a maximum or minimum racquet weight.

Balance Point

You probably know what the balance point of a tennis racquet means, but do you know its significance? The balance point is the place near the center of the racquet where you can balance the frame on one finger (that's why it is called the balance point). Sometimes the balance point is called the center of mass, or the center of gravity. You normally measure the balance point as a distance from the butt end of the racquet. Technically, the distance from the butt end to the balance point is the sum of each little bit of mass in the frame multiplied by that bit's distance from the butt end of the racquet, all divided by the total mass of the racquet. If that resulting distance from the butt is more than half the racquet's length, you say the racquet is head-heavy. If the balance point is closer to the butt end than the geometric center of the frame, you say the racquet is head-light. Often the distance from the balance point to the geometric center of the racquet is specified as 1/2-inch head-light or 1-inch head-heavy, etc.).

Though balance seems straightforward, it is sometimes confusing. For example, since racquets now come in different lengths, two evenly balanced racquets (balance point halfway between the tip and the butt) may have their balance points at different distances from the butt end. What the racquet manufacturers and racquet testers should tell you is the actual distance of the balance point from the butt end of the racquet.

There's another common misconception. Many people think balance point means there is an equal amount of weight on each side of that point. This can be true, but it is not necessarily so. For example, if you balance a meter stick (a ruler 100 centimers long – Figure 3.1a), it balances at 50 centimeters (Figure 3.1b). Now, if you place a 10-gram weight at each end of the meter stick, it still balances (Figure 3.1c). But if you place either weight a different distance from its nearest end than the other weight, the meter stick no longer balances at the same point (Figure 3.1d). To make it balance again at that point, you have to add more weight to the "high" side of the stick. The amount you need to add depends on where you add it. Thus, in this case, to balance the meter stick at 50 centimeters, you need to have a different amount of weight on each side of the balance point (Figure 3.1e). If you have 10 grams at one end (50 centimeters from the balance point) and 10 grams 20 centimeters from the other end (30 centimeters from the desired balance point), then to keep the balance at 50 centimeters, you must make the weight times the distance from the original balance point the same on each side. So if one side is 10 grams times 50 centimeters equals 500 gram centimeters, the other side must be a total of 16.7 grams times 30 centimeters (in other words, we must add 6.7 grams to the existing 10 located at 30 centimeters from the desired balance). Or, we could have added any amount of weight anywhere along that side of the meter stick such that each weight times its distance from the balance together added to 500 (Figure 3.1f). So adding 10 more grams 20 centimeters from the balance would have also worked. That would result in 20 grams on one side of the balance point and 10 grams on the other. Yet, the balance point is the same (50 centimeters) as when we had 10 grams on each end of the meter stick.

A weight feels heavier or lighter depending on its distance from its support point. To feel this, pick up the meter stick (or a yardstick) from one end and place a weight at the middle at 50 centimeters. Now move the weight to the end (100 centimeters away from your hand). It should take about twice as much effort to hold up the end of the stick in this case. The same weight seems heavier. In fact, the weight being twice as far feels twice as heavy. So the balance point is determined by the combination of the mass and location of each little piece of the racquet. The actual formula for the balance point of a racquet is to add each infinitely small racquet piece times its distance from the end of the racquet and divide by the total mass of the racquet:

$$(m_1d_1 + m_2d_2 + \ldots m_nd_n) \div \text{total mass} = \text{balance point} \qquad (3.1)$$

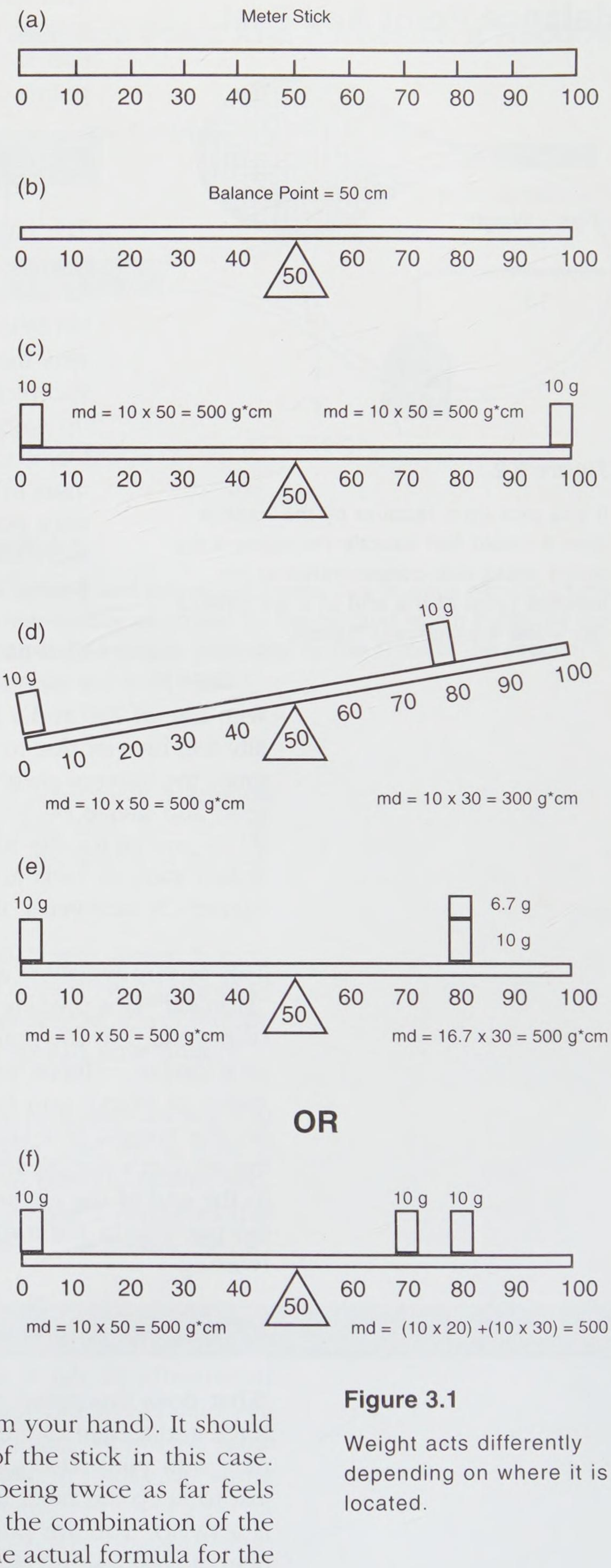


Figure 3.1

Weight acts differently depending on where it is located.

Balance Point And Feel

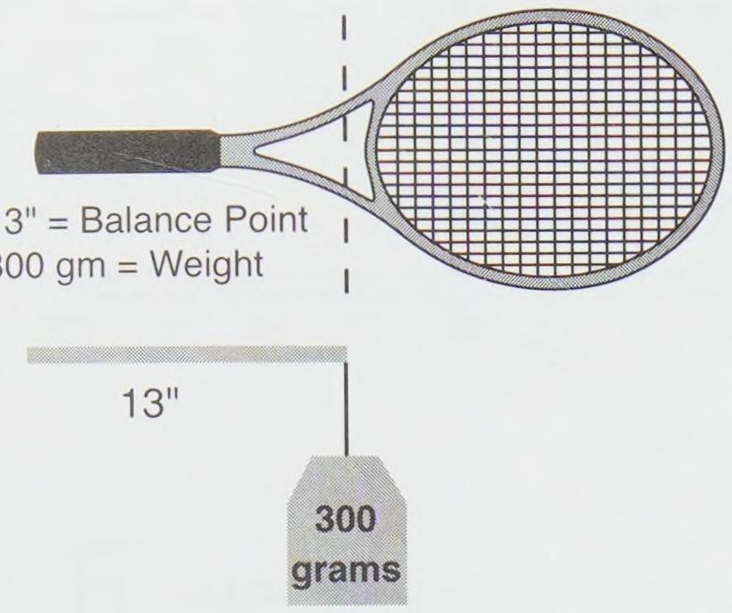


Figure 3.2

If you pick up a racquet by the handle, then it would feel exactly the same if the whole mass was concentrated at the balance point at the end of a weightless rod witha a weightless handle

This formula is not balancing just mass or weight — it is balancing weight multiplied by distance. That, of course, is why two racquets can have the same weight but different balance points or different weights and the same balance point.

Pickup Weight

The balance point is important for another somewhat magical reason. The average of all these individual weights and distances come together in this spot such that the entire racquet behaves as if all its mass were concentrated right there. That is why the balance point is often called the center of mass. When you pick up the racquet from the handle (hence the name "pick up weight") and hold it horizontal to the floor, the weight you feel is exactly what it would feel like to hang the racquet's total mass at the balance point. So, for a 300 gram racquet with balance point of 13 inches, you are feeling the same thing as if you were holding onto a weightless rod with 300 grams hanging off its end at 13 inches (Figure 3.2).

This makes the concept experiential, but how do you compare a racquet of 300 grams with a balance at 13 inches (33.02 cm) with one of 290 and a balance at 13.5 inches (34.29 cm)? Which one will actually feel heavier due to a larger pickup weight? Easy — just multiply the weight times the balance point (convert to centimeters first to measure in the same system) and divide by 100 to make the number manageable. In this case, we get 99.06 gm·cm for the 300 gram racquet vs. 99.44 for the lighter racquet. So the lighter racquet feels just a bit heavier (or more head-heavy) even though it's lighter. Or vice versa, the heavier racquet feels slightly lighter.

This pickup weight is also known as the "first moment" or the "static moment." "Moment" is a physics term related to length of the lever arm from the force (the weight) to the point of application (your hand). That is also the definition of a torque — force times distance (length of the lever arm). A torque tries to make an object spin faster or slower, depending on which way it is applied. So, the weight of a racquet torques or twists your hand by the amount of all the racquet's weight acting as if it were at the center of mass times the distance to the end of the racquet. Your hand must apply an equal but opposite torque (so the sum of the torques on the racquet are zero) to keep the racquet from twisting.

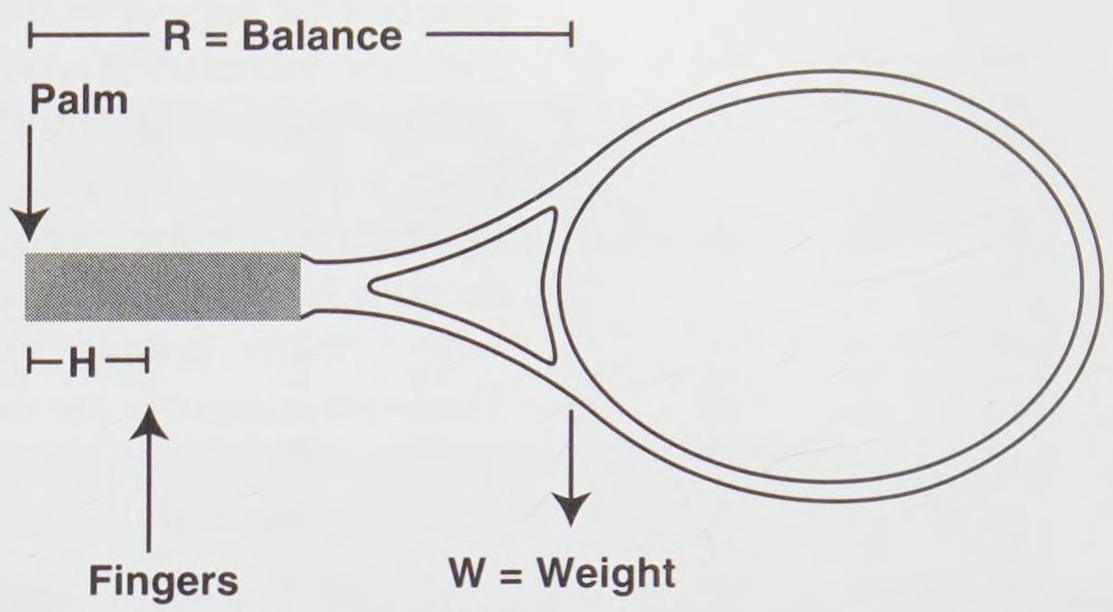
What Balance Point Means to Tennis Players

What does this mean to a tennis player? The player has to supply a counter force and torque to the racquet. When you hold a racquet in the normal manner (with your hand on the grip), you must apply a torque with your fingers just to keep the head up (remember, a torque is a force times a distance — it is a twist). You are pulling up with your fingers and pushing down with the heel of your palm. The amount of torque you must apply to the handle to keep the racquet in equilibrium is the same as the racquet is applying to your hand — the weight of the racquet multiplied by the distance from your hand to the balance point. This torque that you apply to the racquet is the feeling you relate to whether you think a racquet feels heavy, or feels light, when you are just holding, not swinging it. You can always make a racquet feel lighter in

your hand by choking up on it just a bit. This reduces the distance from your hand to the balance point, therefore it reduces the amount of torque your hand must apply to keep the racquet in equilibrium (just holding it steady). However, there is a second way to make the racquet feel lighter in your hand. Spread your fingers out.

If you examine the situation carefully, you will discover that your index finger is pushing up on the racquet handle while the butt of your palm is actually pushing down on the butt end of the racquet. The force your index finger must exert on the handle to keep the racquet head up is the weight of the racquet (W) times the distance from the butt of your palm to the racquet balance point (R) divided by the distance from the butt to your index finger (H) (Figure 3.3). As a formula, this reads:

Force = W x R/H



Balance Point And Forces on a Racquet

Figure 3.3

The upward and downward forces due to your hand produce a counter-clockwise torque on the racquet which balances out the clockwise torque produced by the force of the racquet's weight around your hand.

This ratio of distances multiplied by the racquet weight is about four or five (the distance to the balance point is about 13.5 inches and the upward hand force is applied about 3 to 4 inches from the butt end), so the racquet will "feel" four or five times as heavy when you hold it by the handle as when you hold it at the balance point.

By spreading your fingers, you can increase H from 4 inches to almost 5 inches. With your fingers tight, a 12-1/2 ounce racquet will require 40.6 ounces of upward force to keep the head up, while with the fingers spread, it will require only 32.5 ounces of force. It is clearly advantageous to spread your fingers.

All the tennis books, all instruction, and all the teaching professionals tell you to spread your fingers when you grip the racquet handle, and not to hold it with a tight fist like a baseball bat grip. This gives more control, power, etc., but there is one other major benefit that spreading your fingers produces that never seems to be mentioned in any of these instructions . . . it makes the racquet feel lighter to hold.

You can also reduce the force required to keep the head up by moving your entire hand up the handle toward the head, but this is not as effective as spreading your fingers. Moving your entire hand does not increase H, and you have only reduced R by a small fraction of its original length. To get the same reduction as the spread fingers provide, you would have to choke up about 2.6 inches, which is a considerable distance.

If two racquets have the same weight and the same length, the head-heavy one will require you to apply a greater torque to keep its head up compared to the head-light frame, and you will think the head-heavy frame feels heavier, even though it has the same weight. (To keep the racquet head up, you will have to apply a larger force with your fingers.) If this were the case, why would any-

one want a head-heavy racquet? The bang you get out of a frame (the "power") depends on where the weight is located. As a general rule, a head-heavy frame will give you more ball speed for the same swing speed, compared to a head-light racquet of the same weight. Another correlated general rule says that the power you get out of a racquet falls as the ball impact point to balance point distance increases if you are translating the racquet or moving it in a straight line (as you might in a punch volley as opposed to a groundstroke). In such a case, if you hit the ball closer to the balance point, you will get more power for a given racquet weight and head speed at the impact point. A head-heavy racquet has its balance point closer to the center of the strung area, which is where many people like to hit the ball, so it will produce more power. However, racquets are usually swung about an axis, not translated in a straight line, in which case the maximum power point moves away from the balance point, as we will discuss in the swingweight section.

Customizing the Balance Point

When you buy a racquet, the balance point is set. But a knowledgeable racquet technician can apply lead tape to adjust the weight and balance of a racquet to alter the racquet's feel and peformance. Table 3.1 shows how much the balance point will change by adding specific amounts of weight at specific distances from the existing balance point. The heavier the racquet, the less effect per gram of added weight, since it is a smaller percentage of the total weight of the racquet.

If the weight is added on the handle side of the balance point, the change is negative (towards the handle). The table shows increments up to 5 grams. For increments greater than five grams, simply add gram increments together. For example, 10 grams (2 x 5 grams) at 12 inches for a 300 gram racquet adds 0.4 inches to the balance point (0.20 + 0.20). However, these calculations assume all the weight will be added at the same spot, but lead tape will cover a few inches, so you may have to do some averaging.

Swingweight

Racquet Speed

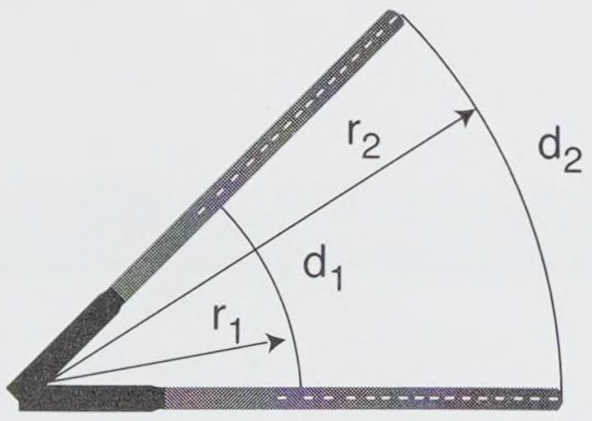


Figure 3.4

The racquet tip travels farther (d_2) than the throat (d_1) in the same time. Speed depends on distance from pivot point (r_1, r_2). But mass at greater distances from the pivot point is harder to accelerate. Why? Because it travels a greater distance in the same time, so it accelerates faster and the force needs to be larger.

So far, all we have done is picked up the racquet and held it steady. It felt heavier or lighter, depending on where we held it. But weight feels differently when we put it in motion with a swing. We've just established that the effect of weight depends on how far it is from the handle. The same holds true for the velocity of any point on the racquet. Velocity varies with distance from a pivot point because a racquet swing travels in a circular motion. (Swingweight is usually calculated at 4 inches from the butt end. This is primarily because the only commercial machines available to measure swingweight swing the racquet from that location. See Chapter 5 for more on measuring swingweight.) All points on an object that is translating or traveling in a straight line are moving at the same speed. But if the object is rotating or traveling on a curved path, like a swinging racquet, then the mass points of the object far from the pivot are moving faster than those closer to the pivot. Figure 3.4 makes this clear.

The tip of the racquet travels a greater distance than the throat or the handle, but they all travel their respective distances in the same

Table 3.1

Balance Customization

Amount balance will change in inches for racquet of specific weight by adding 1-5 gram weights in 1-inch increments from the balance point. For increments greater than 5 grams, add results of separate increments together. Changes are negative if weight is added on handle side of balance.

Added Grams	Inches From Balance	Racquet Weight in Grams												
		230	240	250	260	270	280	290	300	310	320	330	340	350
1	1" From Balance	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
2		.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
3		.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
4		.02	.02	.02	.02	.01	.01	.01	.01	.01	.01	.01	.01	.01
5		.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.01	.01
1	2" From Balance	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
2		.02	.02	.02	.02	.01	.01	.01	.01	.01	.01	.01	.01	.01
3		.03	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02
4		.03	.03	.03	.03	.03	.03	.03	.03	.03	.02	.02	.02	.02
5		.04	.04	.04	.04	.04	.04	.03	.03	.03	.03	.03	.03	.03
1	3" From Balance	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01	.01
2		.03	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02
3		.04	.04	.04	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
4		.05	.05	.05	.05	.04	.04	.04	.04	.04	.04	.04	.03	.03
5		.06	.06	.06	.06	.06	.05	.05	.05	.05	.05	.05	.04	.04
1	4" From Balance	.02	.02	.02	.02	.01	.01	.01	.01	.01	.01	.01	.01	.01
2		.03	.03	.03	.03	.03	.03	.03	.03	.03	.02	.02	.02	.02
3		.05	.05	.05	.05	.04	.04	.04	.04	.04	.04	.04	.03	.03
4		.07	.07	.06	.06	.06	.06	.05	.05	.05	.05	.05	.05	.04
5		.09	.08	.08	.08	.07	.07	.07	.07	.06	.06	.06	.06	.06
1	5" From Balance	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.01	.01
2		.04	.04	.04	.04	.04	.04	.03	.03	.03	.03	.03	.03	.03
3		.06	.06	.06	.06	.06	.05	.05	.05	.05	.05	.05	.04	.04
4		.09	.08	.08	.08	.07	.07	.07	.07	.06	.06	.06	.06	.06
5		.11	.10	.10	.10	.09	.09	.09	.08	.08	.08	.08	.07	.07
1	6" From Balance	.03	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02
2		.05	.05	.05	.05	.04	.04	.04	.04	.04	.04	.04	.03	.03
3		.08	.07	.07	.07	.07	.06	.06	.06	.06	.06	.05	.05	.05
4		.10	.10	.10	.09	.09	.09	.08	.08	.08	.07	.07	.07	.07
5		.13	.12	.12	.11	.11	.11	.10	.10	.10	.09	.09	.09	.08
1	7" From Balance	.03	.03	.03	.03	.03	.03	.02	.02	.02	.02	.02	.02	.02
2		.06	.06	.06	.05	.05	.05	.05	.05	.04	.04	.04	.04	.04
3		.09	.09	.08	.08	.08	.08	.07	.07	.07	.07	.06	.06	.06
4		.12	.11	.11	.11	.10	.10	.10	.09	.09	.09	.08	.08	.08
5		.15	.14	.14	.13	.13	.13	.12	.12	.11	.11	.11	.10	.10
1	8" From Balance	.03	.03	.03	.03	.03	.03	.03	.03	.03	.02	.02	.02	.02
2		.07	.07	.06	.06	.06	.06	.05	.05	.05	.05	.05	.05	.04
3		.10	.10	.10	.09	.09	.09	.08	.08	.08	.07	.07	.07	.07
4		.14	.13	.13	.12	.12	.12	.11	.11	.10	.10	.10	.09	.09
5		.17	.16	.16	.15	.15	.14	.14	.13	.13	.12	.12	.12	.11
1	9" From Balance	.04	.04	.04	.03	.03	.03	.03	.03	.03	.03	.03	.03	.03
2		.08	.07	.07	.07	.07	.06	.06	.06	.06	.06	.05	.05	.05
3		.12	.11	.11	.10	.10	.10	.09	.09	.09	.08	.08	.08	.08
4		.15	.15	.14	.14	.13	.13	.12	.12	.12	.11	.11	.10	.10
5		.19	.18	.18	.17	.17	.16	.15	.15	.14	.14	.14	.13	.13
1	10" From Balance	.04	.04	.04	.04	.04	.04	.03	.03	.03	.03	.03	.03	.03
2		.09	.08	.08	.08	.07	.07	.07	.07	.06	.06	.06	.06	.06
3		.13	.12	.12	.11	.11	.11	.10	.10	.10	.09	.09	.09	.08
4		.17	.16	.16	.15	.15	.14	.14	.13	.13	.12	.12	.12	.11
5		.22	.21	.20	.19	.19	.18	.17	.17	.16	.16	.15	.15	.14
1	11" From Balance	.05	.05	.04	.04	.04	.04	.04	.04	.04	.03	.03	.03	.03
2		.09	.09	.09	.08	.08	.08	.07	.07	.07	.07	.07	.06	.06
3		.14	.14	.13	.13	.12	.12	.11	.11	.11	.10	.10	.10	.09
4		.19	.18	.18	.17	.16	.16	.15	.15	.14	.14	.13	.13	.12
5		.24	.23	.22	.21	.20	.20	.19	.18	.18	.17	.17	.16	.15
1	12" From Balance	.05	.05	.05	.05	.04	.04	.04	.04	.04	.04	.04	.03	.03
2		.10	.10	.10	.09	.09	.09	.08	.08	.08	.07	.07	.07	.07
3		.15	.15	.14	.14	.13	.13	.12	.12	.12	.11	.11	.10	.10
4		.21	.20	.19	.18	.18	.17	.16	.16	.15	.15	.14	.14	.13
5		.26	.25	.24	.23	.22	.22	.20	.20	.19	.19	.18	.17	.17
1	13" From Balance	.06	.05	.05	.05	.05	.05	.04	.04	.04	.04	.04	.04	.04
2		.11	.11	.10	.10	.10	.09	.09	.09	.08	.08	.08	.08	.07
3		.17	.16	.16	.15	.14	.14	.13	.13	.12	.12	.12	.11	.11
4		.22	.21	.21	.20	.19	.19	.18	.17	.17	.16	.16	.15	.15
5		.28	.27	.26	.25	.24	.23	.22	.21	.21	.20	.20	.19	.18
1	14" From Balance	.06	.06	.06	.05	.05	.05	.05	.05	.04	.04	.04	.04	.04
2		.12	.11	.11	.11	.10	.10	.10	.09	.09	.09	.08	.08	.08
3		.18	.17	.17	.16	.16	.15	.14	.14	.13	.13	.13	.12	.12
4		.24	.23	.22	.21	.21	.20	.19	.18	.18	.17	.17	.16	.16
5		.30	.29	.28	.27	.26	.25	.24	.23	.22	.22	.21	.20	.20
1	15" From Balance	.06	.06	.06	.06	.06	.05	.05	.05	.05	.05	.05	.04	.04
2		.13	.12	.12	.11	.11	.11	.10	.10	.10	.09	.09	.09	.08
3		.19	.18	.18	.17	.17	.16	.15	.15	.14	.14	.14	.13	.13
4		.26	.25	.24	.23	.22	.22	.20	.20	.19	.19	.18	.17	.17
5		.32	.31	.30	.29	.28	.27	.26	.25	.24	.23	.23	.22	.21
1	16" From Balance	.07	.07	.06	.06	.06	.06	.05	.05	.05	.05	.05	.05	.04
2		.14	.13	.13	.12	.12	.12	.11	.11	.10	.10	.10	.09	.09
3		.21	.20	.19	.18	.18	.17	.16	.16	.15	.15	.14	.14	.13
4		.28	.26	.26	.24	.24	.23	.22	.21	.20	.20	.19	.19	.18
5		.34	.33	.32	.30	.30	.29	.27	.26	.26	.25	.24	.23	.22

time. So, racquet speed depends on where on the racquet you are talking about, and that velocity increases with the distance from the butt end of the racquet. All points on the racquet obviously travel the same angle in the same time. So all points on a racquet have the same angular velocity, no matter their distance from the handle; whereas every point has a different linear velocity depending on that same distance.

The feel of a swinging racquet is twice dependent on the average distances of masses on the racquet from the handle — once for the effect of the mass without motion (balance point) and once for the velocity of the mass in motion. This is distance times distance, or distance squared. Swingweight is an objects resistance to change in its rotation. Mass in circular motion is what swingweight is about, so we end up with:

$$\textbf{Swingweight} = \textbf{mass} \times \textbf{distance} \times \textbf{distance} = md^2 \qquad (3.2)$$

This is swingweight for a point of mass at a given distance from its point of rotation — we're not up to a racquet yet. To equate this to an entire racquet, let's do the same exercise we did with balance point — consider the racquet as an accumulation of small bits of mass. Each bit of mass has a swingweight of md^2, and the sum of all these individual swingweights is the total swingweight of the racquet. The units of swingweight, which are usually left out in the popular tennis literature in favor of unitless numbers, are kilogram times centimeters squared (Figure 3.5) or kilogram times meters squared in the physics literature.

Figure 3.5

How Swingweight Is Determined

(Example: 324 gm, 27", even balance)
Note: Distance is determined from 4-inch (10.16 cm) mark

Inches from end	Weight per inch (0.012 kg) x (distance cm -10.16)^2
1	0.70
2	0.31
3	0.08
4	0.00
5	0.31
7	0.70
8	1.24
9	1.94
10	2.79
11	3.79
12	4.95
13	6.27
14	7.74
15	9.37
16	11.15
17	13.08
18	15.17
19	17.42
20	19.82
21	22.37
22	25.08
23	27.95
24	30.97
25	34.14
26	37.47
27	40.95

Swingweight = 335.84 kg·cm^2

Depending on the distribution of mass, a racquet's swingweight will be the sum of each bit of mass times the distance squared to the pivot point at 4 inches on the handle. (Figure 3.5 is just an approximation because it is pretending all the mass of each bit exists only at inch intervals along the racquet and not spread out between each inch.) Swingweight is ultimately the sum of every atom's mass times its distance squared from the pivot point.

What Swingweight Means to the Player

The swingweight of a racquet is a measure of how much torque (twist) you must apply to the racquet handle to get the racquet to swing. (You do not apply a force to the handle of a racquet to cause it to swing, you apply a torque.) Swingweight can only be measured by an experiment where the racquet is moving. You cannot distinguish one swingweight from another by just holding a racquet in your hand. To determine a racquet's swingweight you must swing the frame or put it into a machine that moves it. A racquet with a large swingweight will be difficult to swing and a racquet with a small swingweight will swing easily. As a general rule, for a given racquet weight, a head-heavy racquet will have a larger swingweight than a head-light frame of the same total length and weight.

While most people have a "feel" for weight and balance, few people have a good feel for swingweight. In tests run with non-tennis players, the participants could only distinguish racquets having one swingweight from racquets having another swingweight when the difference was about 25 percent. When the same tests were run with college level varsity players (who practice or play about 2 hours every day) and teaching pros, they could distinguish frames that differed by as little as 2 percent in swingweight.

A racquet with too large a swingweight will be sluggish, hard to swing, and not feel maneuverable. When racquet manufacturers were contemplating producing longer (28-, 29-inch length) racquets they realized that if all they did was to add the extra inches to the butt end of their existing frames, the swingweights would have come out too high. Players would not have immediately switched over to the longer racquets. A complete redesign of the frame was needed to provide the extra length, yet have a feel and swing similar to the racquet players were used to and liked playing with. That is one of the reasons why sales of the extended length racquets did so well initially, and are still doing well. The Rules of Tennis do not specify a minimum racquet length, but do specify 29 inches as the maximum length.

Why would anyone not want a racquet with the smallest swingweight if it were easier to swing and maneuver? Racquet manufacturers have tried very hard to keep swingweights down, since they will swing much easier. However, there is often a correlation between swingweight and the speed of the ball as it leaves your racquet. In general, for a given swing speed, the racquet with the larger swingweight may give you a bit more ball speed on your shots. Of course, if the swingweight is too high, you may not be able to achieve that given swing speed.

If you find that your present racquet seems to be sluggish when you swing it and you cannot generate the head speed you want, the problem may be too large a swingweight. Just because a racquet has been made lighter, that does not guarantee that the swingweight has been reduced. It is swingweight, not weight, that determines your swing speed. To find out if this is your problem, try choking up on the handle just a bit when you swing. Since choking up reduces the effective swingweight markedly, see if this solves your problem.

Customizing Swingweight

Just as you can alter the balance of a racquet, you can alter its swingweight by adding lead tape. Table 3.2 demonstrates the effects on swingweight of adding weights at different locations along the racquet. (The table is in units of kilograms x centimeters squared. These are the units racquet technicians are most familiar with and the units that the automated measuring machines use.) The swingweight of a racquet depends very critically on where the racquet's weight is located and how far the weight is from your hand. Adding a few grams of lead tape to the tip of your racquet will increase the swingweight much more than adding the same weight at the throat of your frame. Adding extra weight to the handle region will hardly change the swingweight at all, but it will change the racquet's total weight and balance point.

The real value of the chart is for customizing. Each number in each column signifies the amount the swingweight will be changed if you add that much weight at that distance.

Table 3.2

Swingweight Customization

Columns are the amount swingweight changes (in kg*cm^2) by adding the specified weight at the top of each column at distances from the butt equal to the far left column. Add columns together for larger weights (measured 4" from the butt).

Inches From Butt to Add Wt.	Cm From Butt	Amount of Weight Added (Grams)									
		1	2	3	4	5	6	7	8	9	10
0	0.00	0.10	0.21	0.31	0.41	0.52	0.62	0.72	0.83	0.93	1.03
1	2.54	0.06	0.12	0.17	0.23	0.29	0.35	0.41	0.46	0.52	0.58
2	5.08	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.21	0.23	0.26
3	7.62	0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.05	0.06	0.06
4	10.16	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	12.70	0.01	0.01	0.02	0.03	0.03	0.04	0.05	0.05	0.06	0.06
6	15.24	0.03	0.05	0.08	0.10	0.13	0.15	0.18	0.21	0.23	0.26
7	17.78	0.06	0.12	0.17	0.23	0.29	0.35	0.41	0.46	0.52	0.58
8	20.32	0.10	0.21	0.31	0.41	0.52	0.62	0.72	0.83	0.93	1.03
9	22.86	0.16	0.32	0.48	0.65	0.81	0.97	1.13	1.29	1.45	1.61
10	25.40	0.23	0.46	0.70	0.93	1.16	1.39	1.63	1.86	2.09	2.32
11	27.94	0.32	0.63	0.95	1.26	1.58	1.90	2.21	2.53	2.85	3.16
12	30.48	0.41	0.83	1.24	1.65	2.06	2.48	2.89	3.30	3.72	4.13
13	33.02	0.52	1.05	1.57	2.09	2.61	3.14	3.66	4.18	4.70	5.23
14	35.56	0.65	1.29	1.94	2.58	3.23	3.87	4.52	5.16	5.81	6.45
15	38.10	0.78	1.56	2.34	3.12	3.90	4.68	5.46	6.25	7.03	7.81
16	40.64	0.93	1.86	2.79	3.72	4.65	5.57	6.50	7.43	8.36	9.29
17	43.18	1.09	2.18	3.27	4.36	5.45	6.54	7.63	8.72	9.81	10.90
18	45.72	1.26	2.53	3.79	5.06	6.32	7.59	8.85	10.12	11.38	12.65
19	48.26	1.45	2.90	4.35	5.81	7.26	8.71	10.16	11.61	13.06	14.52
20	50.80	1.65	3.30	4.95	6.61	8.26	9.91	11.56	13.21	14.86	16.52
21	53.34	1.86	3.73	5.59	7.46	9.32	11.19	13.05	14.92	16.78	18.65
22	55.88	2.09	4.18	6.27	8.36	10.45	12.54	14.63	16.72	18.81	20.90
23	58.42	2.33	4.66	6.99	9.32	11.65	13.97	16.30	18.63	20.96	23.29
24	60.96	2.58	5.16	7.74	10.32	12.90	15.48	18.06	20.65	23.23	25.81
25	63.50	2.85	5.69	8.54	11.38	14.23	17.07	19.92	22.76	25.61	28.45
26	66.04	3.12	6.25	9.37	12.49	15.61	18.74	21.86	24.98	28.10	31.23
27	68.58	3.41	6.83	10.24	13.65	17.06	20.48	23.89	27.30	30.72	34.13
28	71.12	3.72	7.43	11.15	14.86	18.58	22.30	26.01	29.73	33.45	37.16
29	73.66	4.03	8.06	12.10	16.13	20.16	24.19	28.23	32.26	36.29	40.32
30	76.20	4.36	8.72	13.08	17.45	21.81	26.17	30.53	34.89	39.25	43.61

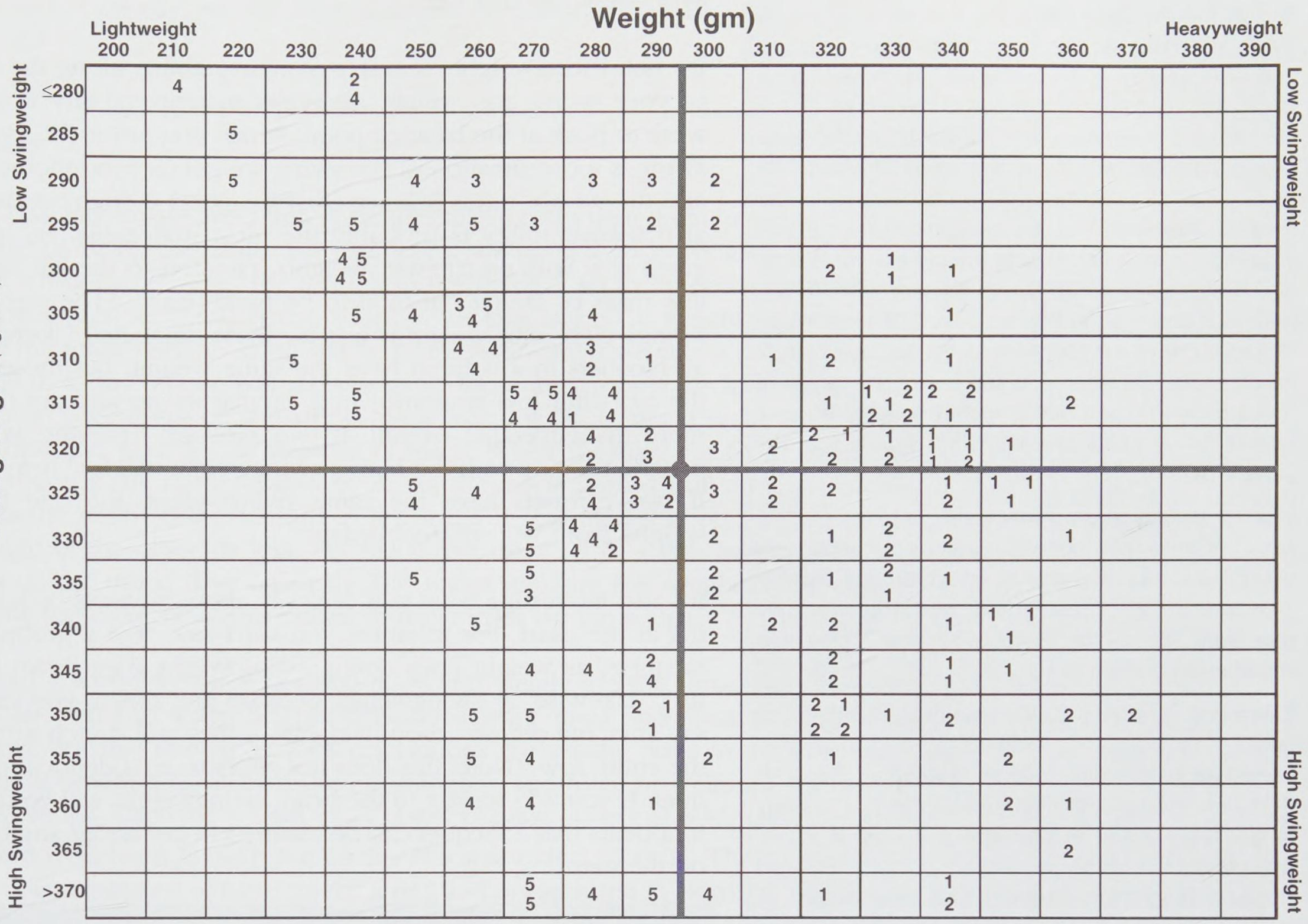
For example, if you add 10 grams (= 0.010 kilograms) at 23 inches from the butt (23 inches from the butt is 23-4 = 19 inches = 48.26 centimeters from the pivot point), you will add approximately 23.29 swingweight units to the existing swingweight (0.010 x 48.26 x 48.26 = 23.29). At 27 inches, it would be 34.13. And if you add 15 grams at 27 inches, the additional swingweight is 51.19 (the 5 and 10 gram columns added together, or the 8 and 7, etc.). Also, notice the effect of adding weight vs. adding distance. Doubling the added weight doubles the increase in the swingweight. Doubling the distance from the pivot point (subtract 4 inches from whatever row you are in and double the result to locate the point twice the distance from the pivot point) while keeping the weight the same will add four times the additional swingweight. Where you add weight makes much more difference than how much you add. And notice too, there is no such thing as lowering swingweight, even though you can lower the balance point.

The Relationship of Weight, Balance and Swingweight

Figure 3.6 is a weight, balance, and swingweight map. It plots all performance racquets of a recent year (2000) by weight and swingweight with balance num-

Figure 3.6

Weight, Balance & Swingweight Map



✦ Center of Swingweight/Weight Universe (includes all performance frames — i.e., over $100 suggested retail price — in year 2000. Approximately half the racquets are to each side and half above and below the center axis cross.

Balance point (% of length): **1** Super Head-light ≤ 46% **2** Head-light = 47-49% **3** Even = 50% **4** Head-heavy = 51-53% **5** Super-Head-heavy ≥ 54%

ber coded as follows: 1 = super head-light (balance at less than or equal to 46% of length); 2 = head-light (balance equal to 47-49% of length); 3 = even (balance at 50% of length); 4 = head-heavy (balance equal to 51-53% of length); 5 = super head-heavy (balance greater than or equal to 54% of length). The balance is figured as the percentage of racquet length from the handle. That allows comparing head-light and head-heavy racquets of all lengths on the same map.

The map reveals slightly different nuances to the relationship between these three components of weight distribution. Some are obvious, some surprising, and all are interesting. The map is designed to gain a better overall understanding of how these variables interact and affect racquet performance.

Is It the Weight of the Swing, or the Swing of the Weight?

What immediately stands out is the range of racquets represented in each row and column. For example, in the weight column of 290 gram racquets, there

Swingweight and Customizing Length?

You have been playing with a standard length (27-inch) racquet for your entire life, and you wonder what it would be like to move up to a longer (28- or 29-inch) frame. You have read all the ads, read the reviews, and talked to (and played against) people who have the longer frames. You have even hit once for five minutes with a longer racquet. However, you are reasonably happy with your present racquet. It cost a bundle 3 years ago, and it still plays great. Could another inch or two take your tennis up to the next level? You would like to know. Do you really want to go out and buy all new frames? So when you see an ad telling you of a company that will add 2 inches to your present racquet for a reasonable fee, what should you do? DON'T DO IT!

You will usually end up with a racquet that you will have difficulty playing with, and as a result, you won't like it. The modification could turn the carefully balanced, well-tuned frame that you presently play with, into a club that maybe only a caveman would enjoy playing with.

There is a parameter called swingweight that determines how difficult it is to swing a racquet and maneuver it from one location to another. The larger the swingweight, the more torque you must apply to get the racquet to do what you want it to do. Swingweight can be measured using a Babolat Racquet Diagnostic Center (RDC), and almost all modern racquets have a swingweight in the 300 to 360 range (in units of kilogram x centimeters squared). If you take a typical 27-inch racquet that has a swingweight of 330 and a weight of 320 grams (11.3 ounces), adding 2 inches to the handle will increase the swingweight to at least 425. This is greater than any other modern racquet on the market, except the 32-inch long Gamma Big Bubba. The added 2 inches will increase the swingweight by about 30%, which means it will be 30% more sluggish than the racquet you started with. The increased benefits of the longer frame may be cancelled by the extra swingweight you will be forced to drag around on your shots.

However, if you buy a typical 28- or 29-inch frame that has been designed from scratch to handle the extra length, you probably will be pleased with the way it feels and the way it plays. The manufacturers have cleverly redesigned their longer racquets and redistributed the weight so that the swingweights come out very similar to the values found in the average 27-inch frame. This will give you all the advantages of a longer racquet (since it is longer), yet when you swing it, the racquet will feel like the standard 27-inch frame that you are used to.

are swingweights ranging from 290 to over 370. Similarly, in the 315 swingweight row, weights vary from 230 to 370 grams. That's a 140 gram difference, yet they both swing equally as easy. How can that be?

It's not. Swingweight measures maneuverability along the arc of your swing and weight measures maneuverability if you were to push at the balance point, which you never do. Every swing is a combination of the swing weight (second moment) and the weight times balance (first moment) though rotational maneuverability is probably the most noticeable. So in a given row with equal swingweights, racquets to the left have less mass or inertia but tend to be head-heavy. (The sum of swingweight plus weight is less for these racquets.) Likewise, all racquets in a column have the same weight, but those to the top offer less resistance in a circular swing and are thus more maneuverable overall. If two racquets have the same weight, the one with the higher swingweight will feel heavier. If two racquets have the same swingweight, the one that weighs more will still feel heavier.

It's also interesting what you can't say about racquets by looking at the chart. For instance, you can't say, "For a group of racquets, as weight goes down, swingweight goes down (or up)." Likewise, as swingweight goes up and down, you can't say anything definite about weights — they still stretch across the entire row. (Note: this does not apply to an individual racquet. If you add weight to the head, swingweight will go up.) It appears that a racquet can be made to fit just about any box on the map.

As Weight Goes Down, Balance Goes Up

In general, for this universe of racquets, as weight goes down, balance goes up, and as weight goes up, balance goes down. Notice how the number codings drift from super head-light on the right to super-head-heavy on the left.

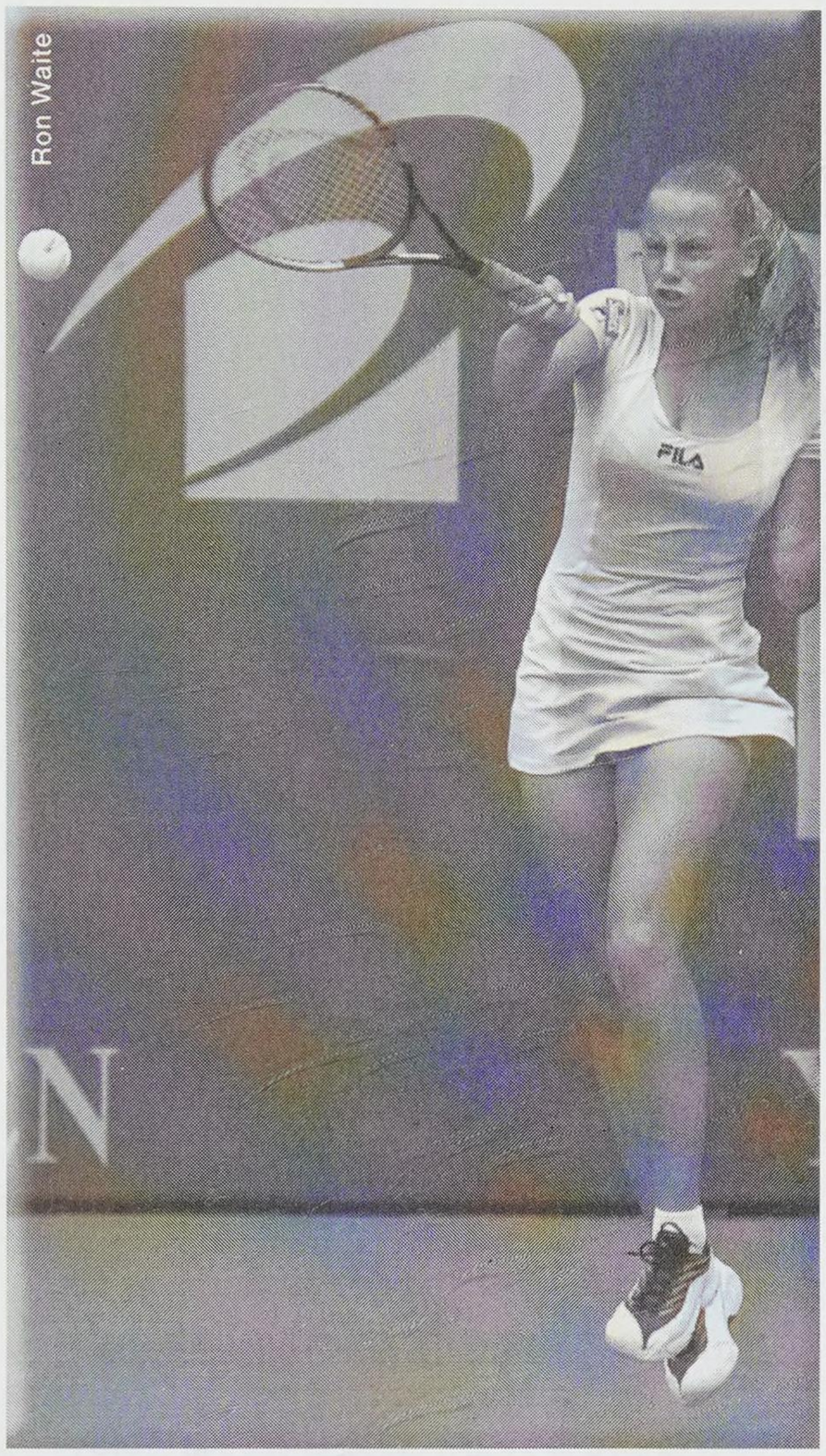
The head of a racquet is under tremendous stress because of the tension in the strings and the constant pounding due to hitting the ball. With less weight (material), you have to use what's available in the head to keep it strong. Still, it's tempting to think that if two racquets have the same swingweight, a heavier racquet will have at least as much mass in the head as the much lighter head heavy racquets. For example, look at the 315 swingweight row. There's almost a 140 gram difference between the racquet on the left and the racquet on the right. You'd think that all the extra weight would spread out so the heavier racquet would have more material everywhere, even though the balance is toward the handle, not the head. But if they both had the same mass located at the same distance in the head, plus the heavier racquet had more mass everywhere else, then it would have a higher swingweight than the lighter but higher balanced racquet. It follows that

the lighter racquet has more absolute (not just relative) mass in the head. This makes perfect sense, it's just difficult to imagine. (Note: you can think of bizarre combinations of weight distribution that violate this statement, but at the end of the day, we still have to have a racquet that does its job and remains intact.)

Notes

1. Mass and weight are different things but we often treat them in everyday language as being the same. For example if we hang a 60 lb weight on the end of a string then we say the tension in the string is 60 lb. In doing so we have made several technical errors. Suppose we take our 60 lb weight to the moon. Will the tension in the string still be 60 lb? And what if we take a racquet strung at 60 lb to the moon? Will the strings be loose and floppy? Technically, the 60 lb object we hung on the string has a mass of 60 lb, not a weight of 60 lb. Technically, weight is a measure of force. When we take our 60 lb object to the moon it still has a mass of 60 lb but the moon doesn't pull on it as strongly as the earth so the tension in the string is less than it is on earth and the weight of the object is less. Put the object on a set of scales and it will read less on the moon than on earth. The scales measure the weight of the object or the force on the scales rather than the mass on the scales.

Back on earth, we often say that mass and weight are the same, but then we get into trouble when doing calculations since mass and force are different things. For that reason, physics calculations are usually done in the SI system of units where mass is measured in kilogram and force or weight is measured in Newtons. Then the weight of an object of mass M (in kg) is Mg (in Newtons) where g on Earth is 9.8 but g on the Moon is only 1.67 m/s^2. The tension in a string is a measure of force on the string and is measured in Newton. The string tension won't change when you take your pre-strung racquet to the moon, and neither will the mass of your racquet, but the racquet will feel a lot lighter. The rules of tennis will need to be changed drastically when we start living and playing on the moon.



Jelena Dokic, Yugoslavia

4

Racquet Stability

Twistweight and Recoilweight

By Crawford Lindsey and Howard Brody

Making a lightweight, stable, maneuverable, and powerful racquet is not easy. Some of these features benefit from more weight, some from less. So, adding, subtracting, and moving weight around is what racquet design is all about. The history of major racquet innovations is, to a large extent, a story of weight distribution and doing more with less.

Weight In Past Racquet Developments

The introduction of metals, and then graphite, were billed as materials stories. But what made these materials so important (especially graphite) was the ability to eliminate and relocate weight because of these materials' higher stiffness and strength-to-weight ratios. You could now build the same structure with greater stiffness and strength and use less material weight to do it.

The first oversize racquet (produced commercially by Prince in the 1970s, but handmade experiments had been around for 50 years) was also indirectly about weight. It was marketed as a "headsize" story, but not only did the larger head's longer strings mean more power in the middle of the sweetspot, but a wider head also meant more power on off-center hits due to the stabilizing influence of weight at greater distances from the center line of the racquet.

Wilson's Hammer system was also about weight. It was the most radical elimination and redistribution of weight ever, removing it from the handle and resulting in radical head heaviness.

Even the widebody (racquet frames with a thick cross-section perpendicular to the stringbed) revolution could be interpreted as weight related because it was graphite's strength-to-weight and stiffness-to-weight ratios that made the radical shape of widebody construction structurally possible.

And, of course, the era of super lightweight frames of the present is, by definition, about weight.

Stability By Weight Polarization

In the last chapter, we discussed how the amount and location of weight affects maneuverability and the feel of the racquet through the air. Amount and location of weight also becomes important at impact. Here we discuss how weight affects stability. Recently, Prince started polarizing the weight at 2 and 10 o'clock and in the butt of the handle. This strategy stabilizes the racquet in two directions — along the width and length of the racquet (Figure 4.1). True, the story sounds familiar — perimeter weighting and butt weighted racquets are

Three Kinds of Swingweight and Their Relation To Stability

Figure 4.1

Swinging and impacting a ball torques the racquet in three ways and each torque rotates the racquet around a different axis. The resistance to rotation around any axis is the "swingweight" around that axis.

Swingweight about the handle axis (swing axis) is called just that — "swingweight." This measures the distribution of weight along the length of the racquet, which in turn determines maneuverability and stability around that axis when you swing.

Swingweight around an axis drawn from butt to head (twist axis) can be called "twistweight," and it measures weight distribution from side to side and its effect on maneuverability and stability to twisting by you or the ball.

Swingweight around an axis parallel to the racquet face and through the balance point (recoil axis) can be coined "recoilweight." This measures weight distribution from tip to butt and the resulting resistance to recoil.

Parallel Axis Theorem: If you know the moment of inertia about any given axis, then you can calculate it about any other parallel axis by using the parallel axis theorem (see Note 1)

not new concepts, but they have never been integrated into, nor marketed as, a philosophy of racquet design. The emphasis on head to butt polarized weighting is a new twist.

What, How, When, Where and Why of Polarization

1. Why stabilize? Lightweight racquets require extra stabilization because they have less inertia, get pushed around more, and incur more shock. Impact causes a racquet's motion to change primarily in two directions. First, there is backwards recoil, like a door on a hinge (on lengthwise off-center hits). Second, the racquet twists like a top in your hand (on widthwise off-center hits). The amount of recoil and twist represents wasted energy.

2. Weight stabilizes. More weight stabilizes more, and less weight, less. But the greater the distance the weight is from its center of rotation, the more it stabilizes. So, if you double the weight at a certain location, you double its effect on stability. If instead, the weight's distance from the racquet's swing pivot point is doubled, the effect on stability is quadrupled. Likewise, adding one-fourth the weight at twice the distance has the same effect as all the weight at the original location. In short, weight stabilizes and polarized weight stabilizes more with less.

3. What does stabilization do? Stabilization reduces recoil and twist and their effects. Maximum stability against both kinds of torque is achieved by placing weight at 3 and 9 o'clock for twist resistance, and 12 o'clock and the butt for recoil resistance. These are the greatest polarized distances on the racquet. This is why bigger head sizes and longer racquets are more stable, all else being equal.

Twistweight: The Stability of a Racquet Against Twisting
Howard Brody

Hit a ball off-center, and you know what happens. The racquet tries to twist in your hand. The ball comes off at the wrong angle, and your shot does not have the speed that you hoped it would have. Chalk up another unforced error on your stat sheet.

You know that if you can only get the ball to hit closer to the axis of the racquet, the frame will not twist as much. Then your shot will land in the court, usually closer to the location where you want it to go. Making sure that the ball hits near to the central axis of the frame requires good hand-eye coordination, concentration, and practice — lots of practice. Is there anything else you can do to keep the racquet from twisting so much on miss-hits? Yes, and here science comes to your rescue.

You can use a racquet with a wider head. The width of the head determines something called the polar or roll moment of inertia (physics for twistweight). Wider racquets (as distinguished from "widebody" racquets which have a thicker frame) have a larger polar moment and provide more stability against twisting. One great virtue of the original Prince oversize racquet was that it had almost a 40% greater stability against twisting than the old small head racquet that was in use at that time. To measure a racquet's stability against twisting (the moment of inertia about its long axis) is a rather technical exercise that can be performed with something called a torsion pendulum or by turning the frame into a pendulum with the handle horizontal (see Chapter 5). The definition of the moment of inertia is the sum of every little mass piece in the racquet multiplied by that mass piece's distance from the axis of rotation squared. However, the racquet weight multiplied by its head width (or diameter) squared will give you a very good approximation of the stability factor. This is shown in Figure 4.2 for a number of racquets of different mass and width. As a specific example, a 0.3 kg (300 gram) frame that has a 25 cm inches wide head will have a stability factor of 187.5 (weight x width x width = 0.3 x 25 x 25 = 187.5). Selecting a racquet with the same weight, but a head just three centimeters wider (28 cm instead of 25 cm), will produce a stability factor of 235.2 (0.3 x 28 x 28 = 235.2), which is an increase of 25%.

There are corrections to this due to the exact weight and shape of the head. A square head (sides parallel) will have slightly more stability than an oval or egg shaped head, which in turn will have somewhat more stability than a triangular ("yield sign"

shaped) head, for the same total head width. A head-heavy frame will be a bit more stable than an evenly balanced racquet of exactly the same dimensions and total weight.

A racquet with an increased stability factor will have a more uniform response across its face (less variation in rebound ball speed for slight miss-hits) as well as less twisting on the miss-hits. This may be thought of as having an enlarged sweetspot and being a more forgiving racquet.

The reduction in the twisting may have a very beneficial effect in reducing the probability of developing the dreaded tennis elbow. When you hit a ball well off-axis, the racquet will twist regardless of how tightly you are gripping the handle. But the racquet rarely twists out of your hand. Your hand and your forearm twist along with the racquet, but your upper arm does not rotate that way. That is what causes the problem in the elbow where your forearm and upper arm meet — one rotates, the other does not. A racquet that is more stable and twists less on off-axis impacts will be kinder to your arm and elbow.

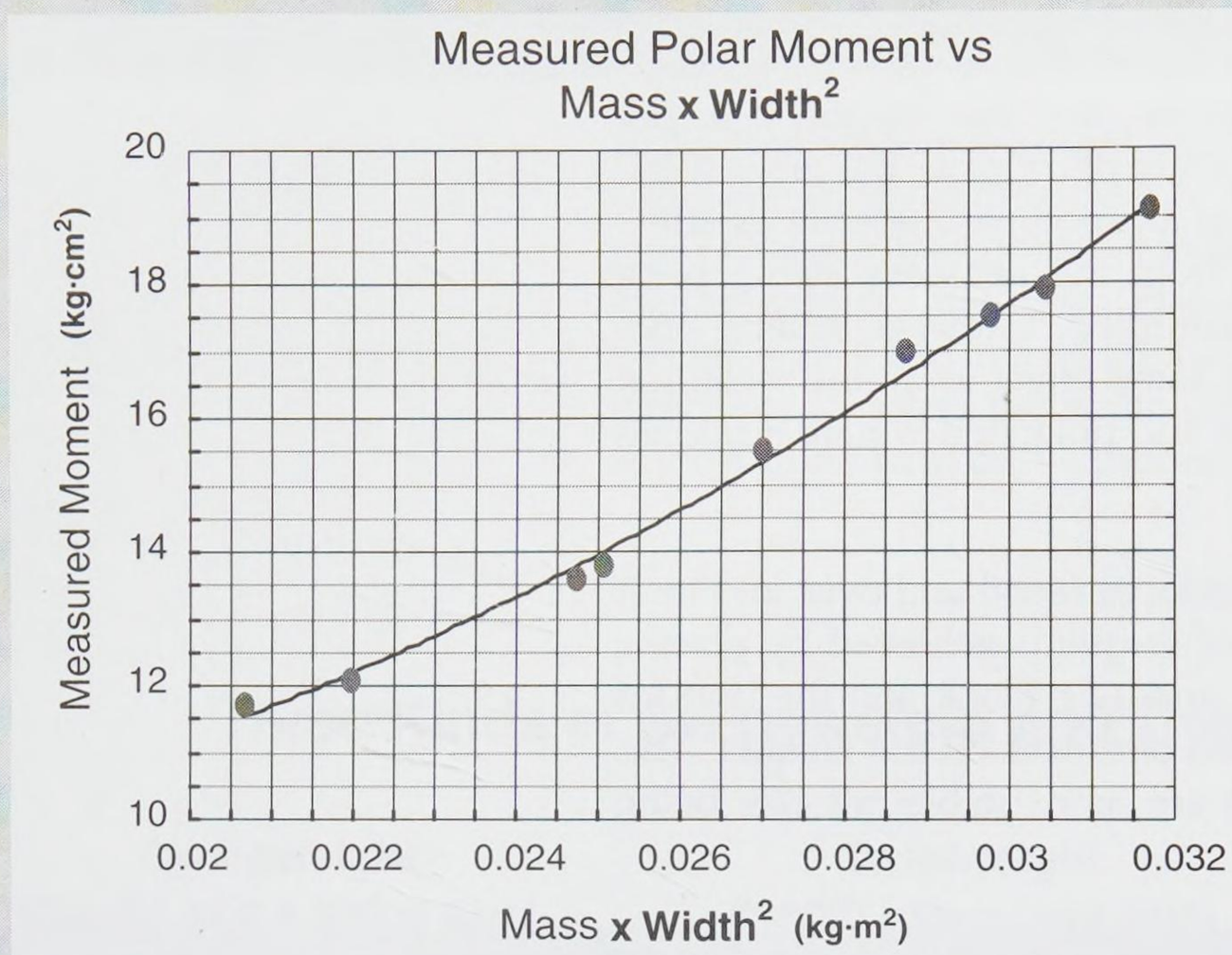


Figure 4. 2 Measured Polar Moment of Inertia versus Mass x Width x Width.

The measured polar or roll moment of inertia for a number of tennis racquets with different masses and widths is plotted against the mass of each racquet multiplied by that racquets head width squared. Note that the data point for the different racquets seems to lie along a smooth curve.

From these arguments, it seems that everyone should play with a racquet that is constructed at the very limit of width that the ITF allows (31.75 cm). Why don't they? There is the matter of maneuverability. More stability usually leads to less maneuverability. However, this is a particular type of twisting maneuverability, and it may not affect all players the same way. If you hit very flat shots, or if you chop and slice a great deal, the larger stability factor about the long axis of your racquet may not be a major detriment. If, on the other hand, you use a great deal of arm pronation (rotating about the long axis of your arm) on your serve and on your groundstrokes, this extra frame stability will slow down your racquet head and consequently take some of the snap out of your shots.

You have to make the decision on your racquet depending on whether you have an aggressive, arm pronating style of play, or a more conservative, less error prone stroking. If your arm is very strong, and the extra stability factor does not slow down your stroke, then the answer is simple. For the rest of us, a decision (or compromise) must be made.

Remember: A racquet with a wider head will be more stable against twisting, give a more uniform response across the head, and provide more control. It also will be somewhat less maneuverable on groundstrokes and it will not be as user-friendly on your serve and volley game as a narrow head racquet.

4. Stability adds power. Recoil and twist steal energy and reduce power.

5. Stability adds comfort. Energy siphoned off by twist and recoil goes into your hand and arm, not the ball.

6. Stability adds control. Twisting and recoil reduce control because the racquet face rebounds at uncontrollable angles from which the ball also rebounds in undesired directions.

7. Stability expands the power zone. Sweetspot size is an indication of the degree of stabilization. The most widely used definition of sweetspot is the area of the stringbed that produces an outgoing ball velocity that is at least 45 percent (40-50 percent, depending on whom you ask) of the incoming velocity.

8. Torque robs energy. The further off center you hit, the more the racquet will twist, recoil or both. The more the racquet is stabilized, the further off-center you can hit with less energy loss and, therefore, more power, comfort, and control. The sweetspot is therefore larger. Unless you hit dead center every time, the size and location of the sweetspot area is more important than the maximum "sweetness" at the center.

9. Stability raises the sweetspot. If stability follows weight and sweetspot follows stability, then sweetspot follows weight also. Weight toward the racquet tip stabilizes and drags the sweetspot to the upper half of the stringbed where most balls are hit.

10. It's more difficult to polarize weight and stabilize a light racquet. That's because there is less weight to distribute, and in the extreme case, all the weight goes into the bare-bones structure just to keep the racquet from breaking. There is little leftover to move around for additional benefits. The lighter the racquet, the more it needs polarization, but the more difficult it is to accomplish and the less you have to do it with. Thus, at a certain point, a lighter racquet can never compete with a much heavier one in terms of stability.

The lightweight race (culminating, so far, with the lightest racquets in the neighborhood of 200 grams strung) has taught manufacturers how to remove weight where it isn't needed in order to build the lightest possible foundation structure. Then, some of that weight can be added back to produce added benefits besides simple structural integrity. Getting the most out of every fraction of a gram is important.

The idea of weight polarization is that recoil does not occur around the handle axis. Ball impact causes the racquet to rotate around the balance point, not the handle. This is illustrated by looking at two extremes. If a ball hits an unheld racquet, it flies off rotating around its balance point. If a steel rod were inserted through the handle and cemented into the floor, the racquet would instead rotate around the rod (as in Figure 4.1). You may be tempted to say a player holding a racquet would be more like the steel rod case, but you'd be wrong. Studies have shown that a person is not strong enough to stop rotation around the balance point during the 5 ms of impact. The player's hand itself recoils at the wrist because, it too, is rotating around the balance point. The player slows the rotation about the balance point, but does not relocate it to the handle like a secure steel rod would until after the impact is over.

Thus, to maximize resistance against recoil, weight must be polarized around the balance point. So, putting weight in the butt does not make it substantially more difficult to swing around the handle axis because it is located close to the axis. But that weight does significantly increase the swingweight for swinging around the balance point (recoilweight) because it is at a much greater distance to the balance point.

Interpreting the Stability Table (Table 4.1)

Swingweight around any axis (whether it be the swing, twist, or recoil axis) depends on the amount and distance of the weight from the axis. Thus, all things being equal, a longer, wider or a heavier racquet will be more stable. For these reasons, racquets being compared must be the same length and with-

in a certain weight range. That's the only way to make an "apples-to-apples" comparison. To that end, the stability table compares 28-inch racquets below 300 grams.

The table ranks recoil stabiilty in two ways. The first is "polarization index." This measures the racquet's degree of weight polarization. It is calculated as the recoilweight divided by the racquet weight. This tells you "pound-for-pound" how polarized the weight is. It is an indication of what is done with the weight available. The top half of the table ranks racquets by polarization.

The bottom half of the table ranks the same racquets in order of recoilweight (the higher the number, the more stable to recoil). For a given weight range, the recoilweight depends on the degree of polarization. The recoilweight is the feature the player is looking for (at a certain weight or swingweight). The polarization index helps explain how the racquet's weight and distribution combine to determine the recoilweight rankings.

Notes

1. Parallel axis theorem: If you know the moment of inertia about any given axis, then you can calculate it about any other parallel axis by using the parallel axis theorem. For example, if you know the moment of inertia about the balance point, to determine the moment of inertia about the butt, use the formula $I_{butt} = I_{cm} + md_{butt}^2$, where d_{butt} = distance from butt to cm, m = mass of racquet, and I_{cm} = moment of inertia about balance point (recoilweight). If you know the swingweight about an axis 4 inches from the butt and want to know the recoilweight the formula is $I_{cm} = I_{swt} - md_{swt}^2$, where d_{swt} = distance from 4-inch axis to cm, m = mass of racquet, I_{cm} = moment of inertia about balance point, and I_{swt} = moment of inertia about the 4-inch axis. If you know the swingweight about the 4-inch axis and want to know the swingweight about the butt, the formula is $I_{butt} = I_{swt} - md_{swt}^2 + md_{butt}^2$, where d_{swt} = distance from 4-inch axis to cm, dbutt = distance from butt to cm, m = mass of racquet, I_{swt} = moment of inertia about 4-inch axis, and I_{butt} = moment of inertia about the butt end.

Table 4.1

Stability 1998: 28" Racquets Under 300 Grams

Polarization Index Ranking = Recoilweight/Weight (measures the degree of weight polarization). If two racquets are close in weight, the one with the highest polarization will be more stable. If two racquets have the same, or very close polarization indexes, the heavier one will be more stable. (For example, see the four racquets with a 0.51 index.)

Mfg.	Racquet	Polarization Index	Recoil-Weight (kg·cm²)	Weight (gm)	Swingwt (kg·cm²)	Headsize (in.²)	Bal. (cm)	COP* (in.)
Prince	Ring	0.56	150	267	339	125	36.75	20.50
Prince	Rip	0.55	146	268	332	115	36.50	20.25
Pro Kennex	Kinetic Pro Ti 20G Reach	0.55	153	279	336	110	35.75	20.13
Prince	ThunderStorm LongBody OS	0.54	142	263	328	120	36.75	20.25
Dunlop	Max Superlong +1.00 OS	0.51	149	291	347	105	36.25	19.83
Gamma	Diamond Fiber C-3.0 OS	0.51	112	220	292	116	38.75	20.44
Head	Ti.S7	0.51	124	243	316	124	38.25	20.32
Prince	ThunderLite LongBody MP	0.51	149	292	326	95	34.75	19.48
Prince	Thunder Extreme Titanium OS	0.51	147	289	322	115	34.75	19.45
Dunlop	Revelation Lite +1.00	0.50	129	260	313	100	36.75	19.79
Prince	Thunder Extreme Titanium MP	0.50	143	287	310	100	34.25	19.23
Wilson	Hammer Titanium 3.0 Stretch	0.50	130	259	327	118	37.75	20.09
Gamma	Diamond Fiber C-3.0 MP	0.49	110	222	288	105	38.50	20.21
Wilson	Hyper Sledge Hammer 2.0 St. 115	0.48	112	236	312	115	39.25	20.23
Yonex	Super RQ Ti-600 Long	0.48	136	286	331	110	36.25	19.45
Prince	Thunder Superlite Titanium OS	0.47	101	213	266	115	38.00	19.87
Prince	ThunderLite LongBody OS	0.47	139	296	325	110	35.25	19.11
Wilson	Hammer 6.2 95 Stretch	0.47	131	278	346	95	38.00	19.83
Yonex	Super RQ Ti-800 Long	0.47	111	238	316	110	39.50	20.20
Gosen	Twin Pro	0.46	122	264	342	120	39.00	20.04
Wilson	Sledge Hammer 3.8 Stretch OS PH	0.46	125	270	350	112	39.00	20.04
Wilson	Hyper Hammer 2.3 Stretch SOS	0.46	112	243	318	120	39.25	20.09
Wilson	Hyper Sledge Hammer 2.0 St. 125	0.45	115	254	326	125	39.00	19.91
Wilson	Hammer 6.2 110 Stretch	0.45	128	285	349	110	38.00	19.62
Yonex	Super RQ Ti-900 Long	0.45	115	255	335	120	39.50	20.07
Yonex	Super RQ Ti-450 Long	0.45	128	283	332	110	37.00	19.38

Recoil Stability Ranking = Recoilweight (swingweight taken at the balance point). Within a given weight range, high polarization can make a racquet more stable than heavier less polarized ones. However, after a certain point, light racquets (polarized weighting or not) are less stable than heavier ones.

Pro Kennex	Kinetic Pro Ti 20G Reach	0.55	153	279	336	110	35.75	20.13
Prince	Ring	0.56	150	267	339	125	36.75	20.50
Dunlop	Max Superlong +1.00 OS	0.51	149	291	347	105	36.25	19.83
Prince	ThunderLite LongBody MP	0.51	149	292	326	95	34.75	19.48
Prince	Thunder Extreme Titanium OS	0.51	147	289	322	115	34.75	19.45
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Prince	ThunderLite LongBody OS	0.47	139	296	325	110	35.25	19.11
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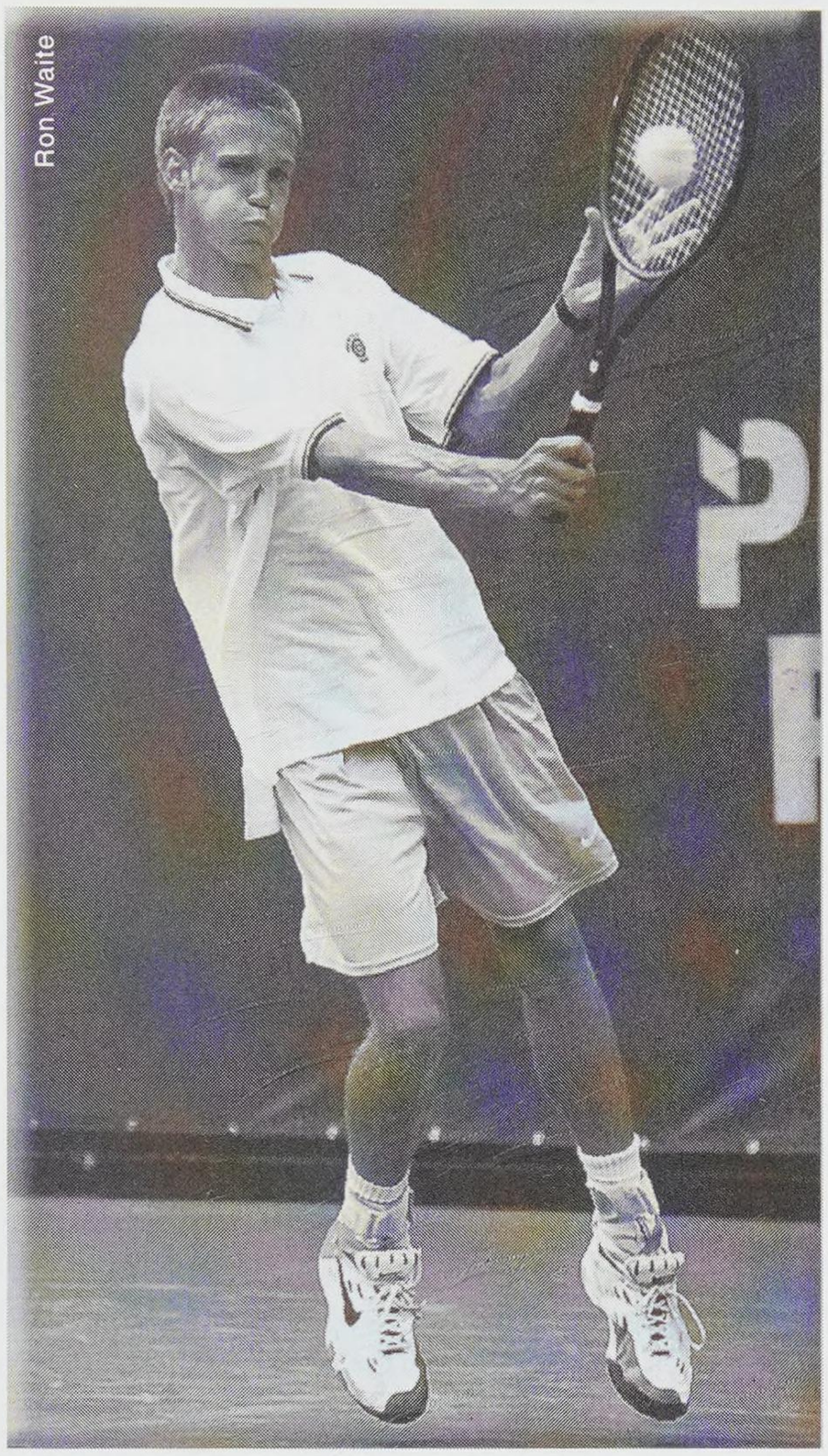
Gamma	Diamond Fiber C-3.0 MP	0.49	110	222	288	105	38.50	20.21
Prince	Thunder Superlite Titanium OS	0.47	101	213	266	115	38.00	19.87

*COP is distance from butt end. It is the impact location that causes a "no shock" point at the butt end of the handle during the



Jonathan Beardsley, United States

5

Measuring Swingweights

Do-It-Yourself Swingweight, Twistweight and Recoilweight

By Rod Cross and Crawford Lindsey

The weight of a racquet can be measured by placing it on a set of scales. The swingweight is different and can be measured only by swinging the racquet. This is typically done using a commercial swingweight machine like the Babolat RDC or Alpha Accuswing. But not everyone wants to invest in these. Hence, this chapter offers do-it-yourself techniques for determining the different kinds of swingweights (swingweight, twistweight, and recoilweight). A swingweight is different from weight as we usually know it. The resistance of an object to motion in a straight line is called its inertia and is measured by its mass (or its weight). The resistance to rotation is called rotational inertia and is measured by its moment of inertia with respect to a certain axis of rotation. Depending on which axis is chosen (perpendicular to the handle or balance point, or along the racquet's entire length), we call the moment of inertia the "swingweight," "recoilweight" or "twistweight" respectivly.

Ideally, one should swing the racquet about the actual axis of rotation of interest (near the end of the handle), but it is not easy to mount a racquet in this way. It is much easier to mount the racquet as a pendulum so that it rotates about an axis in the string plane. The swingweight about a parallel axis in the handle can then be calculated using the formula on the next page. (Note: The industry standard is to publish swingweights measured 4 inches from the butt end on the handle. This has become standard since that is where the commercial machines calculate the swingweight. However, using the parallel axis formula, you can calculate the swingweight about any parallel axis.)

Figure 5.1

Measuring Swingweight

As shown in Figure 5.1, a racquet can be supported vertically by resting one of the cross strings on two horizontal prongs about 6 inches apart. A suitable arrangement is to use two rods (about 1/4-inch diameter) placed on a table with weights on top so that each rod protrudes about 2 inches over the edge of the table. Pushing the handle gently allows the racquet to swing back and forth smoothly as a pendulum and it will keep swinging for several minutes if the rods do not not rub against any of the main strings.

If the racquet is supported on the bottom cross string, it will take about 3 seconds to complete one cycle of oscillation. This is quite a slow oscillation and the time can be measured accurately with a stopwatch. The racquet swings faster if it is supported on the top cross string or on a string near the middle of the stringbed. In that case, it is best to measure the time for 10 oscillations and then divide by 10 to get the time (T) for each oscillation. An interesting property of a pendulum is that the time for one oscillation does not depend on the size of the swing, provided the size is not too large. In practice, this means that you should keep the swing angle less than about 15 degrees each side of the vertical.

Swingweight Setup

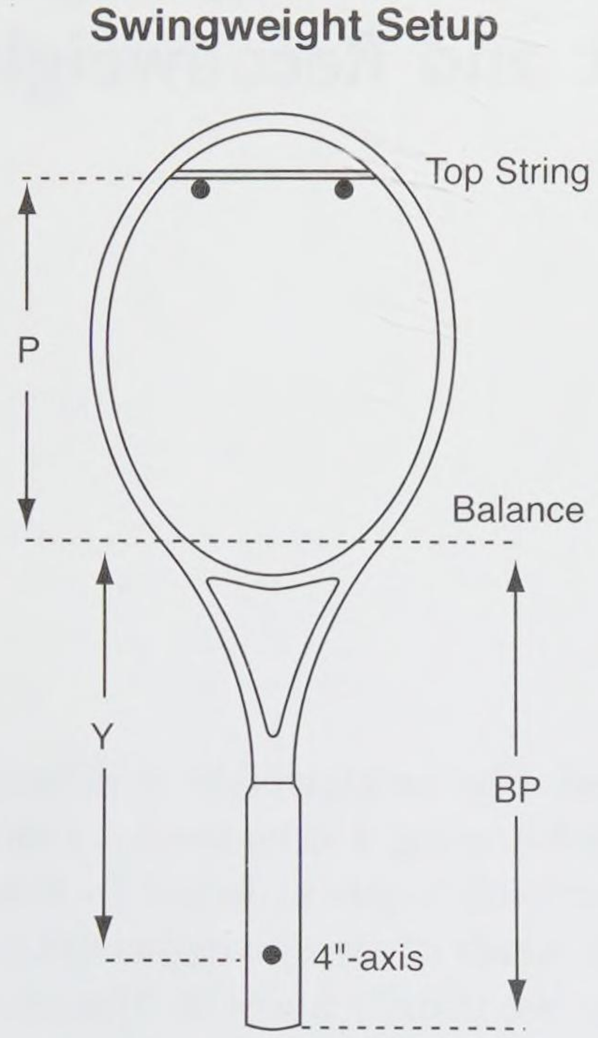


Figure 5.2

The formula for the swingweight is a bit complicated since the axis of rotation used to measure the swing time is not the same as the axis normally used to swing the racquet (Figure 5.2 and 5.3). The formula is:

$$\text{Swingweight} = M\left(\frac{PT^2}{40.28} - \frac{P^2}{1000} + \frac{Y^2}{1000}\right) \qquad (5.1)$$

in $kg{\cdot}cm^2$ where,

M = the mass of the racquet in grams.

T = time in seconds for one oscillation.

P = distance in cm from the balance point to the rotation axis (the bottom edge of the cross string used to support the racquet).

Y = BP - 10.16 = the distance in cm from the balance point to a point 4 inches (10.16 cm) from the butt end of the handle.

BP = the distance in cm from the balance point to the butt end of the handle.

Swingweight Axis

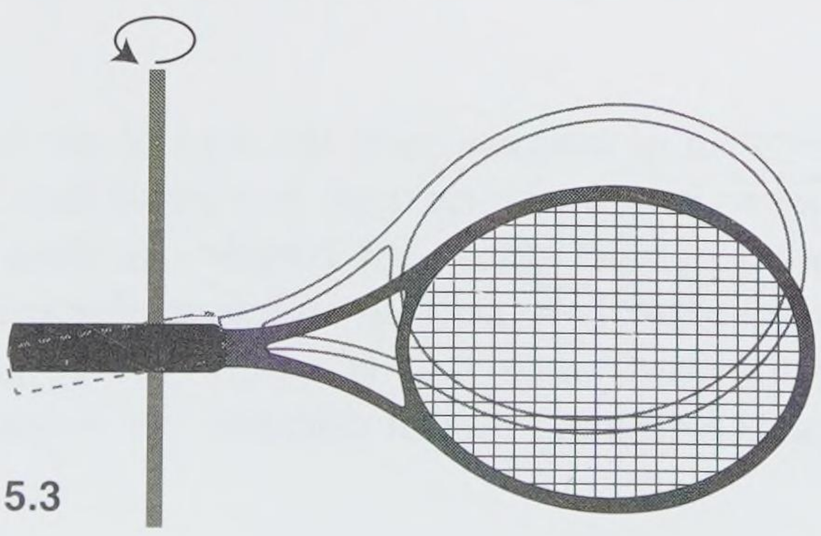


Figure 5.3

The above formula gives the swingweight in units of $kg{\cdot}cm^2$ for a racquet that is swung about an axis 4 inches from the butt end of the handle. This is the standard way of specifying the swingweight, or of comparing one racquet with another, even though the actual axis of rotation in normal use is beyond the end of the handle. A distance of 4 inches is used since racquets are normally positioned in swingweight machines so that the actual axis of rotation is 4 inches from the butt end.

Example

Measurements were made using a racquet with M = 291 gm, BP = 35.7 cm and Y = 35.7 - 10.16 = 25.54 cm. The time for ten complete cycles of oscillation back and forth on the bottom cross string was 32.72 sec for the first measurement and then 32.90, 32.72 and 32.78 sec for the next three measurements. The average time T for one cycle of oscillation was T = 3.278 sec. The distance P from the balance point to the point of support (the bottom edge of the bottom string) was 1.3 cm. These figures give

Swingweight = 291 x (1.3 x 3.278 x 3.278/40.28 - 1.3 x 1.3/1000 + 25.54 x 25.54/1000) = 291 x (0.3468 - 0.0017 + 0.6523) = 290.2 kg·cm^2.

In order to check if the answer is the same by swinging the racquet on a different string, the racquet was supported with the top cross string (nearest the tip) resting on the two prongs. The bottom edge of the top string was a distance P = 30.2 cm from the balance point. This time, the average period of oscillation was T = 1.323 sec. The formula then gives

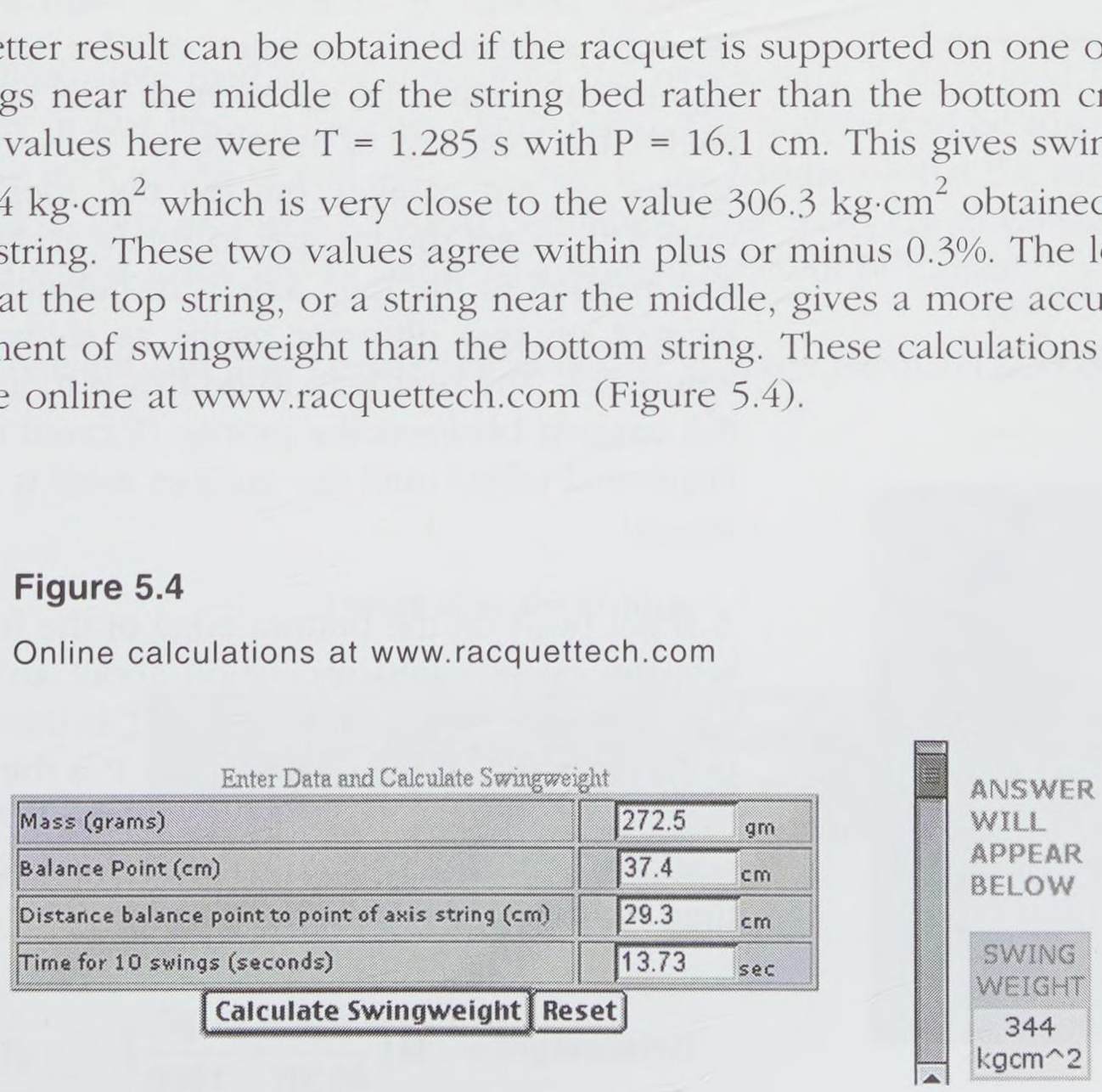
Swingweight = 291 x (30.2 x 1.323 x 1.323/40.28 - 30.2 x 30.2/1000 + 25.54 x 25.54/1000) = 306.3 kg·cm^2.

This is a bit higher than the previous value, but if we take an average, we get 298.1 plus or minus 7.9. This amounts to an error of plus or minus 2.6%. The two values should be exactly the same in theory, but small errors in measuring T and in measuring the distances contributed to the two different answers. A small error in measuring P has a relatively large effect when P is small. For example, if P was actually 1.4 cm in the first case rather than 1.3 cm, then the swingweight is 298 which is much closer to the second value. An error of 0.1 cm in P in the second case is not so important. For example, if P was 30.3 cm instead of 30.2 cm, then swingweight = 305.8 kg·cm^2.

A better result can be obtained if the racquet is supported on one of the cross strings near the middle of the string bed rather than the bottom cross string. The values here were T = 1.285 s with P = 16.1 cm. This gives swingweight = 308.4 kg·cm^2 which is very close to the value 306.3 kg·cm^2 obtained using the top string. These two values agree within plus or minus 0.3%. The lesson here is that the top string, or a string near the middle, gives a more accurate measurement of swingweight than the bottom string. These calculations can all be done online at www.racquettech.com (Figure 5.4).

Figure 5.4

Online calculations at www.racquettech.com



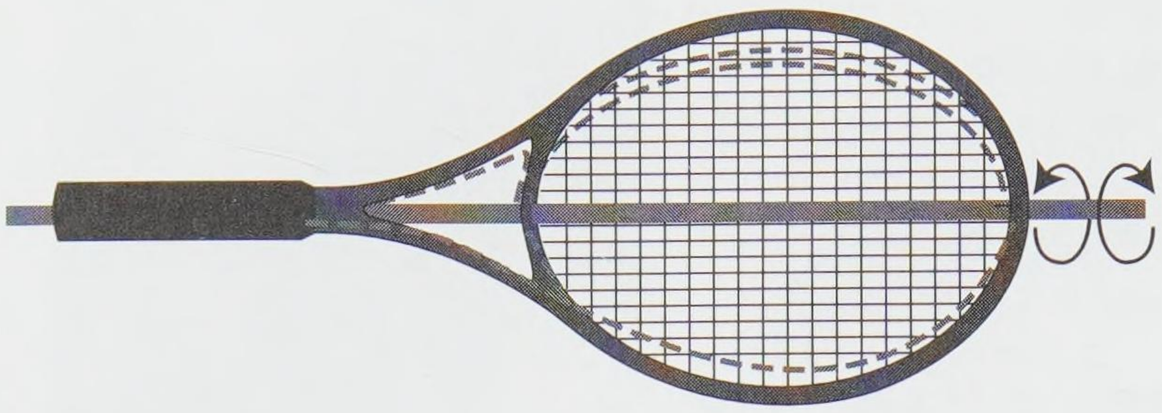


Figure 5.5

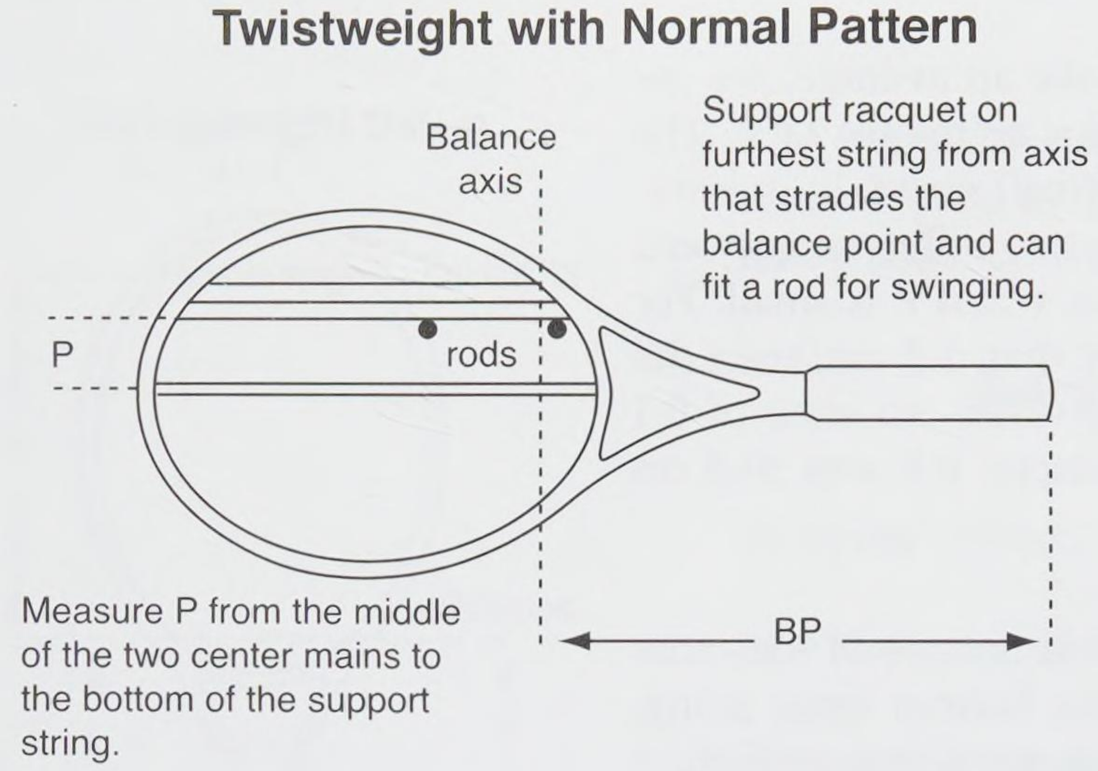


Figure 5.6

Figure 5.7

Measuring Twistweight

The above method is easily extended to include a measurement of the polar moment of inertia. This is just a technical name describing the resistance or inertia of a racquet for rotation around its long axis (Figure 5.5). If a racquet strikes a ball towards the top or bottom edge of the frame, then the racquet will tend to twist in the hand and the ball will come off the strings at the wrong angle. The amount of twist will be small if the racquet has a large polar moment of inertia. By analogy with swingweight, the polar moment can be called the "twistweight." Extra mass at the 3 and 9 o'clock positions on the frame is often used to increase the twistweight.

In theory, all that needs to be done to measure the twistweight is to rotate the racquet through 90 degrees, replace it on the prongs resting on a main string, and measure the time for 10 oscillations. In order to get the racquet to balance on the prongs in a horizontal position, both prongs must be located on opposite sides of the balance point (Figure 5.6). It is not necessary that the prongs be an equal distance on either side, but the balance point must be somewhere in the throat area of the strings and not outside the stringbed. If the main strings are all parallel, then the same string must rest on both prongs. If necessary, the main string used to support the racquet can be moved slightly to ensure that the rotation axis is parallel to the long axis of the racquet. If the main strings are not parallel, but fan out, then the only way to get the racquet to rotate about an axis parallel to the long axis is to balance the racquet on two different mains as shown in Figure 5.7. The best way to do this is to adjust the spacing between the prongs (located on a horizontal table) until the racquet itself is horizontal.

A slight push on the bottom edge of the frame sets the racquet into oscillation about an axis that is parallel to the long axis. If T is the time in seconds for one oscillation, and P is the distance (in cm) from the long axis to the bottom edge of the strings used to balance the racquet, then twistweight is calculated as:

$$\text{Twistweight} = M\left(\frac{PT^2}{40.28} - \frac{P^2}{1000}\right) \qquad (5.2)$$

The best string to use is the one that is furthest from the axis but still allows the prongs to be inserted on each side of the balance point. The Wilson Triad 3.0 uses a fan pattern and can be balanced on the 6th and 7th main strings from the middle, at P = 5.90 cm. In this case, T = 0.767 s, giving twistweight = 15.0 kg·cm^2. On the 5th and 6th strings, P = 4.95 cm, T = 0.791 s and twistweight = 15.0 kg·cm^2, exactly the same as the first measurement. A sawn-off ruler is helpful in getting a good measurement of P. If the ruler is too long it extends over the frame and it doesn't sit nicely on the string plane.

On some racquets, the balance point lies outside the string plane. In that case, a solution is to suspend the racquet horizontally using two lengths of string tied to a horizontal support frame as shown in Figure 5.8. The formula for the twistweight is the same as before, but P is the distance from the long axis of the racquet to the actual axis of rotation near the top end of the string.

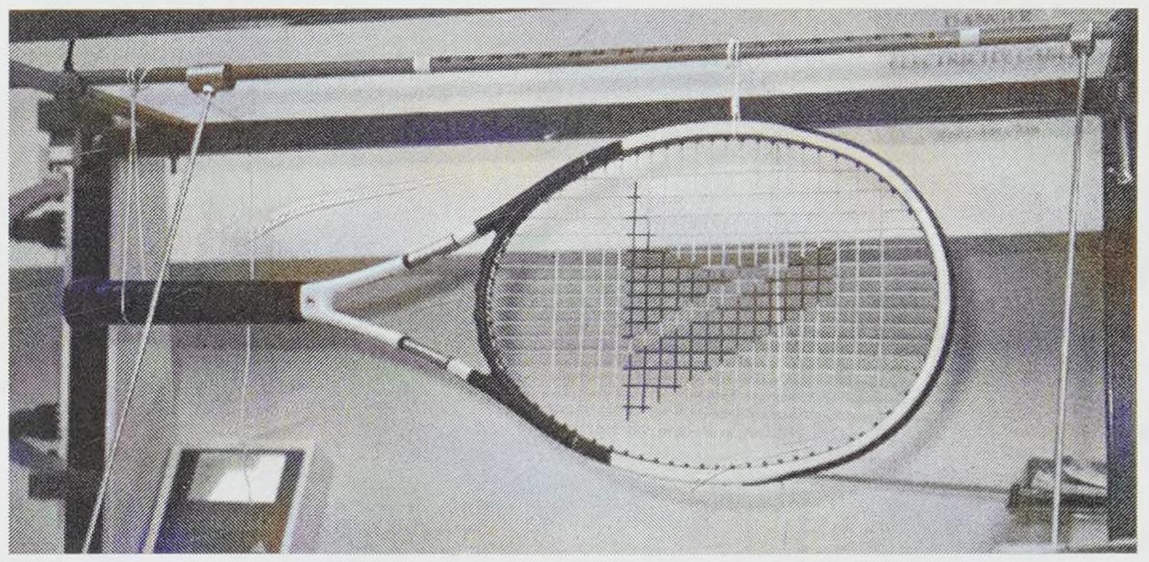


Figure 5.8

The twistweight of a racquet is a lot smaller than its swingweight. The twistweight can be increased by adding lead tape of mass M at 3 o'clock and additional tape of mass M at 9 o'clock. If the tape is located a distance R (in cm) from the long axis (passing through the middle of the handle) then the twistweight will increase by MR^2/500 (in kg·cm^2) where M is in gm. For example, if M = 10 gm and R = 14 cm then the twistweight increases by 3.9 kg·cm^2. This amounts to about a 30% increase for most racquets.

An alternative method of measuring the twistweight is described by Howard Brody in The Physics Teacher (April 1985). The racquet can be suspended vertically by a length of stiff wire attached to the handle so that the racquet twists about its long axis as a torsional pendulum. It is necessary to calibrate the system using a metal rod or bar with a known moment of inertia.

Twistweight can also be calculated online at www.RacquetTECH.com (Figure 5.9).

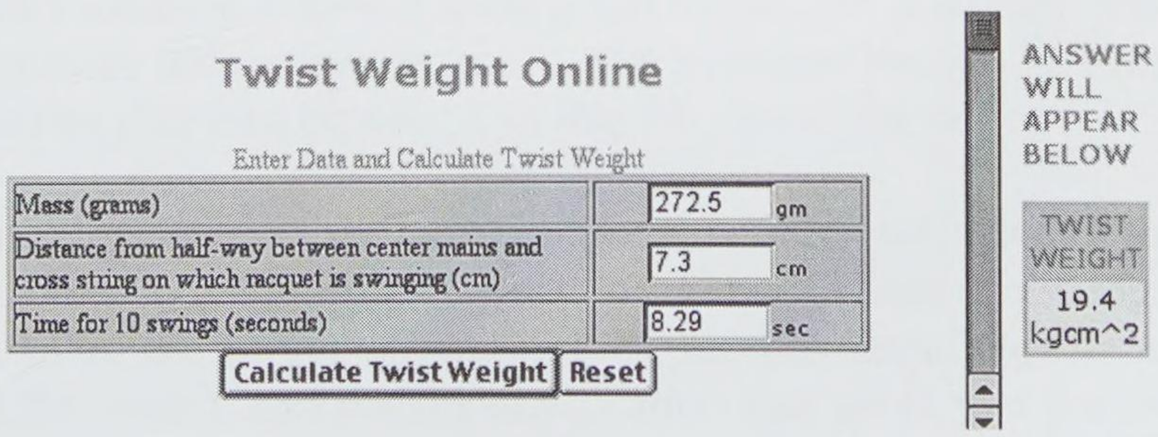


Figure 5.9

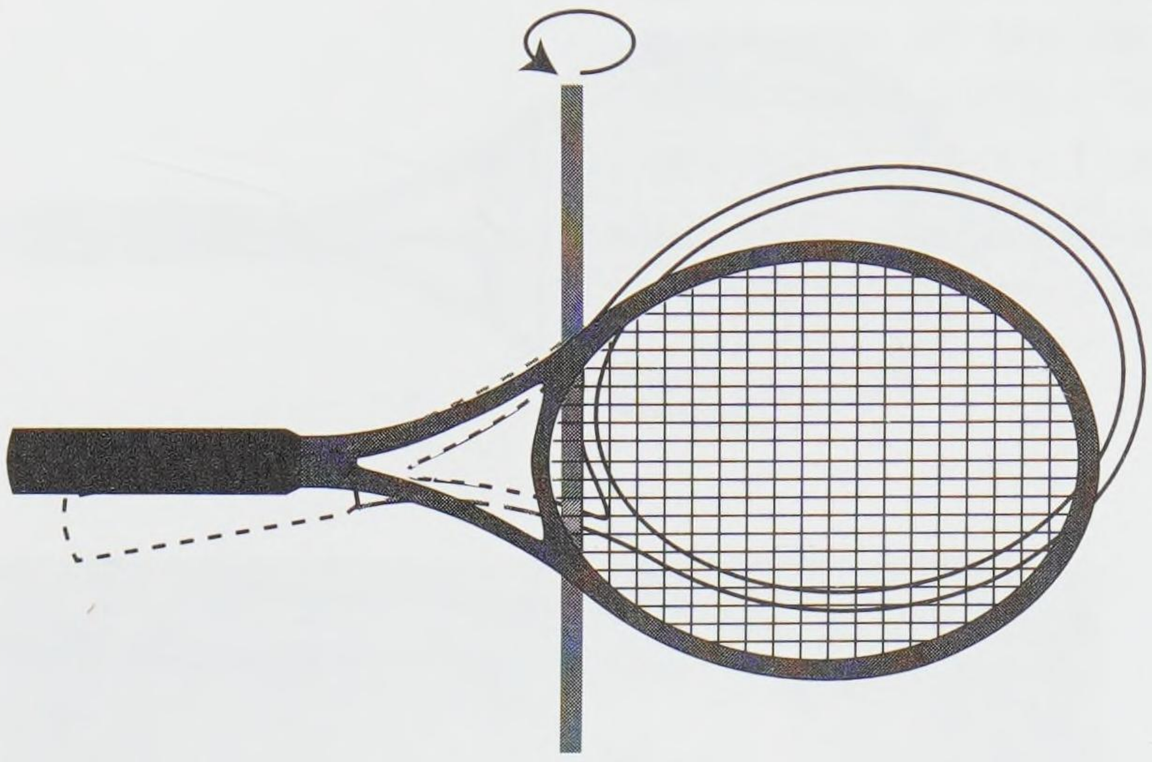


Figure 5.10

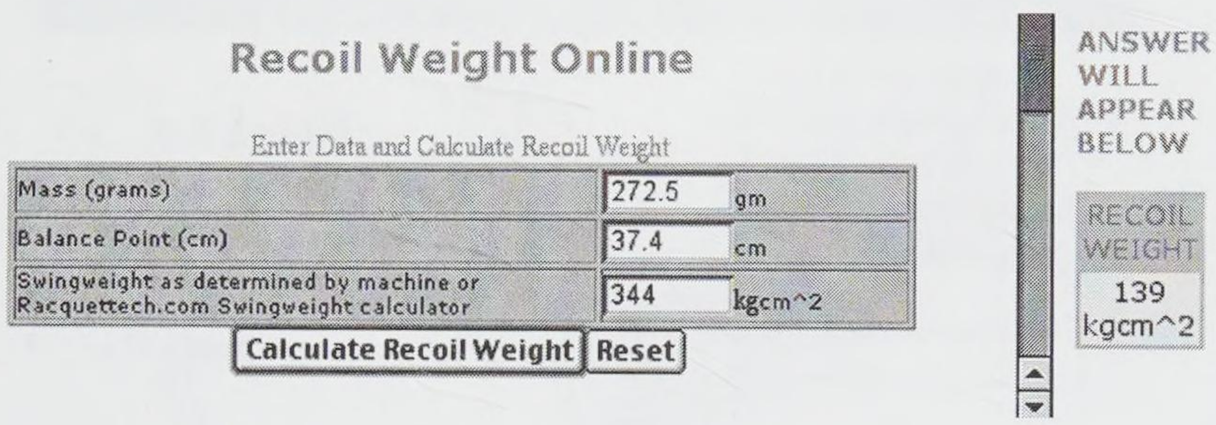


Figure 5.11

Measuring Recoilweight

Recoilweight is the swingweight around the balance point or center of mass (Figure 5.10). Once you have the moment of inertia about any axis (in this case we already have the swingweight about the 4-inch point on the handle), you can determine the moment of inertia around any other axis parallel to the swingweight axis. The formula is the parallel axis formula and its general form is as follows:

$$I = I_{cm} + MY^2 \qquad (5.3)$$

where:

I = moment of inertia (swingweight) about axis parallel to the axis of the balance point.

I_{cm} = recoilweight (swingweight about the center of mass or balance point).

M = mass of racquet in grams.

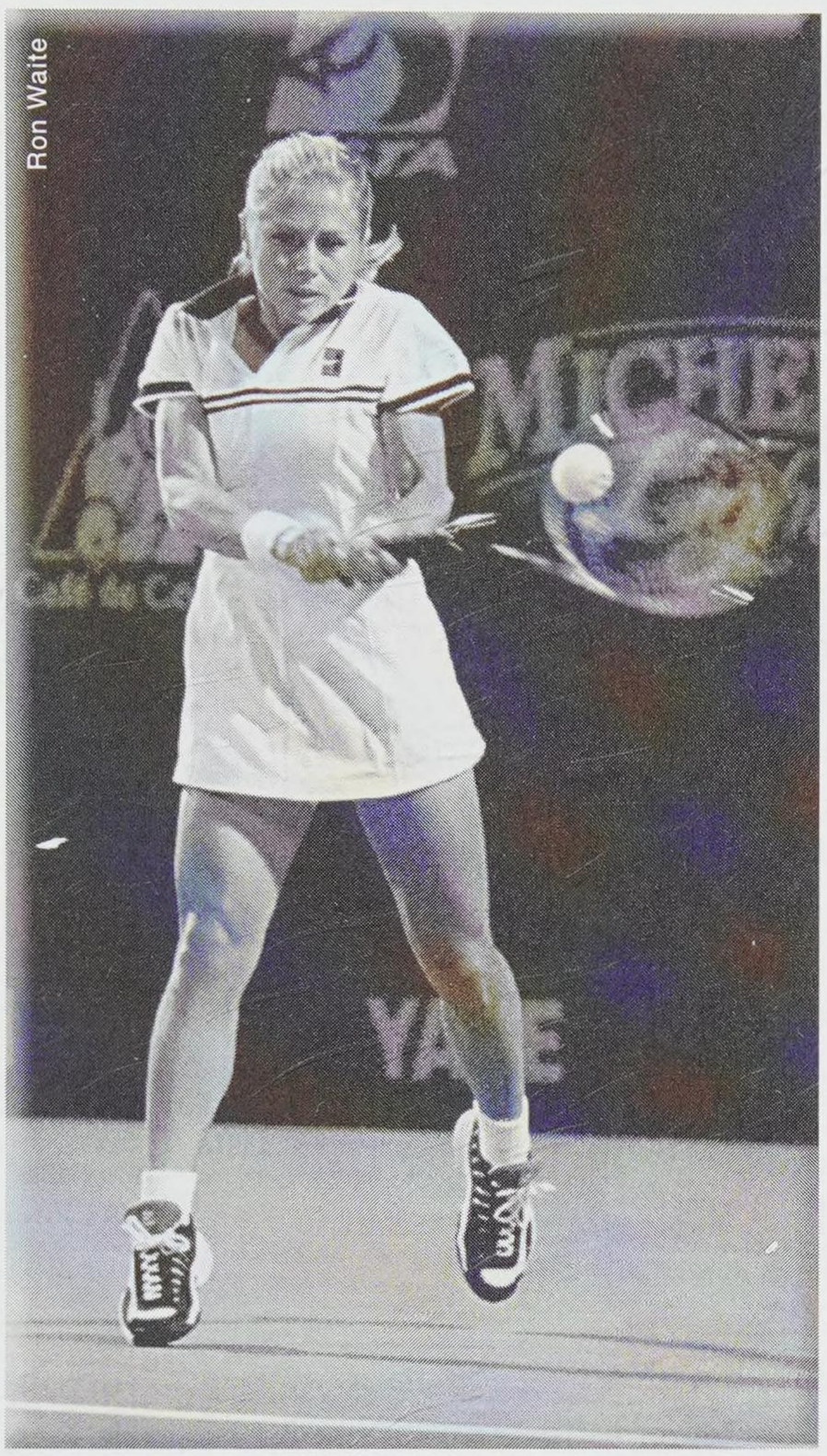
Y = distance in cm from swingweight axis (10.16 cm from butt end) to the balance point.

So, since the recoilweight is the moment of inertia about the cm, we solve for I_{cm}

$$\text{Recoilweight} = \text{Swingweight} - MY^2 \qquad (5.4)$$

The units are $kg \cdot cm^2$.

The recoilweight can also be calculated at www.racquettech.com (Figure 5.11).



Amanda Coetzer, South Africa

6

Sweetspots

The COP, Node, Power Point, and Dead Spot

By Howard Brody, Rod Cross, and Crawford Lindsey

One of the terms often used in the tennis world is "sweetspot," particularly by racquet manufacturers in advertising their products. What is a sweetspot? Where on the racquet is it located? Can one location be sweeter than another? Are some sweetspots bigger than other sweetspots? How can you find the sweetspot on your own racquet?

Ask a tennis player what or where a sweetspot is, and the answer you will probably get is, "The sweetspot is the ball impact location where it feels good to me," or possibly, "It's the location where the ball seems to fly off of the strings with little effort by me." The problem then is to turn these "touchy-feely" definitions into measurable quantities that can be tested in the lab or on the court.

When the problem is analyzed, it turns out that there are actually three separate sweetspots on a tennis racquet (Figure 6.1). They are the impact location of the ball on the strings where the shock or jar your hand feels is minimized (called the center of percussion), the ball impact location that produces the least vibration of the racquet (called the node), and the impact location that gives you the most return ball speed or "power" (where the coefficient of restitution is a maximum). It is clear that each of these sweetspots, in one way or another, fits or agrees with the player response quoted above. If the ball impacts at a point well away from the sweetspots, the impact jars. The maximum power point is not a jar-free location, but it delivers the sweet feel of "oumph."

Spots on a Racquet

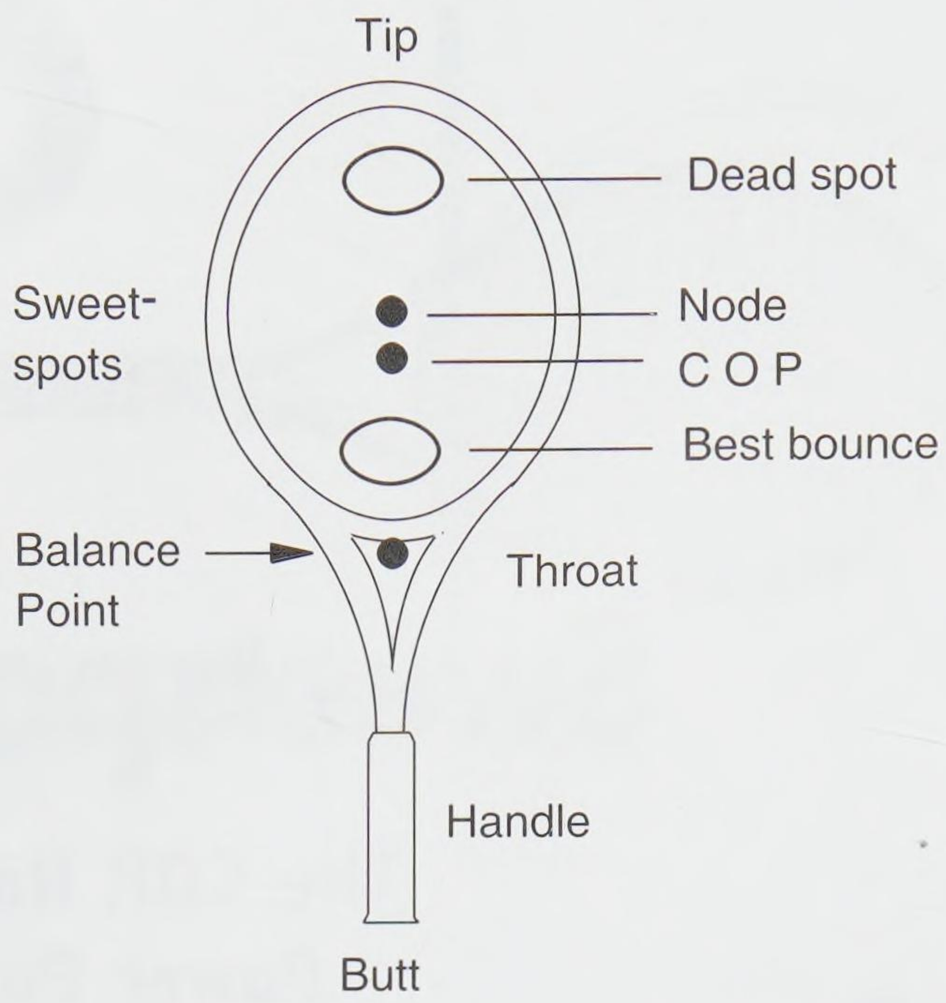


Figure 6.1
The ball bounces best near the throat only if the racquet is initially at rest.

How a Racquet Recoils

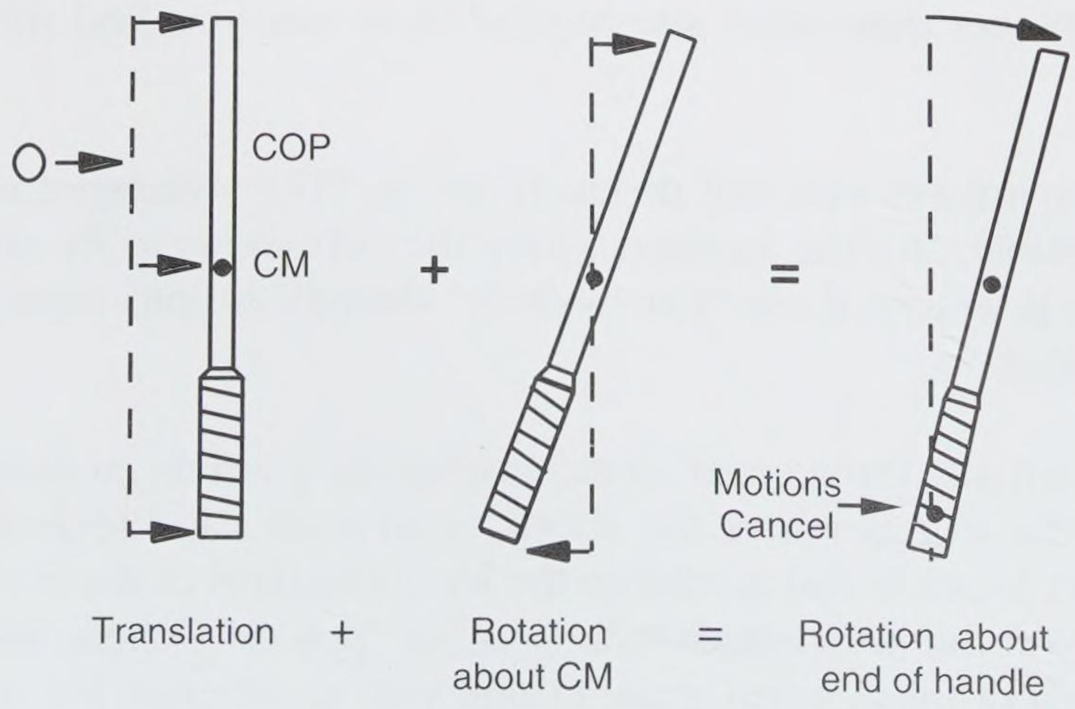


Figure 6.2

The Center of Percussion

Sometimes, when you hit the ball, there is a sharp shock, or jar, to your hand and wrist. Sometimes you feel almost no sensation at all due to the ball impact on the racquet's strings. What is it that determines whether you will have pain or pleasure?

Suppose that the racquet is initially at rest. If a ball strikes a racquet at the racquet balance point (also known as the center of mass, CM, or center of gravity) and then rebounds, the entire racquet will recoil in the direction of the original ball motion in order to conserve linear momentum. All parts of the racquet will recoil at the same speed V. The faster the ball is moving, the faster will be the recoil. Since the CM, or balance point, is usually in the throat of the racquet, you will probably not want to hit the ball there. If the ball strikes the racquet well above the balance point, near the center of the head (a more likely location to hit a ball), the racquet will still recoil to conserve linear momentum, but now it will also twist or rotate about the CM or balance point in order to conserve angular momentum (Figure 6.2).

This rotation will cause the head of the racquet to recoil backwards in the original ball direction, and the handle to rotate forward. The actual motion of the racquet will be the sum of both of these effects. Since the motion of the entire racquet due to linear momentum conservation is backward, and the motion of the racquet handle or grip due to angular momentum conservation is forward, they are in opposite directions. The whole racquet then moves away from the ball with a speed V_1 due to the recoil, but the handle simultaneously moves towards the ball with speed V_2 due to rotation of the racquet. If there is any point in the handle where $V_1 = V_2$, then that point will remain stationary and the rest of the racquet will rotate about that point as shown in Figure 6.2.

Therefore, it is possible for a particular ball impact location on the strings to have the two motions exactly cancel at the point where the hand holds the racquet. If this were to happen, and the two motions cancelled, then the shock or jar that the hand experiences due to the ball impact on the strings would be minimized.

The particular ball impact location on the strings where the two motions cancel at the hand is called

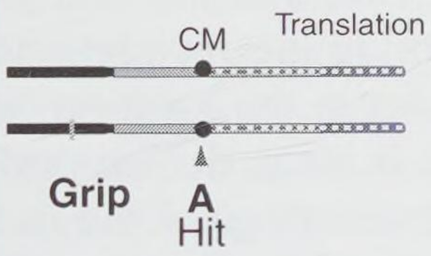
the center of percussion (COP), and its location depends upon how the weight is distributed along the racquet as well as where the hand is located on the handle. The axis of rotation is called the conjugate point with respect to the impact point, and the impact point is called the center of percussion (COP) for that particular axis of rotation. The axis and the COP form a pair of conjugate points. For an impact near the tip of the racquet, the axis of rotation is about half way between the end of the handle and the CM. For an impact near the throat of the racquet, the axis of rotation is beyond the end of the handle.

If the ball hits the strings above the COP point (closer to the tip of the racquet), the racquet handle will try to come out of your hand by causing a force on your fingers. If the ball hits the strings between the COP and the throat, there will be a sharp force pushing on the palm of your hand. It is clear that if you hit the ball at or very close to the COP point that corresponds to where you are holding the racquet, you will feel less shock or jar from the ball impact. You will have hit that sweetspot (Figure 6.3).

Related Pairs: COPs and Pivot Points

Every hitting location A has a related pivot point B. And vice versa—every gripping point B is a pivot point for some hitting point A. If you hold the racquet about 4" from the end, there is a point in the racquet face that when hit, will cause no rotation into or out of the hand. That hitting point is the center of percussion (COP) for that gripping location. Because the handle does not slam into or pull out of the hand here, the COP is called a sweetspot.

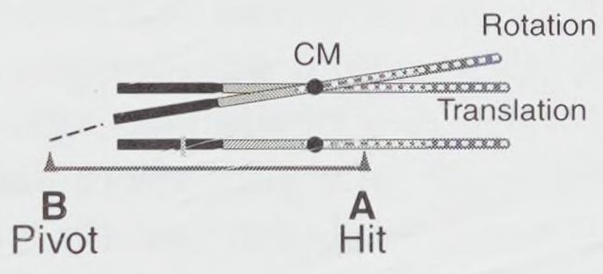
CASE 1
Hit at CM



No COP

Hitting at CM causes only backward motion. There is no off-center torque in this case. The racquet goes straight back into the hand.

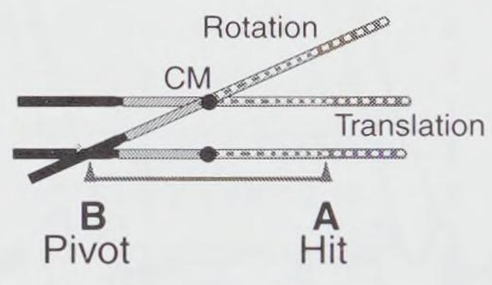
CASE 2
Hit near CM



A = COP of B

Hitting at A, close to CM, causes the racquet (at your gripping location) to move backward faster than it rotates. The result is a pivot at B beyond the end of the handle. You'll feel some pressure of the handle into your hand.

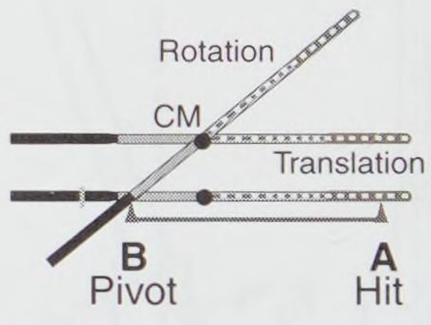
CASE 3
Hit middle of face



A = COP of B

Hitting at A, near the center of the strings, causes the racquet (at your gripping location) to translate backwards and rotate forwards at the same speed. The result is a pivot at B in the grip right near the base of your index finger. The handle will be motionless at this spot.

CASE 4
Hit near tip



A = COP of B

Hitting at A, near the tip, causes the racquet (at your gripping location) to rotate faster than it translates. The result is a pivot at B closer to the CM. The handle will try to pull out of your hand.

Notes: (1) The amount and speed of translation is the same regardless of the impact point. It is only the amount of rotation that varies since the torque increases from case 1 to case 4. (2) In Case 3, if the impact point A moves closer and closer to the CM, the pivot point will move to different points under the hand and ultimately to the end of the racquet. Besides the base of the index finger, the end of the racquet is most often seen as the most desireable spot for the pivot to occur. Regardless of the impact point, the racquet will rotate and tend to bend the wrist backward. Soon after the initial impact, the hand takes over from the ball as the primary force on the racquet, with the result that the axis of rotation shifts to the wrist immediately after the ball leaves the strings.

Figure 6.3

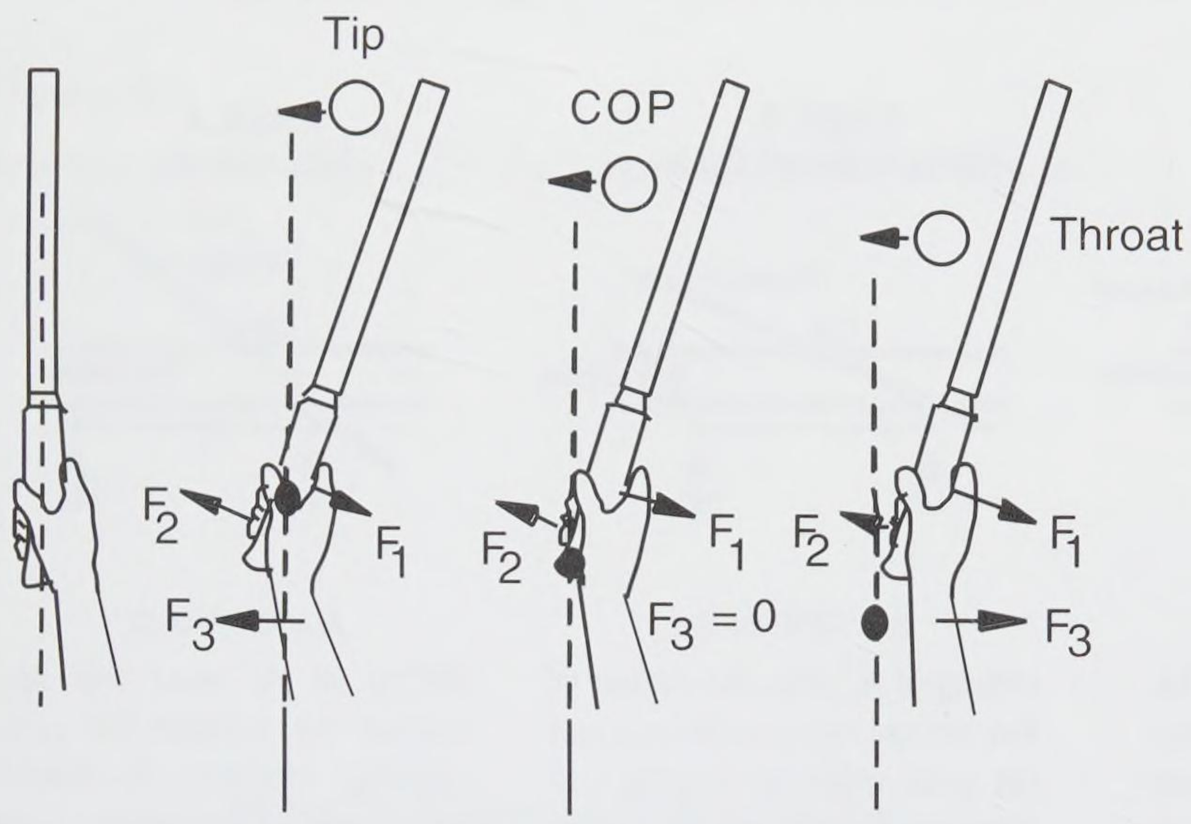
It is important to understand that this COP pivot is momentary. For a brief moment during impact, the racquet is rotating or pivoting around a point on the handle and not around the CM. Pivoting implies a stationary position while the rest of the racquet moves around it. But this pivoting about the handle is

just a snapshot in time. But it lasts just long enough that if you're holding the racquet there, the impact will involve a minimum jolt to your hand. After a few milliseconds, the racquet would be rotating about the CM again, if it weren't for your hand restraining it and relocating the axis to your wrist after impact.

It seems that no matter where the ball strikes the strings, there may well be a point under the hand where the force is zero, but there will always be other points where the force is not zero. In fact, that is exactly what is measured. The impact causes the racquet head to recoil, so the whole racquet rotates in the hand, exerting a force on the upper part of the hand and a force in the opposite direction on the lower part of the hand. The torque causes the hand to rotate about an axis through the wrist. However, if the impact is at the COP for rotation about the end of the handle, then the force on the upper part of the hand is equal and opposite the force on the lower part of the hand, so there is no net force on the hand or the forearm. Therefore, the forearm will not receive a sudden jolt if the ball impacts at this COP (Figure 6.4).

Figure 6.4.

F_1 is the force on the upper part of the hand, F_2 is the force on the lower part of the hand, and $F_3 = F_1 + F_2$ is the net force on the forearm.



The location of the COP on the head of the racquet does not depend on how hard you hit the ball or the type of stroke you are using. It is a property of the racquet frame itself and where you are holding the racquet. If you move your hand up and grip the racquet higher on the handle (choke up), the corresponding COP point will also move upwards on the head.

You can determine the location of the COP of your own racquet by turning the racquet into a pendulum, with the pivot at the location on the grip where the hand is located. The period (the time for a complete swing) of the racquet-pendulum is obtained by allowing the racquet to swing through a full cycle. The distance between the pivot point and the COP location (in inches) is given by

$$\mathbf{COP = 9.78 \times T^2} \qquad \mathbf{(6.1)}$$

where T is the period of the racquet-pendulum (in seconds). If it takes the racquet-pendulum 1.25 seconds to swing through a full cycle (T = 1.25 sec), then the distance from the pivot point to the conjugate COP point where you want to hit the ball is 9.78 x 1.25 x 1.25 = 15.3 inches.

There's another quick method to determine the COP location. It is to use the following formula:

$$\mathbf{x = I_{cm} / (Mb)} \qquad \mathbf{(6.2)}$$

where

I_{cm} = the moment of inertia about the CM with units of $kg \cdot cm^2$,

b = the distance in centimeters from the impact point to the CM, and

M = mass in kilograms of racquet,

x = distance from conjugate point to CM. The conjugate point is usually taken at the butt end of the handle or at some point underneath the hand.

Another way to approximately locate the COP is by holding the end of the handle between your finger and thumb and throwing a ball onto the strings. If the handle jumps out of your hand, then you missed the COP. It is usually located about 5 cm away from the center of the strings, as shown in Figure 6.1. A more accurate measurement of the COP can be made with a piezo disk (extracted from a piezo buzzer) between the thumb and the handle to measure the force acting on the thumb.

Assuming the base of your index finger is about 3 to 4 inches from the butt end of the racquet, the COP for a typical, old, classical, wooden racquet came out near the throat of the frame, and not near the center of the head, where players tend to hit the ball. With the introduction of the oversize head, the COP point came out close to the center of the head. This was not because the COP was a different distance from the hand, but because the center of the enlarged head has been moved down toward the racquet handle. It is also possible to raise the COP location higher in the head by adding weight to the tip of the racquet, or by removing weight from the throat area.

The Node

A tennis racquet, even when made with the most advanced composite material, is not a completely rigid body — it can flex. When the racquet hits the ball, the frame deforms, then snaps back, and it vibrates. The principal mode of vibration of a hand-held racquet is shown schematically in Figure 6.5. Note that in the fundamental mode the racquet vibrates so that it has a large amplitude (big oscillation) in the middle and at each end. Since you hold the racquet at the end, you will feel those vibrations. The other dotted lines are the next highest mode of vibration. This mode is not stimulated by a tennis ball, but might be if you hit your racquet on a hard court or tap the frame with a hammer. Looking at Figure 6.5, you can see that there are also two locations along the racquet where the racquet vibrating in the fundamental mode has no amplitude. Those locations are called nodes.

If the racquet is not struck at one of the two nodes, the frame will vibrate. The amount of vibration depends how hard the ball is hit, how flexible the racquet is, and how far from the node the ball strikes. When a racquet vibrates, most of the vibration occurs at the tip, the throat, and at the butt end of the handle. There is a spot near the middle of the strings that doesn't vibrate at all. It is actually not a spot, but a curved line extending from the middle of the strings to points at about 2 and 10 o'clock on the frame (Figure 6.6). The spot that most people talk about is the intersection of this curved line with the long axis of the racquet. It turns out that if a ball hits this spot (or anywhere else along the curved

Vibration of a Uniform Beam

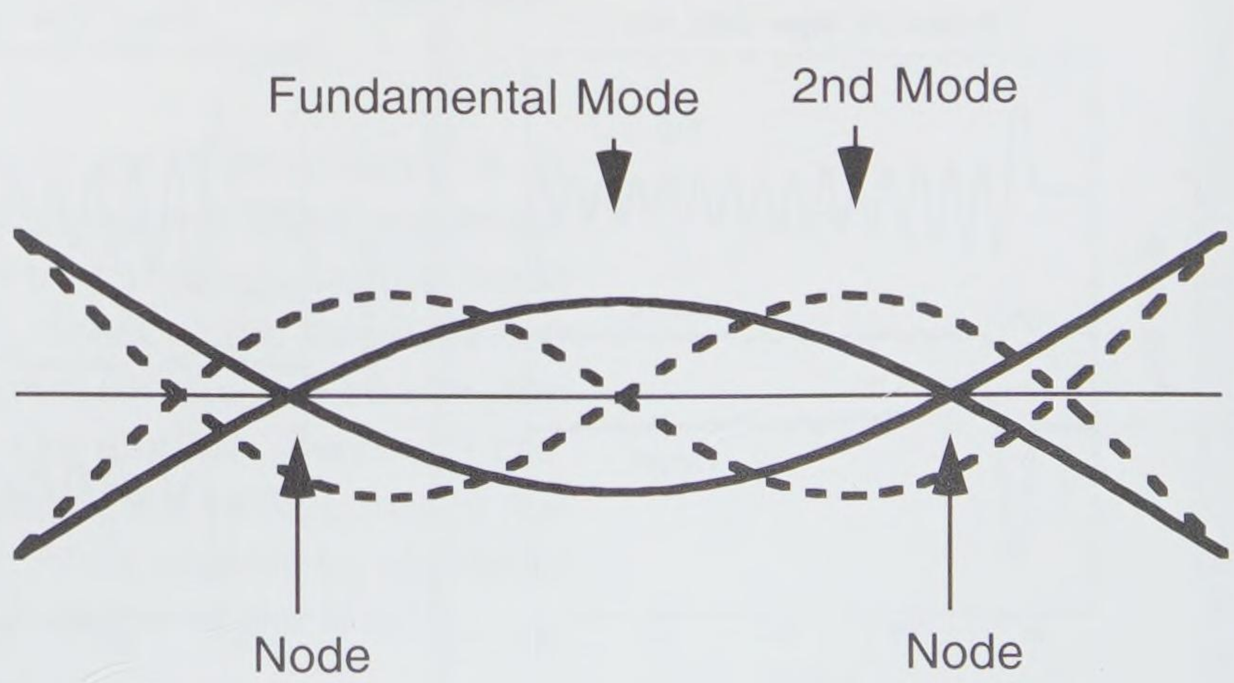
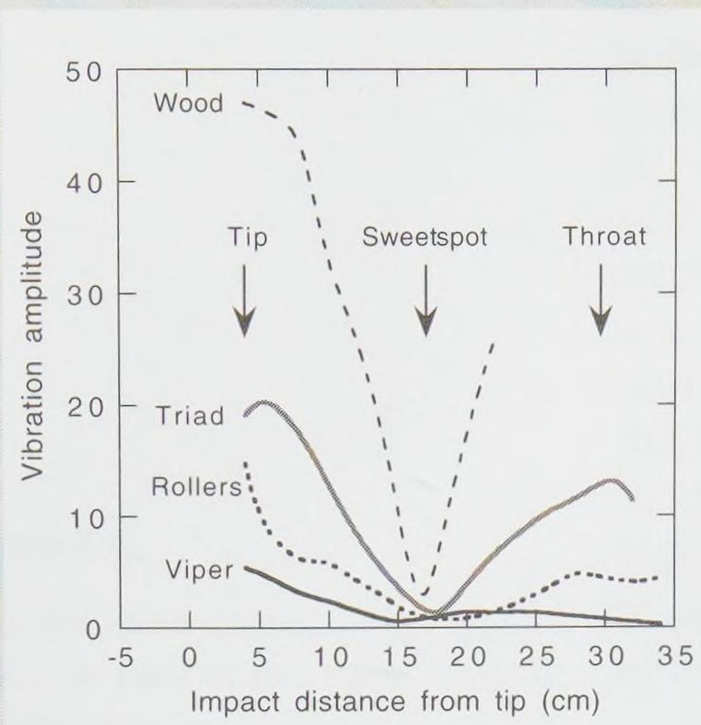


Figure 6.5

Vibrations at Different Locations on Racquet

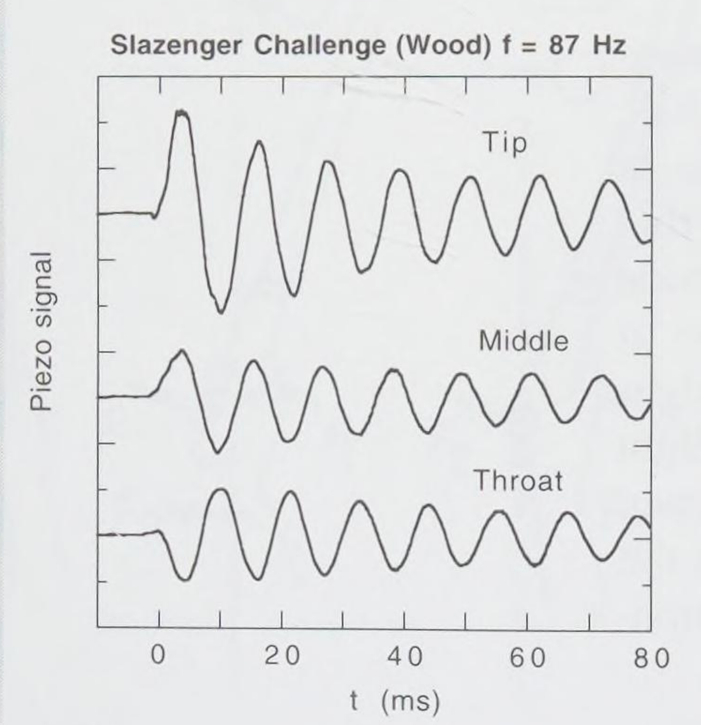
Typical voltage signals from the piezo are shown in Figure 6.7a, for impacts at the tip, middle, and throat of (a) an old Slazenger wood racquet, (b) a Wilson Triad 3.0 racquet, and (c) a Prince Viper racquet. Each of the racquets was suspended freely from a horizontal beam by two lengths of string so that it was free to rotate and vibrate. A tennis ball was also suspended from the beam, as a pendulum, so that it could swing towards the stringbed and impact at the same speed on every racquet and at the exact spot we wanted.



6.7a

The results in Figure 6.7a are typical of all racquets in that (a) the largest vibrations of the handle occur for impacts at the tip and throat of a racquet, and (b) the smallest vibrations occur at the sweetspot near the center of the stringbed. The initial motion of the handle is either towards the incoming ball for an impact near the tip, or away from the incoming ball for an impact near the throat. This can be seen from the piezo traces where the first part of the signal is positive for an impact near the tip and negative for an impact near the throat.

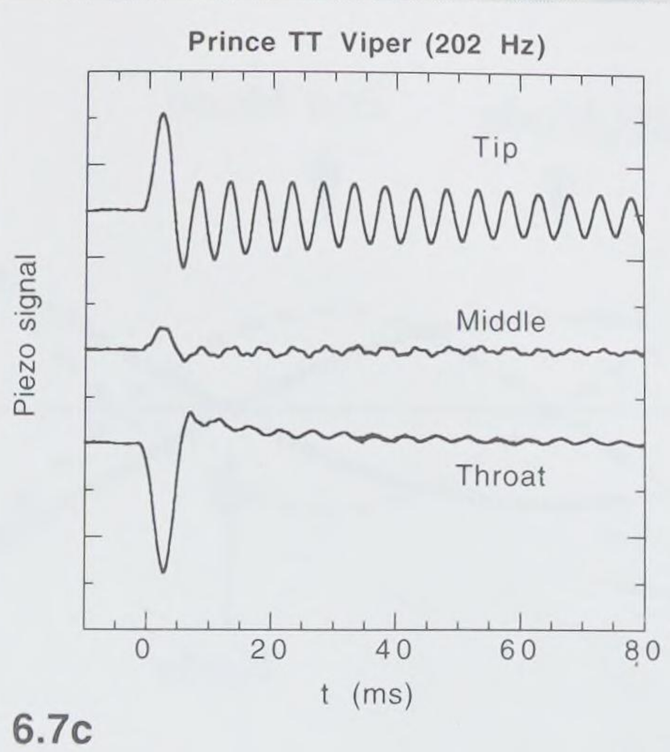
There is another sweetspot near the first, called the center of percussion (COP). If a ball impacts at the COP, then the racquet rotates about an axis through the butt end of the handle. In that case, the butt end does not move towards or away from the incoming ball. The node sweetspot results in minimum vibration of the handle. The COP sweetspot results in minimum sudden motion of the handle and hence minimum shock. The first spike in waveforms is the intial acceleration of the handle due to handle rotation and translation. The rest of the spikes are the vibrations at the handle. In these examples, we missed hitting exactly at the node or COP. That's because the impact locations were preset to be the same for each racquet.



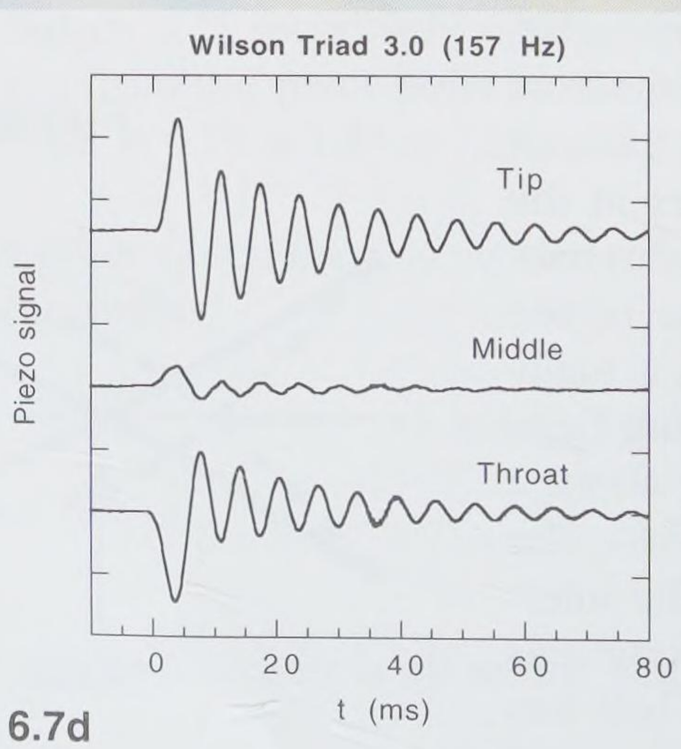
6.7b

The four racquets shown in Figure 6.7a were chosen because they each illustrate some interesting differences between racquets (Figure 6.7b-6.7d). Old wood racquets vibrate a lot more than modern graphite racquets and they vibrate at a lower frequency. Both of these effects are due to the fact that old wood racquets are heavier and more flexible than graphite racquets. The Triad is interesting because the vibrations are strongly damped by a rubber Iso-Zorb strip glued between the head and the handle. Additional strong damping occurs in all racquets when the handle is held by hand. Damping doesn't mean that the first vibration cycle is significantly reduced in amplitude. Damping refers to the fact that the subsequent vibrations get smaller quicker, and it may also act to reduce the amplitude of the first vibration to some extent.

The Viper had the smallest vibrations of all the racquets tested. It was also one of the stiffest and it vibrated at the highest frequency (202 Hz with the wood block attached or 227 Hz without the wood block). Frame vibrations are almost completely absent in this racquet for an impact near the throat. The single large spike in the piezo waveform is due to the sudden acceleration of the handle when the ball strikes the strings. That cannot be avoided and it therefore occurs in all racquets. It results in shock, but no vibration in the Viper (or in any other racquet that vibrates at 200 Hz or above). With other racquets, the effects of rotation and vibration merge together right from the beginnining of the impact, increasing the overall acceleration of the handle and the shock felt by the hand.



6.7c



6.7d

line), then the racquet will not vibrate at all, even at the tip or the butt end. This spot is therefore a sweetspot, technically known as a vibration node. If the ball hits the strings, say one inch away from this spot, then the racquet will vibrate a small amount. The further away from the spot, the more the racquet vibrates. If the ball impacts at a point on the node line near the 2 or 10 o'clock position, then the frame will not vibrate but it will twist in your hand. Consequently, the sweetest point on the cuved node line is the one that intersects the long axis.

However, when a ball strikes the racquet at the node, that mode of oscillation does not occur, and the hit feels sweet. The vibrations of a hand-held racquet frame have a frequency in the range of 100 to 200 hertz (vibrations/second), depending on the frame stiffness, frame length, and the racquet weight. Stiffer frames and lighter racquets tend to vibrate at a higher frequency (more oscillations per second). You can determine the location of the node of your own racquet without fancy electronic equipment by holding the racquet between two fingers at the approximate location of the node in the handle (the top of the grip) and striking the strings at various places with a ball held in your other hand. You will be able to feel which impact locations cause vibrations and which location on the strings does not cause the frame to vibrate. That point will be the node.

Figure 6.6

Curved node line. An impact anywhere on the line will not result in vibration.

Much design effort has gone into both reducing vibrations and damping out vibrations. A stiffer racquet will have a smaller amplitude of vibration for the same off-node ball impact. Various methods have been tried to damp out those vibrations, which do occur, and the best one seems to be the human hand. In fact, the tighter the racquet handle is gripped, the quicker the vibrations damp out (with more of the energy of the racquet vibration going into the hand and arm).

The small, soft objects that are placed in the stringbed close to the throat with the hope of reducing or eliminating vibrations, do essentially nothing to prevent, reduce, or damp out vibrations of the frame of the racquet. However, they do quickly kill off the 500 to 600 hertz string vibrations. (They turn the "ping" that you hear when the ball hits into a "thud" sound.) The energy contained in string vibrations is quite small compared to the energy in frame vibrations, because the frame has about twenty times the mass of the strings.

It is obviously quite desirable to hit the ball at the node of the frame, or as close to it as possible, to eliminate or minimize the subsequent annoying vibrations of the frame. With the old, small headed, wooden racquets, the node location tended to be below the center of the head, closer to the throat. When racquet heads were enlarged by extending them down toward the handle, the center of the head was moved closer to the hand. On modern, oversized racquets, the node is now located at or above the center of the strung area of the head. It is also possible to move the location of the node slightly by changing the stiffness or mass distribution of the frame or the shape of the head.

While vibrations are annoying and bothersome, there is no clinical evidence that vibrations of either the frame or the strings can cause any type of hand,

wrist, and arm or elbow damage. There is some experimental evidence (wear pattern on the strings, high speed photography, etc.) that on groundstrokes, good tennis players tend to hit the ball at the location of the node of their racquet, rather than at one of the other two sweetspot locations.

The Power of the Tennis Racquet: COR and ACOR

The term power, as used in tennis, refers to the speed of the outgoing ball relative to the effort the player has exerted in swinging the racquet. A racquet which provides high ball speeds for small effort on the part of the player is said to be powerful. This is to be distinguished from the definition of power used in physics and engineering where the term means the rate at which work is done or energy transformed.

As important as the maximum power of a racquet is, the variation of the power with respect to where the ball hits on the strings may be more important. If the ball strikes the strings at a small distance from the desired location and the result is a very different ball rebound speed, the shot will probably fall short and a point will be lost. When this happens, the power sweetspot is considered to be quite small and the racquet will be difficult to play with. If, on the other hand, some miss-hits do not lead to an appreciable difference in rebound ball speeds and they result in acceptable ball trajectories (the shot still goes in), the racquet is said to have a large power sweetspot, be "forgiving", and enjoyable to play with.

A heavier racquet will be more forgiving and it will also be more powerful, if you swing it at the same speed as a lighter racquet. A lighter racquet will give you more power if you can swing it at a higher speed. When you try to swing a racquet at a higher speed, you run the risk of a loss of control. Most players do not swing their racquets as fast as is physically possible when hitting groundstrokes, so as to avoid a loss of control. On the other hand, on a serve, players often swing the racquet as fast as they can.

To measure the effective power of tennis racquets, a manufacturer will fire balls at various locations on the head of a freely suspended racquet and measure the rebound ball speed. The ratio of ball speed out (rebound) to ball speed in (incident) is called the apparent coefficient of restitution (ACOR). The term ACOR is used instead of the commonly used term, coefficient of restitution (COR), because in this case, the recoil velocity of the racquet is neglected. The larger this ACOR ratio, the more powerful the racquet. A map of the ACOR value across the strung portion of the racquet will show where it is lively and where it is dead — where you will get power and where you will not.

As a general rule, the ACOR is a maximum near the balance point of the racquet and it falls off as you approach the tip or move to the sides of the head. The fall-off to the sides can be reduced somewhat by making the racquet wider (as Howard Head did when he introduced the oversize) or adding weight to the head at 3 and 9 o'clock. Some of the loss of power as the impact point moves toward the tip is overcome by the fact that a racquet is swung in an arc, not translated or pushed in a straight line. This means that the tip is moving faster than the throat, which tends to move the maximum power point up into the head and away from the throat. By enlarging the head on the modern racquet, the center of the strung area has been brought closer to the maximum power point.

However, for groundstrokes, unless you have a very wristy swing, for many racquets, the maximum power point is often still well below the center of the head and below the other two sweetspots. To raise the power spot higher into the head, racquet designers have done a number of things. They have moved the balance point up (by removing weight from the handle or adding it to the head), made the head stiffer (wide body construction), or made the head get wider toward the tip, with the strings fanning out as well. The resulting racquets allow players to hit the ball at the node or at the center of the head and not pay a penalty of a loss of power.

For a typical serve (where the incoming ball speed is zero), the modern, stiff, graphite composite racquet has its maximum power point well above the center of the head. This has the added advantage of allowing you to hit the ball when it is higher, thus increasing the chances of the serve going in (greater height gives you a bigger acceptance window). The position of the maximum power point is illustrated in Figure. 16.1 for a typical serve and in Figure 16.2 for a typical groundstroke. The position is different in each case and it depends on how the weight is distributed in the head. For a groundstroke, it is possible for the maximum power point to coincide with one or other of the other two sweetspots, or at least be reasonably close, in which case all three sweet spots will contribute to the fact that the shot feels good. For a serve, the maximum power point is closer to the tip. Since locations in the upper half of the stringbed are further away from the COP and the node, there will be more shock and vibration. In that sense, the feel will not be so "sweet" but the outcome should be a slightly faster serve.

The Deadspot

Clamp the end of the handle on a table, using your hand to press on the handle, so the rest of the racquet hangs over the edge of the table. Then drop a ball onto the strings at various points. The ball will bounce best near the throat. There is a spot near the tip where the ball doesn't bounce at all. That's the deadspot. At the deadspot, all of the energy of the ball is given to the racquet, and the racquet does not give any energy back to the ball. The reason is that the effective mass of the racquet at that point is equal to the mass of the ball. The effective mass is the ratio of the force at that point to the acceleration at that point ($F = ma$, so $m = F/a$). In a completely elastic collision where no kinetic energy is lost, if a ball of mass m collides head-on with another ball of mass m at rest, then the incident ball stops dead and gives all its energy to the other ball.

If a moving racquet strikes a stationary ball at the deadspot, then all the rotational energy of the racquet is given to the ball. One can serve a ball at high speed when serving from the deadspot. However, when returning a fast serve, the deadspot is the worst place to hit the ball. The best spot is nearer the throat of the racquet since that's where the ball bounces best.

Sergi Brugera, Spain

7

Introduction To Shock & Vibration

What It Is and How It Happens

By Howard Brody and Crawford Lindsey

Introduction

There are two different types of vibrations that occur in a tennis racquet – vibrations of the frame of the racquet and vibrations of the strings. The vibrations of the strings are discussed in Chapter 26, so it will not be dealt with here. We will restrict this article to vibrations of the frame of the racquet.

The frame of a racquet seems like a rigid body with no elastic properties, yet on the time scale of a ball impacting and with the forces involved in such an impact, the racquet frame is flexible. When the strings are hit by a ball, the local tension around the impact region can increase to several hundred pounds. This causes the racquet to deform and then the frame snaps back, overshoots its equilibrium configuration, and oscillates for a period of time, depending on how it is dampened. The frequency at which this happens depends on the stiffness of the racquet frame and its moment of inertia or mass distribution. The racquet can vibrate in a number of different modes of oscillation – bending, torsional, etc. We will restrict our discussion to the lowest bending mode

of oscillation because this is the mode with the largest amplitude and it involves the most energy.

WHAT IS VIBRATION?

Before the ball hits your racquet, it is in equilibrium. At impact it starts bending, shaking, and twisting in all manner of being. In short, it goes into vibration. Vibration is everywhere in our lives, but what is it?

It's nothing but movement. In the simplest case, it's back and forth movement. When the racquet bends, a restorative force attempts to return to equilibrium. This force is nothing but the attractive forces between atoms which gets stronger in proportion to how far the atoms get pulled apart (up to a point). This is how it works. Imagine the racquet composed of little billiard ball particles (mass) all connected together by rubber bands (stiffness). When the ball hits, it knocks the first particle backward stretching the rubber band. This in turn pulls the next particle, and so on right down to the other end of the racquet. This occurs as a force that begins at the point of impact and radiates out at a speed depending on the type of material. After a certain amount of stretch, the rubber bands get stiffer and pull the particles back (restoring force). But in heading back to its starting position, each particle gains momentum and can't put on the brakes. By the time it reaches its equilibrium position (no bend), it has gained maximum velocity and momentum and it flies past until the rubber bands stretch and reverse its direction again — and so on. This motion finally dies out when energy is transferred and "lost" to the air by fighting drag, to heat by internal structures rubbing against each other and getting "tired" from frictional resistance, and to your hand by expending energy compressing flesh. The impact force sets vibration in motion, the restoring force keeps it going, and damping interactions dissipate it.

Bending Shapes (Modes)

The modes of oscillation of a racquet (or any other object) depend on how the racquet is held or clamped. If the frame is clamped near the butt end, the lowest bending mode of oscillation is about 25 to 40 hertz. If the racquet is freely suspended, the frequency of its lowest mode of oscillation is between 100 and 250 hertz. These are schematically shown in Figure 7.1a. The question then arises: when in play, does a hand-held racquet act like a clamped frame or as a free frame?

When the vibration frequency of the lowest mode of a hand-held racquet is measured, it comes out very close to, but slightly lower than the frequency of a free racquet. Looking at oscilloscope traces of the vibration signal of a hand-held racquet, there is essentially no sign of the low frequency pattern one would get with a clamped racquet. This clearly shows that a hand-held racquet behaves as if it were a free racquet. We conclude that the hand is not strong enough to act as a rigid clamp, and the racquet handle can still shake back and forth even when the hand is gripping as tightly as possible.

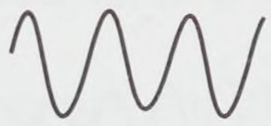
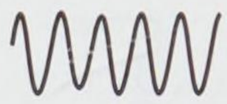
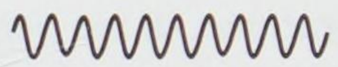
Amplitude of Vibration and the Node

The amplitude of the oscillation of the frame depends very strongly on where on the head the ball impacts. There is one impact location on the head where the amplitude of the oscillation is reduced to zero – no measurable amplitude.

Figure 7.1

a. Bending Shapes During Vibration

Clamped Handle — Free or Hand-held Racquets

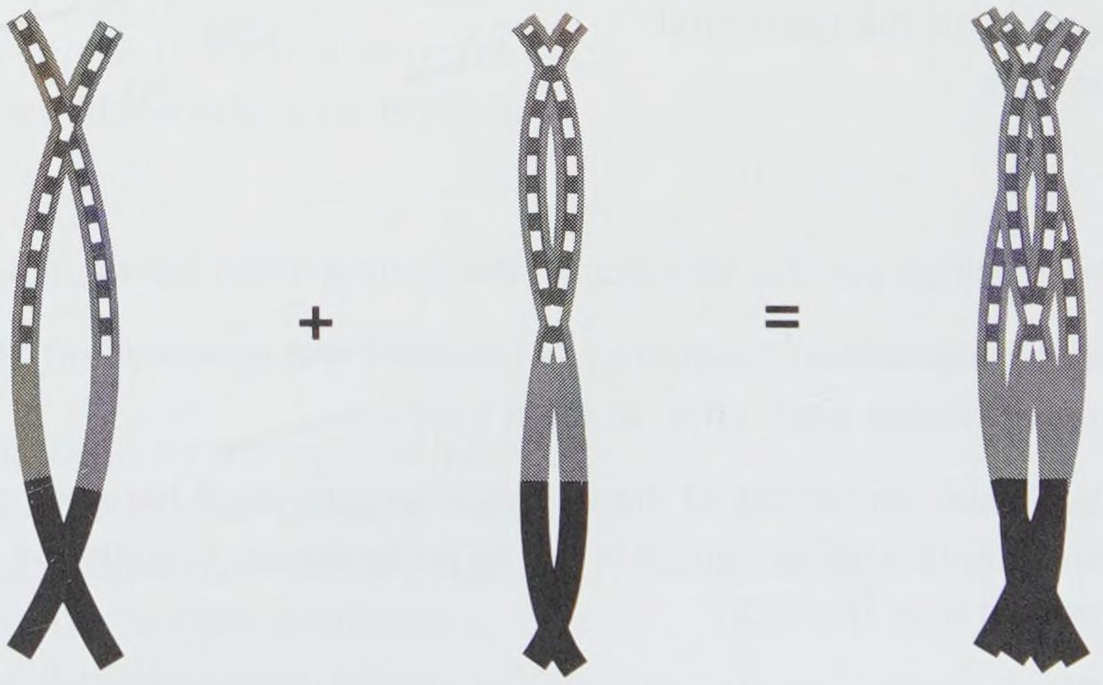
Equilibrium	Diving Board Mode	First Bending Mode	Second Bending Mode
Frequency is number of times per second the racquet oscillates between one extreme of the mode's shape and back again.	25-40 Hz 	125-200 Hz 	250-450 Hz 

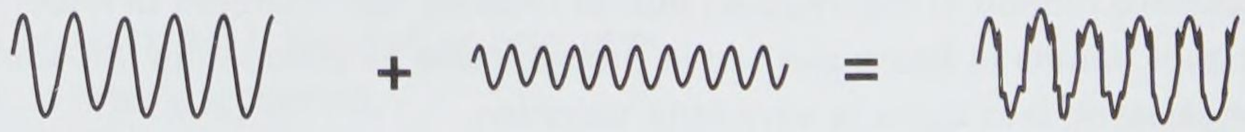
- The Diving Board Model is not appropriate because the hand cannot hold the racquet rigid.
- The first or fundamental mode is the predominant shape for racquet vibration.
- The second mode is a higher vibration mode. It has so little energy and occurs so quickly that the mass of the ball dampens virtually all this vibration before it leaves the racquet. However, it will appear if you hit the racquet with a golf ball or a hammer. In these cases, the impact occurs so quickly that the higher frequency vibrations will show up. Still, higher frequencies die out much faster, anyway.

b. Nodes

- Nodes are locations where there is no movement, no vibration.
- If you are holding at a node, you will feel no vibration at that point.
- If you impact at the node in the head, the racquet will not vibrate anywhere.
- Each point on the racquet travels a different "back-and-forth" distance.

c. Higher Frequency Modes





- When impact is quick enough to cause a higher frequency, the racquet actually vibrates in a composite wave form that is the sum of the two separate frequencies.

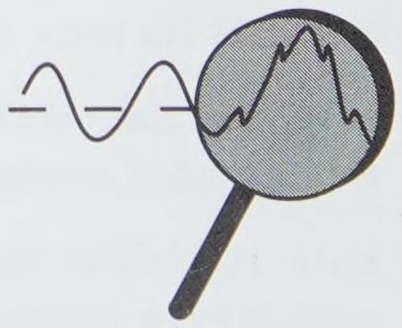
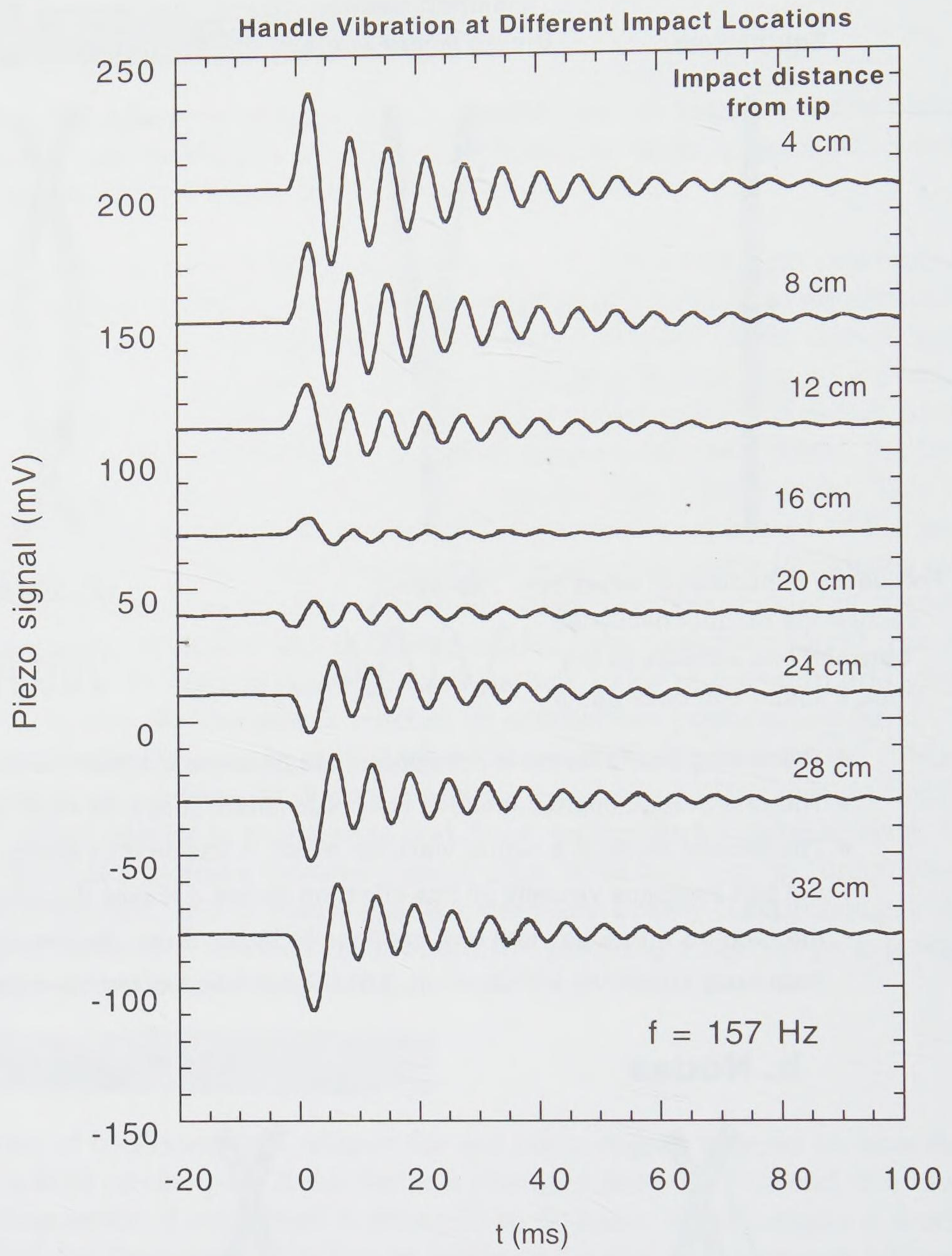


Figure 7.2

Anatomy of a Vibration



- **What is an oscilloscope tracing?** The tracings are the result of piezo accelerometer located on the handle that measures the acceleration of the handle at impact and after. It shows the rate and direction of velocity change, not the magnitude of displacement in space. That can be derived, however.
- **Shock:** The first peak is shock – acceleration of the racquet backward by translation and rotation. The ball has left the racquet by the second peak – 5 ms.
- **Vibration:** All the peaks and troughs after the first are racquet bending of vibration.
- **Frequency:** One cycle of back-and-forth vibration is represented by going from peak to peak. The number of full cycles that occur in one second is the frequency. One cycle per second is known as a Hertz (Hz).
- **Amplitude:** The height of the crest above or below the horizontal starting axis.

- **Impact:** Tracings are the vibration at the handle when the impact is at the designated distance from the racquet tip.
- **Frequency signature:** Frequency is constant and independent of impact location. You can see this because the crests line up right above each other at each location.
- **What you feel:** Amplitude of the vibration at your hand depends on the impact location. Shock and vibration felt at the hand are greatest when impact is at the tip or throat. Vibration is least when impact is at the node. Shock is least when impact is at the COP.
- **COP:** The COP is located somewhere between 16 and 20 cm here. This is evident because the first peak (the backward motion of the racquet due to impact) has changed direction from positive to negative. That means the racquet handle is being pushed in the opposite direction than impact above the COP. At the COP, the first peak is nonexistent, and there is very little vibration.
- **Node:** At the node, the first peak is small (still some backward movement), but the vibration component is flatlined.
- **Damping:** Each successive crest has a smaller amplitude. This is due to damping – energy dissipation due to pushing air molecules around, internal friction between atoms, and with a hand-held racquet, fighting against your hand.
- **Note:** (1) Neither the COP or node was exactly hit in this series of impacts. (2) Each signal shows the voltage generated by a piezo disk located on the handle when the racquet was freely suspended. The ball impacted at the same speed in each case. The tracings have been artifically moved under each other, so the signal readings on the left axis opposite each tracing do not show the actual voltage of each respective trace.

This location is called the node of the first bending mode, and it is usually located near the center of the strung area of the head (Figure 7.1b). There is a second node of this mode of oscillation and it is located in the handle, slightly above the grip. Since the ball rarely strikes this node, we will neglect it. The magnitude or amplitude of the oscillation of a racquet due to the ball impact is directly related to the distance between the impact location and the node. If the ball hits on the head far from the node, the oscillations will be relatively large in amplitude and can produce an uncomfortable feeling. Figure 7.2 shows the vibrations of the frame of a racquet for different ball impact locations on the head. Note that for a ball impact close to the center of the head (where the node is located), the resulting vibrations have a very small amplitude. The amplitude of vibration increases as the impact location moves away from the node.

The amplitude of the vibration depends on three factors:

1. impact location,
2. racquet stiffness, and
3. how hard the ball is hit (the relative ball-racquet velocity).

It is obvious that the harder the ball is hit, the more the racquet frame will deform, and the bigger the amplitude of the oscillation will be. It is also obvious that a stiffer racquet will deform less upon impact, so a stiffer frame will have a smaller amplitude of oscillation, if all else is the same. A flexible frame will deform or distort more on impact, so its amplitude of oscillation will be bigger. Because the energy in the oscillation goes as the stiffness times the square of the amplitude, a more flexible frame, having a larger amplitude of oscillation, will have more energy in its vibration than a stiff racquet will have.

Frequency

The frequency of vibration of the lowest bending mode does not depend on where the ball impacts the head (assuming it misses the node), or how hard the ball is struck. The frequency depends on:

1. racquet stiffness, and
2. racquet mass (actually the moment of inertia).

To first order, the frequency of this mode is proportional to the square root of the stiffness divided by the moment of inertia about the center of mass. When the frequency of the oscillation of the first bending moment is plotted against the square root of the racquet stiffness divided by the racquet's moment, the result is a fairly smooth curve, with little scatter of the data over many different racquets. Because a racquet frame can be stiff in some areas (like the handle) and flexible in other regions (such as the throat or tip), the value one gets from a stiffness measurement depends on how it was measured.

If the racquet is held fixed at one end (like a cantilever), and a force is applied to the tip, you may get a different value of stiffness than if the racquet is suspended at both ends and a force applied in the middle. In addition, there is the problem of the softness of the grip material deforming, which can give an erroneous stiffness reading. An alternate method of specifying racquet stiffness would therefore be to specify the frequency at which the racquet vibrates. This

Higher Frequency Vibrations — What Are They?

The fact that an object can vibrate with multiple frequencies at once is easily demonstrated by two examples. First, consider what happens if you strike middle C on a piano. Middle C has a frequency of 256 Hz. However all notes with frequencies of 2, 3, 4, 5, etc., times 256 Hz also begin to vibrate and no others. The C note created air and piano frame vibrations at all those frequencies that then set the strings into vibration that are tuned to each frequency.

The second demonstration requires a little setup (see Figure 7.3). Loop a rubber band over the head of an unstrung racquet. Hold the racquet horizontally in front of a turned on computer screen (this acts like a strobe light with different horizontal and vertical blinking rates). If you pluck one string of this band (while holding the other off to the side), it appears to vibrate up and down as a whole. If you turn it perpendicular, you see the vibrations occurring in many waves along the rubber band. This is how the racquet also behaves under the proper conditions.

Demonstration
Multiple Frequencies and Shapes

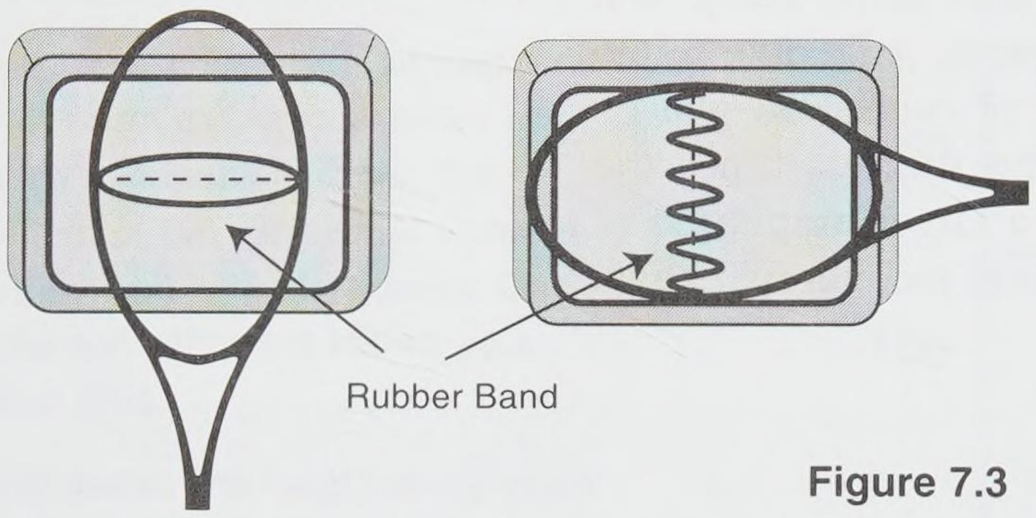


Figure 7.3

method would tend to over emphasize the stiffness of the frame near the center (throat area) and not count the stiffness of the frame near the nodes as much.

You will often hear about higher frequency modes than the first bending (or fundamental) mode. A racquet actually vibrates simultaneously in a theoretically infinite number of modes. However, we don't concern ourselves much with these since they have so little energy that the mass of the ball virtually dampens them all out before it leaves the racquet. All that remains of any significant amplitude (thus, energy) after the impact is the first mode. If you hit the racquet with a light striking hammer or a golf ball, these higher modes will still be visible after impact. That's because these impacts occur so rapidly (because they are stiffer and have less mass), the higher vibrations are not dampened by the very impact that caused them.

Stiffness

The stiffness of a racquet frame (hence the amplitude and frequency of the oscillation) depends on:

1. frame construction,
2. material used in the frame, and
3. frame thickness.

If all else is held constant, the stiffness of a beam goes as the cube of its thickness. Therefore, increasing the thickness of the racquet frame from 19 mm to 38 mm could give you a frame that is twice as thick, and since two cubed is eight, the frame will be eight times as stiff. Even as little as a ten percent increase in thickness can provide as much as a thirty percent increase in stiffness. As an experiment, try bending a ruler or a yardstick, first in one direction, then in the other direction. It will usually bend quite easily when bent against the thin dimension, but it will be very stiff when you try to bend it against the thicker dimension.

What else can influence the frequency of vibration? A strung racquet will be a little more flexible than an unstrung racquet, and in addition, there are 15 grams of string added to the head. Both of these effects will tend to lower the racquet's vibration frequency a little.

Adding weights to the frame at the tip (12 o'clock), the throat (6 o'clock), or the butt end will lower the frequency of oscillation and possibly move the node location a little. Adding weight to the head near the node (3 and 9 o'clock) will not appreciably change the frequency of the first bending moment or the location of the node.

Dampening

Once the racquet is vibrating, it will continue to oscillate until the energy involved is dissipated. For a free, undamped frame, it can take a second or so until the oscillations die out. There are materials that can be built into the frame or added to the frame that will dissipate energy faster than the free racquet alone can. Such devices include hollow spaces with small lead shot or other energy dissipating beads in them and energy absorbing layers of material fabricated into the graphite/epoxy frame. The various vibration dampeners that are placed in the string bed of a racquet do essentially nothing to reduce or diminish frame vibrations. Figure 7.4 shows the vibrations of the racquet with and without a string vibration dampener in place. There is no apparent difference between these two oscilloscope traces.

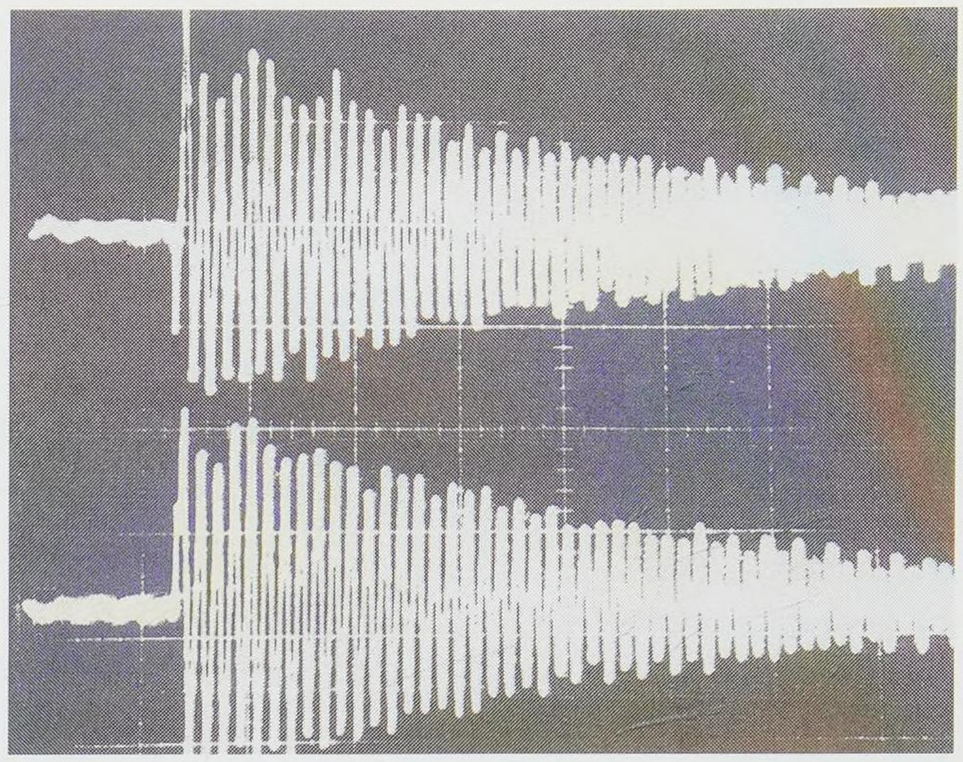


Figure 7.4

Oscillations of a free racquet with and without a vibration dampener in the strings.

Stiffer frames have less amplitude, energy and dampen faster.

Oscillations in stiffer racquets tend to damp out faster than oscillations in more flexible frames for several reasons. The energy involved in a vibrating frame depends on the stiffness of that frame, with stiffer frames storing less energy. Since the frequency of a stiffer frame is higher than the frequency of a flexible frame, within a given time period, the stiffer frame will go through more cycles of oscillation, with each cycle dissipating some of the vibrational energy.

The hand is the best dampener.

What does a marvelous job of dissipating the energy associated with the vibration of a racquet is the human hand, when the racquet is gripped by the handle. The tighter the racquet is gripped, the quicker the vibrations are damped out. Most of the previously mentioned mechanical methods for damping out vibrations work well for a free racquet, but are not that significant compared to the effect of a tight grip. The hand actually dampens most of the vibrations before the other methods have much of a chance to take effect. Figure 7.5 shows the oscilloscope traces of the vibrations of a freely suspended racquet and one that is gripped by a player. It is clear from the figure that the hand does a very good job of damping out vibrations quickly. Wouldn't that be an argument for a very tight grip since it helps to quickly damp out vibrations? Not necessarily, since all the energy quickly taken out of the frame's vibrations end up in the hand and arm, rather than the racquet's frame. If the racquet is gripped further up the handle toward the second node (choked up), then the dampening effect of the hand on the vibrations of the frame is diminished.

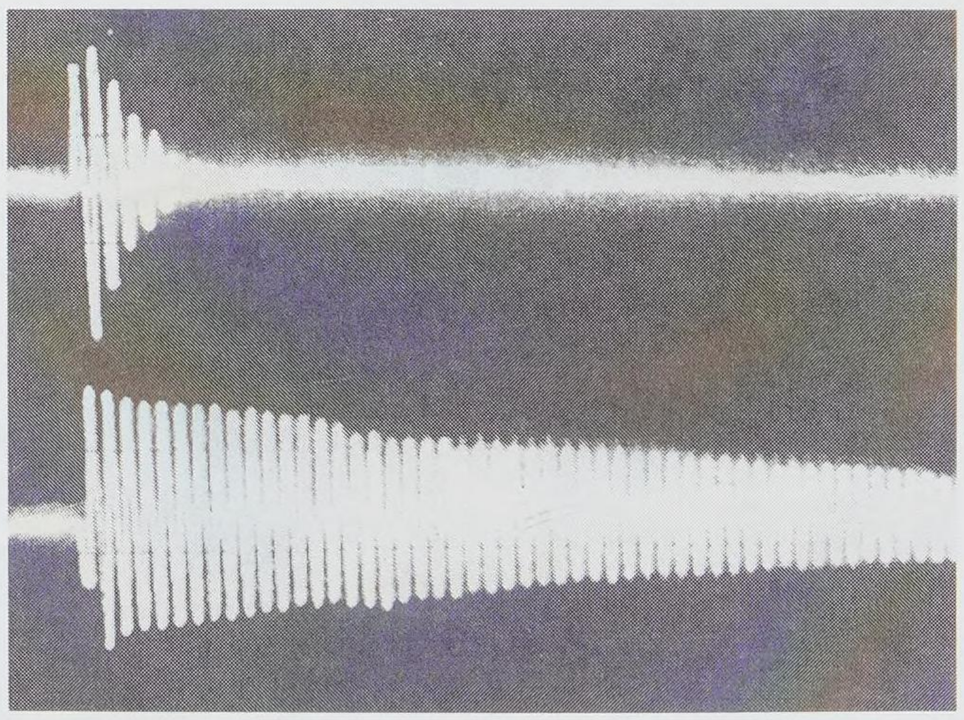


Figure 7.5

Oscillations of a free and a hand-held racquet. Note how quickly the oscillations damp out when the racquet is held in the hand.

Injury

At present, there is no clinical evidence that vibrations of the racquet frame (or the strings) are a cause of any sort of arm, elbow, or shoulder injury. Vibrations

may be irritating, annoying, and uncomfortable, and you may want to eliminate them, but they may not be harmful. Since the node's location is usually fairly close to the center of percussion location, a ball impact that misses the node will usually miss the COP as well. This will produce a shock or jar to the hand and arm, which to many players will be indistinguishable from vibration.

Some tennis articles tell the player to grip the racquet very tightly so that the mass of the hand and arm is added to the mass of the racquet, giving more ball speed (or power). As was mentioned earlier, a hand-held racquet has a vibration frequency slightly lower than a free racquet. As an experiment, a racquet handle was gripped as firmly as possible by this author's hand, and the resultant frequency (for off node impacts) was measured. The frequencies of a free racquet with various masses added on to the butt end were then measured. It was discovered that the frequency shift observed with a very firm grip could be duplicated with 20 grams attached to the butt end of the same racquet, but freely suspended. This means that gripping the racquet as firmly as possible will add less than an ounce to the racquet's weight – and that weight is at the butt end where it does not appreciably change the racquet's hitting power. Another tennis myth is shown to be wrong. There is no need to grip a racquet any tighter than is necessary to get the racquet to its correct location at the correct time and with the correct velocity. It is claimed by some that an excessively tight grip can lead to arm (elbow) injury. This is one tennis myth that may be true.

What Is Shock and What Is Its Relation to Vibration?

Upon impact, the racquet experiences a force. A stiff racquet will experience a high force for a short time and a more flexible racquet will feel a lesser force for a longer time. In either case, the force times the time for each are equal so that the same energy is involved in each case. This force causes three things to happen:

1. The racquet recoils.
2. The racquet rotates.
3. The racquet bends.

This is a complicated array of movements. All three motions contribute to shock. The recoil and bending will slap your hand and the rotation will twist it. This doesn't happen instantly. The racquet head "knows" there has been a collision before the handle does. Granted, it seems instantaneous to us, but the sequence of events is important to the engineering of racquets.

The ball is on the strings for a total time of about 5 ms (0.005 seconds). The bending wave travels up and down the racquet in 8 ms (0.008 seconds) if the racquet's vibration frequency is 120 Hz, or 5 ms if the vibration frequency is 200 Hz (very stiff racquet, but typical of wide body racquets). So, let's take an average racquet somewhere in the middle of this range, say 150 Hz. If the ball strikes the racquet at 10 cm (4 inches) from the tip of a 68.58 cm (27 inches) racquet, the vibration must travel 58.58 cm to the end of the handle and 58.58 cm back to the impact point (total of about 118 cm) if it is to influence the speed of the ball. If the racquet frequency is 150 Hz, that means the wave does one complete cycle (i.e., back and forth, or once up and down the racquet which is about 137 cm) every 0.00067 seconds or 20.5 cm/ms = 205 m/s = 459

mph. If the wave must travel 118 cm back to the impact point at that speed, it will take about 5.8 ms to do so. The ball is already gone. Even for the very stiffest racquets, the wave will be arriving back at the impact point just as the ball is leaving, so it has no, or almost no, effect on the ball.

So, even with the stiffest racquets, it takes 2.5 ms for the bending wave to get to the hand. But what of the rotation and the recoil? How long do they take to get to the handle? The answer is they all arrive at about the same time. At impact, the racquet head rotates with respect to the handle. How does the handle know to move also? In an extended object like a racquet, the message that the head has been hit cannot get to the handle until the bending wave is transmitted to the handle. The bending wave travels quite a bit slower than the speed of sound in a material. (Impact also causes a compression wave that travels at the speed of sound, but the direction of that force is along the length of the racquet and therefore of no consequence to your hand or the ball.) The speed of the bending wave is the "signaling speed" of the racquet and is about 205 meters per second in a racquet with a frequency of 150 Hz. So, for the handle, news of the impact and instructions to recoil, rotate, and bend all arrive at the same time on the wings of the wave.

Because the racquet bends, it is not easy to see that the racquet actually rotates — it rotates at one end but not the other! Imagine throwing a bit of jelly that wobbles as it rotates. How do you tell if it is wobbling, or translating, or rotating? You can't tell until you watch it wobble a few cycles. And it will change during the first few cycles anyway. In a racquet, the initial bend in the head travels up to the handle, reflects up and down the racquet a few times, and eventually sorts itself out into a definite amount of rotation, translation and vibration. But it is impossible to see this in the beginning.

The first spike in a tracing in Figure 7.2 represents this combination translation/rotation/vibration acceleration. That is why this spike is the biggest. The hand resists and stops the translation and rotation, but the vibration continues. All the peaks and valleys after the first spike are racquet accelerations due to vibration only.

Lessons From Vibrations

These arguments raise some interesting questions:

1. Does the whole racquet influence the hit of the ball or does just part of it?
2. If only part of the racquet is involved, what parts and what role does each play?
3. If only part of the racquet is involved, does stiffness along the entire length matter?
4. How much of the racquet's weight is involved in the hit?

How Does Shock Absorption Work?

There is a difference between shock absorption and shock isolation. Also, anything that is advertised as a "shock absorber" is also a vibration dampener. Shock absorption involves turning energy into heat whereas shock isolation changes the way the energy acts on a system. Strictly speaking, most shock absorption systems in racquets work as shock isolation systems, the exceptions being systems with any kind of movable mass. In general, any resilient material can be used as an isolator. "Resilience" means that the material will store and return energy and recover to its original shape without losing any of its properties so that it will be ready to perform again. Depending on the application, the isolator may be a spring, a metal, closed or open-celled foam, or in most cases, an elastomer. An elastomer is another name for any type of natural or synthetic rubber. Each elastomer has different strain, strength, and temperature properties, so choice of material is critical depending on the application.

The key to an elastomer's success is that it be of the proper stiffness to both isolate the shock and not interfere with the performance of the equipment. Shock isolators work by storing the input energy and giving it back at a much slower rate than the input. So the energy is transferred to the handle over a longer period of time than it entered the material. The longer time period means the peak force experienced by the handle will be less. The energy does not magically disappear, it is almost all still transmitted, but it is done so in a kinder, gentler fashion.

Elastomers work differently depending on the direction of the force to their cross-section. Cushion handles or grips are compressed perpendicular to the material. Materials in the frame usually are sheared (like putting a deck of cards between your hands as if you were praying and then pushing up with one hand and pulling down with the other). The secret to most elastomers is that they act 2 or 3 times thicker in the shear direction than their actual dimensional thickness. When you compress an elastomer that is too thin for the force, it will start to react by getting stiffer — exactly what you don't want it to do. In the shear direction, the elastomer can distort 2-3 times further without getting stiffer. This makes material selection very critical when choosing an elastomer. You don't want a material that will actually increase shock at a certain force level. All elastomers do get stiffer above a certain load and many elastomers have different stiffness properties in different directions of force application. For these reasons, elastomeric material does not have to have the same elastic properties in all directions (isoelasticity).

As the questions imply, only part of the racquet is involved during impact. We can refer to a playful experiment by Howard Brody to get an intuitive feel for this answer. Several years ago, Dr. Brody made a racquet with a hinged head, such that the head literally swung freely from halfway down the handle. The surprising results of hitting the ball with this racquet were, first, that you could hit it, and second, that the ball had almost the same velocity as with a real racquet.

The lesson is that it is predominately "local" head stiffness and mass that hits the ball, not the entire racquet. As far as the actual hit and resulting power is concerned, the handle serves primarily to add reach and to accelerate the head to impact speed. Figure 7.6 shows how this can be so.

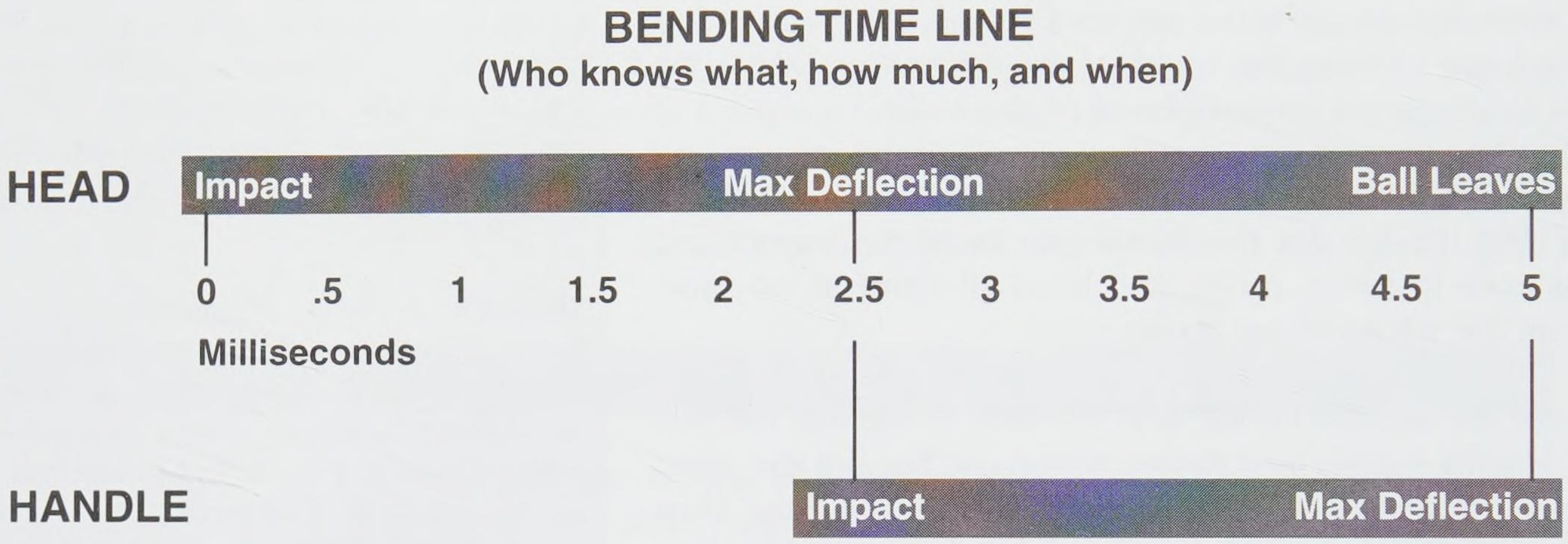


Figure 7.6

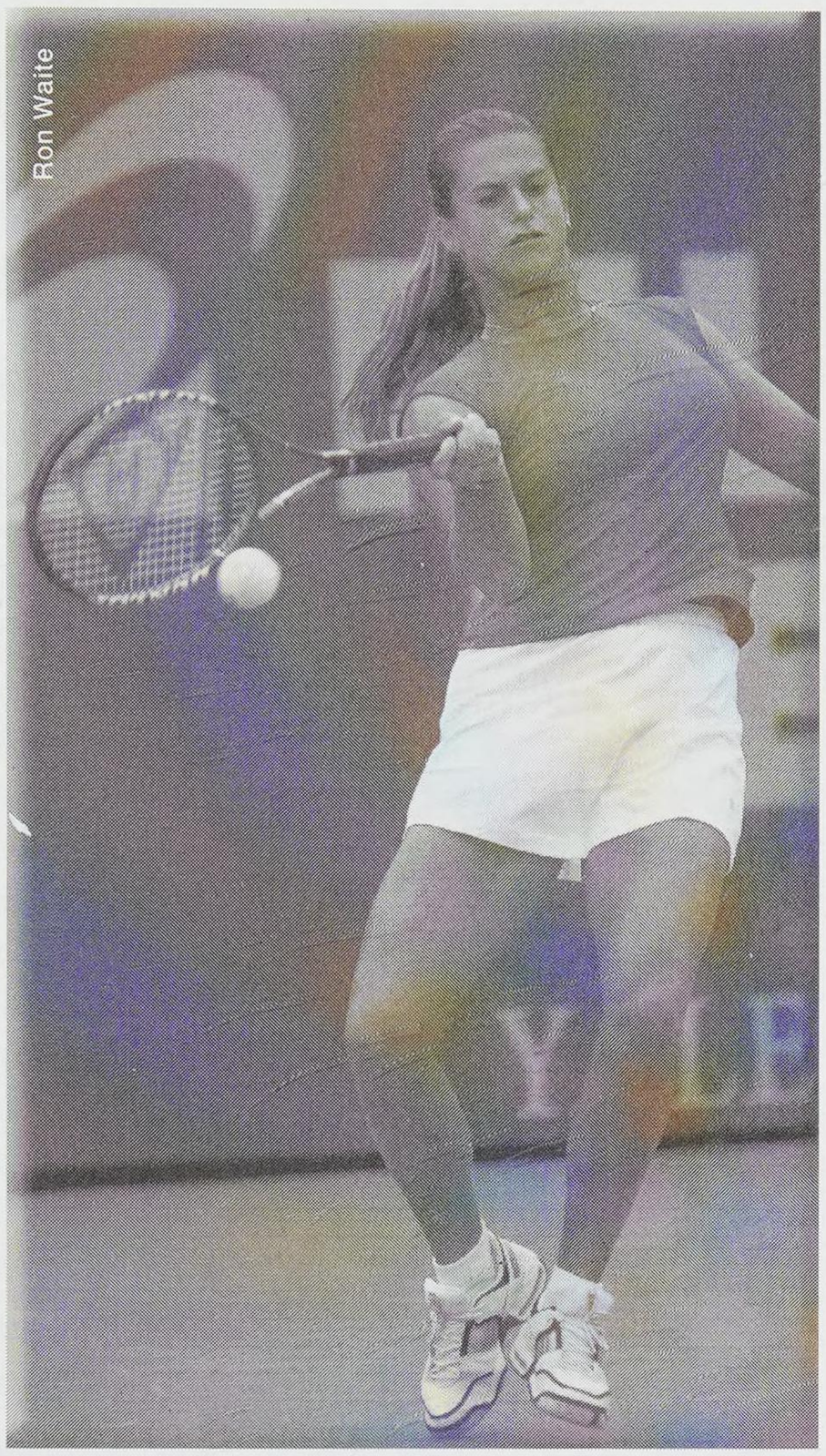
It shows that at any time during impact, each part of the racquet has a different "knowledge" of the impact, ranging from none to everything. Consequently, the so-called "effective hitting mass" and "effective hitting stiffness" are defined only for as much of the racquet to which the impact message can get delivered before the strings start to send the ball back. For example, the amount a racquet will bend (and energy it will store) during the initial backward impact before it reverses its oscillation (2.5 ms) depends on only the stiffness of the length of the racquet over which the bending wave has travelled in those 2.5 ms. That is the only part of the racquet that has been hit in that time. It is the only part of the racquet that knows that it has been hit before the ball changes direction and the strings begin the ball's rebound. The amount of bend also depends on how much of the wave has arrived at a location. If only the front of the wave has arrived, that location thinks it has only been hit by a tiny impact. If the peak of the wave has arrived, that location knows the whole story.

Two objects hitting each other will always satisfy conservation of momentum. If there are no energy losses, they will also satisfy conservation of energy. All

this remains true, even if only one end of a racquet is involved in the collision. In that case, and if the ball begins to rebound before the handle starts to move, then the ball will behave as if it hit the relatively small mass of just the racquet head. Similarly, the racquet head will rebound as if the handle wasn't even there. However, that sets up some significant bending between the head and the handle, so the energy in the head starts to get redistributed along the whole racquet, in the form of rotation, translation, and vibration. After the bending wave has traveled up and down the racquet a few times, the energy gets distributed in exactly the way that you would predict from the conservation equations.

Conclusion

So, that's the theory and the principles behind shock and vibration. But all that said, three things need mentioning. First, the hand still does most of the shock absorbing and continuous jolts to the hand can potentially cause arm problems. Consequently, anything that lessens the hand's task is welcome. Second, the easiest method of shock absorption is lowering string tension (along with using a softer string). However, the significant tension lowering required could have greater negative consequences on performance than positive ones on shock relief. And last, some shock is what "feel" is all about. Some players like a crisp feel and others a softer feel, but everyone wants to feel something.



Amelie Mauresmo, France

8

Racquet Vibration Theory

The Why of Vibrations

By Rod Cross

Frame Bending and Vibrations

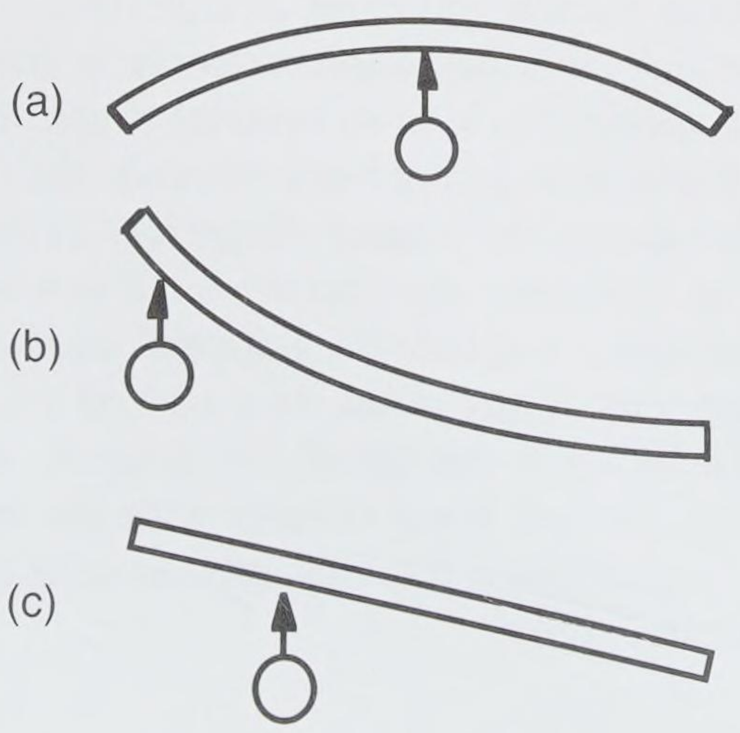
When a tennis player hits a ball firmly with a racquet, the strings stretch and exert a large force on the frame. Even though you won't be able to see it by eye, the ball squashes approximately in half and the frame bends out of shape during the impact. As the ball springs back to its original shape, it propels itself off the strings, assisted by the fact that the strings and the frame also spring back towards their original shape during the latter half of the impact. Ideally, the frame would recover its original shape just as the ball leaves the strings, giving all its stored elastic energy back to the ball. In that case, the frame would not vibrate at all after the impact, and no energy would be wasted in frame vibrations. This, indeed, happens at one spot along the axis of the racquet, near the middle of the strings. It also happens at any point on a curved line joining this spot to the 2 or 10 o'clock positions on the frame, 12 o'clock being at the tip of the racquet. At any other impact point on the strings, the frame remains bent at the instant the ball loses contact with the strings, and the frame vibrates after the collision is over. Frame vibrations are largest when the ball impacts near the tip or the throat of a racquet, but they can be reduced by making the racquet stiffer.

Most tennis racquets vibrate at a frequency between 120 and 150 Hz (cycles per second) taking only 7 or 8 ms (0.008 s) to complete one vibration cycle, depending on the mass and stiffness of the frame. The frame bends backwards and then springs back towards its initial position in about 4 ms, which is a typical time that the ball spends on the strings. One might therefore expect that the racquet should always give back most of its elastic energy, especially if the impact duration coincides exactly with one half of the vibration period. However, this is not generally the case.

When a racquet is used to hit a ball, it bends into a curved arc and tends to straighten out just as the ball leaves the strings. It can straighten out in several different ways, only one of which prevents the racquet vibrating. For example, the racquet may straighten out at sufficient speed that it overshoots the straight position and bends back the other way, with the opposite curvature. If that happens, the racquet will continue to vibrate after the ball has left the strings. The racquet will also vibrate if the head section straightens out while the handle remains bent. Normally, one would consider a modern racquet to be very stiff and it should hardly bend at all during a collision. But it is not infinitely stiff, so it does bend and it does vibrate. A consequence is that the head can bend while the handle remains straight, and then, while the head is straightening out, the handle may start to bend. The latter effect is due to the fact that a bending wave travels down the racquet. The wave reflects off the handle end and arrives back at the impact point just as the ball is leaving the strings. The wave also reflects off the tip of the racquet and arrives back at the impact point well before the ball leaves the strings. The speed at which a ball bounces off the strings is determined partly by the mass of the racquet, partly by amplitude of the bending wave, and partly by the time taken by the reflected waves from each end to get back to the impact point.

Figure 8.1

The impact of a ball with a racquet causes it to bend when the ball hits near the throat (a), or near the tip (b), but not in the middle of the strings (c).



Bending at Different Locations

Even if the whole racquet vibrates as a result of hitting a ball, there is a point near the middle of the strings that doesn't vibrate at all. This point is called a vibration node. The node is a special point and it has the property that when a ball impacts at the node it does not generate any vibrations of the racquet frame, regardless of the string tension or the impact duration. When a ball impacts at the node point, there is very little bending during the impact. Furthermore, the racquet straightens out just as the ball leaves the strings, in such a way that it does not overshoot the straight position and it remains straight along its whole length after the ball leaves the strings.

Vibration Shapes

The effect of an impact at various positions along a racquet is shown qualitatively in Figure 8.1. An impact in the middle of the racquet causes the racquet to bend in the middle with the racquet curved towards the ball. An impact near the tip causes the racquet head to bend with the racquet curved away from the ball. An impact between the tip and the middle results in almost no bending at all.

These effects are consistent with detailed bending calculations for impacts at various points on a racquet, as shown in Figure 8.2. Each curve shows the shape and location of the racquet at 0.8 ms time intervals after the ball first contacts the strings. The results were calculated by modeling the racquet as a uniform beam of length 71 cm, assuming that the racquet was initially at rest and suspended in midair just before the impact without any support at either end. In practice, it is impossible to levitate a racquet this way, but one could support a racquet at one or both ends with a long length of string to give the same effect. The ball was incident at right angles to the racquet, at a speed of 1.0 m/s, and it impacted at a point either near one end, or near the vibration node 15 cm from the end, or at a point 25 cm from the end. An incident speed of 1.0 m/s is, of course, much lower than normal ball speeds in tennis, but the results can simply be proportioned. For example, if the ball is incident at 10 m/s then the y scale should be multiplied by 10. The time intervals are unaffected. The values of e_A represent the ratio of the rebound speed of the ball to the incident speed.

Racquet Motion During Impact

The whole racquet rotates about its center of mass as a result of the impact, and it is also pushed backwards, and it vibrates. The combined effect of rotation about the center of mass (in the middle of the racquet) and translation of the whole racquet is that the actual axis of rotation is shifted towards the handle. For an impact near the throat, the axis of rotation is actually beyond the end of the handle. In this case, the axis is not a well defined point (as it is for an impact at the vibration node) since the axis wobbles due to

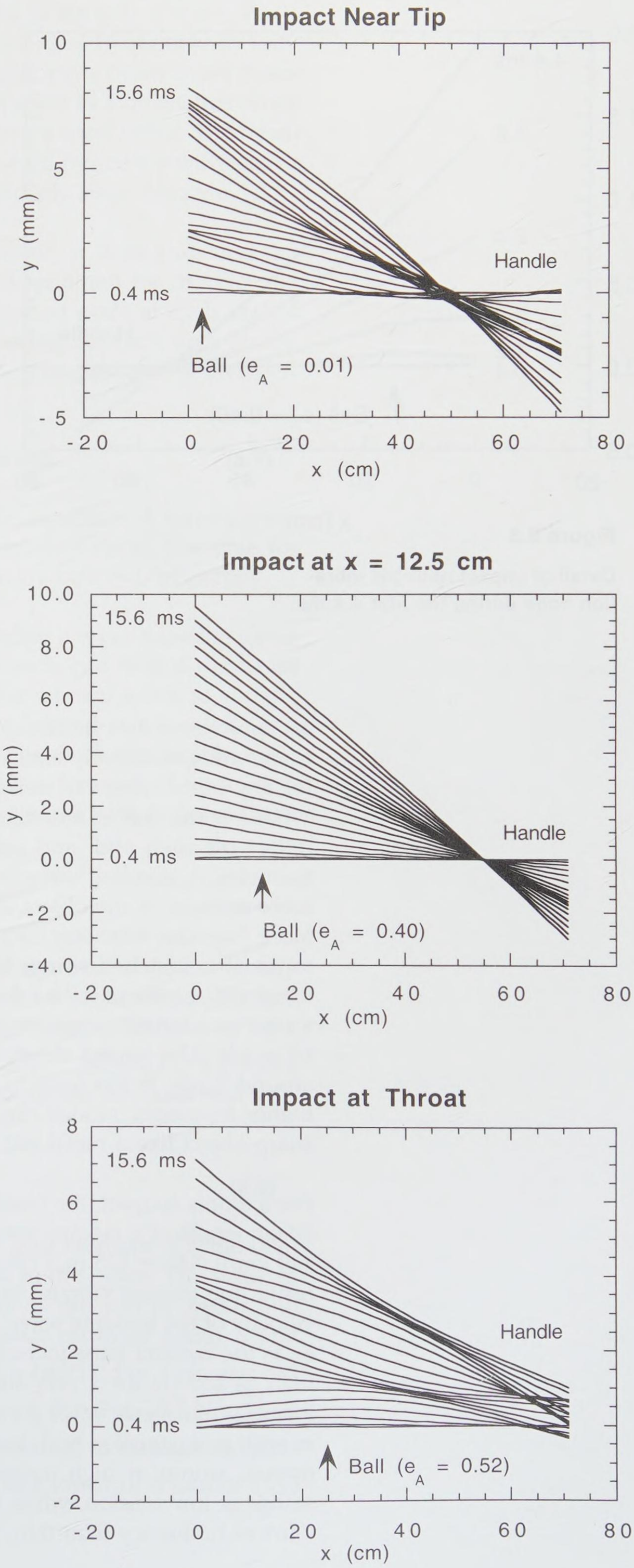


Figure 8.2

Impact of a tennis ball on a freely suspended uniform beam of mass, 360 gm, and length, 71 cm, showing the position of the beam at 0.8 ms intervals. The ball is incident at speed v_{in} = 1.0 m/s and bounces off the beam at a speed $v_{out} = e_A v_{in}$. The impact duration is 5.4 ms near the tip, 5.5 ms at x = 12.5 cm and 5.9 ms near the throat.

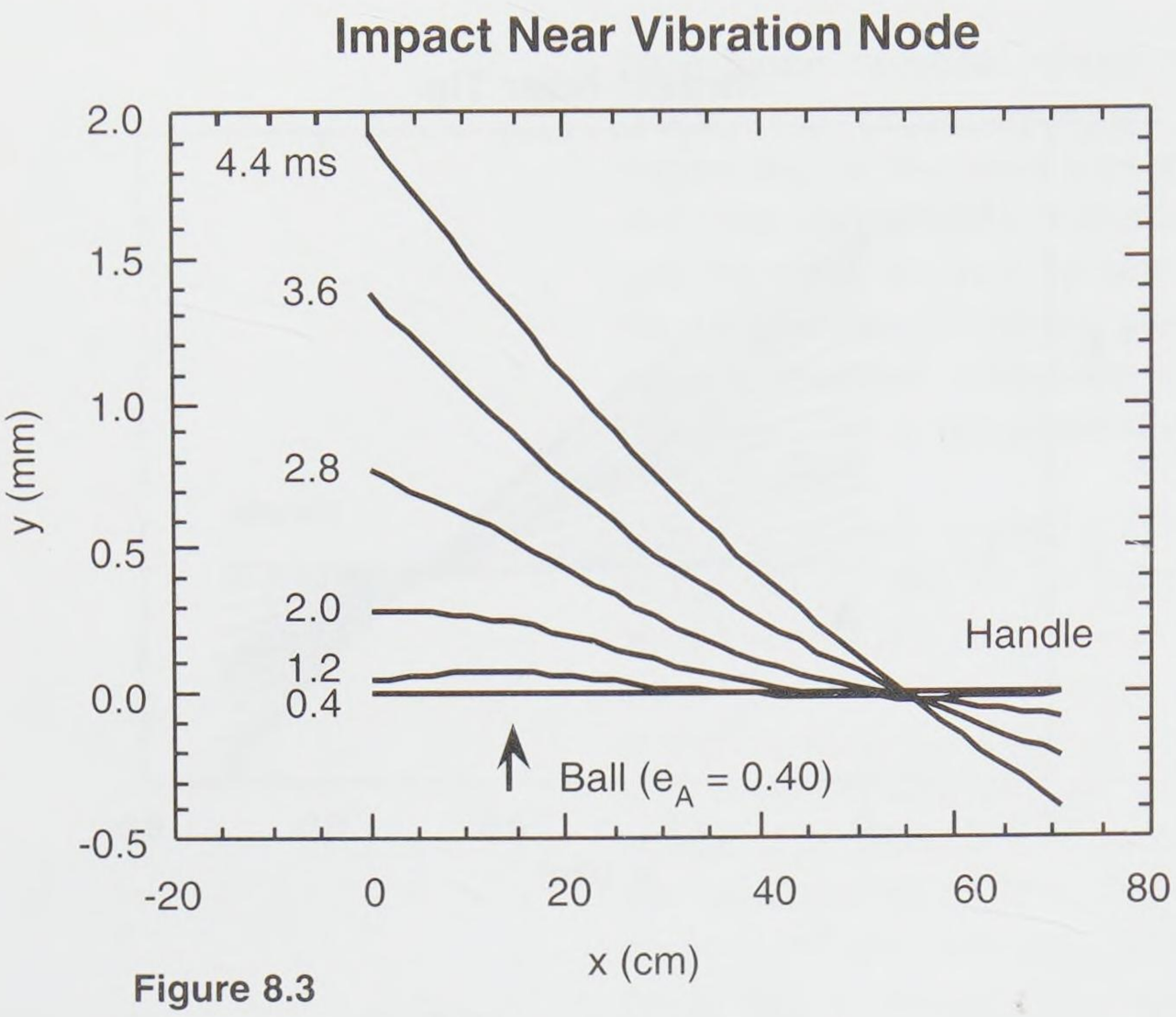


Figure 8.3

Detail of impact near the vibration node during the first 4.4 ms.

the frame vibrations. There is one particular impact point, known as the center of percussion, where the axis of rotation is right at the end of the handle near the wrist. For such an impact, the hand rotates about an axis through the wrist but there is no sudden motion of the forearm either forwards or backwards. At other impact points, the hand still rotates backwards about an axis through the wrist, but the whole forearm will jerk forwards or backwards depending on the impact point.

There is a small residual vibration for an impact near the vibration node, but there is negligible vibration for an impact right at the node. The racquet does bend slightly when the ball impacts near the node, but the racquet quickly recovers. Figure 8.3 shows details of the impact near (but not at) the node point during the impact period itself.

Vibration Frequency

A racquet is not a simple geometric shape, having an approximately elliptical head and an approximately cylindrical handle. Nevertheless, the vibrations of a racquet frame are qualitatively similar to those for a simple rod, or beam, or indeed, for any other relatively stiff object, even a knife, or spoon, or a baseball bat. Whatever the shape, a bending wave can travel along the length of the object, reflect off each end, and set up standing waves at certain well-defined frequencies. A standing wave represents back-and-forth vibrations where one or more sections of the object vibrates "back" while other sections vibrate "forth." More than one vibration frequency can be excited by an impact, like the overtones of a stretched string, but (a) the frequencies for a rod or beam are not integer multiples of each other, and (b) the only vibration mode of any significance in a tennis racquet is the lowest frequency mode called the fundamental mode. The impact duration between the ball and the strings of a racquet, around 5 ms, is too long to excite any of the higher frequency modes. The higher frequency modes can be observed only if the frame is tapped with a sharp object like a metal rod.

For a tennis racquet, the bending wave takes about 8 ms (0.008 s) to travel the 70 cm length of a racquet and back again. The wave travels at a speed of about 140 cm/0.008s = 175 m/s (about three times faster than a very fast serve). As a result, the racquet vibrates once every 8 ms or about 120 times a second. The velocity of the bending wave increases with the racquet stiffness and it decreases as the racquet mass increases. As a result, the vibration frequency can be as high as 200 Hz for a very stiff and light racquet. This is similar to the behaviour of a transverse wave on a stretched string where the wave velocity increases with string tension and decreases as the mass of the string increases. For that reason, strings at high tension vibrate faster, or at a higher frequency, than strings at low tension, while thick, heavy strings of the same length, vibrate at a lower frequency than thin, lightweight strings.

The difference between a racquet and a string is that a racquet has its own built-in stiffness and does not need to be tensioned to support a wave. For a stretched string, the wave speed does not depend on wave frequency. By contrast, high frequency transverse waves propagating along a beam travel faster than low frequency waves. This result can be understood in a qualitative sense from the fact that it is easier to bend a long beam than a short beam. The beam is effectively stiffer for a high frequency wave since high frequency waves have a short wavelength, so the beam is bent over a relatively short distance.

In the case of a stretched string, the string is anchored at both ends and the vibration frequency of the fundamental mode is determined by the fact the string is exactly half a wavelength long, with a vibration node at each end of the string. The speed of a transverse wave on the string depends only on the string tension and the mass of the string. The vibration frequency of a stretched string is given by the formula

wave speed = frequency x wavelength. **(8.1)**

This formula is derived from the fact that speed = distance/time. A wave always travels a distance of one wavelength during one vibration cycle. The time for one vibration cycle (the vibration "period") is given by time = 1/frequency.

The same formula holds for any type of wave, including a wave traveling along a beam. The node points in a beam are located at each end only if the beam is anchored down at each end. If each end of a uniform beam is free to vibrate, then the node points of the fundamental mode are located at x/L = 0.22 and 0.78 where x is the distance along the beam and L is the beam length. Even though the speed of a transverse wave on a beam is a function of frequency, there is only one frequency that has the correct wavelength with nodes at these two spots.

Vibration Modes of a Rod or Beam

The vibration of a rod or beam is a subject that is not treated in elementary physics text books since it is a bit too complicated and specialized. We will simply quote the results here without detailed explanation. The velocity of a bending wave along a rod or beam is given by $v = Cf^{(1/2)}$, where f is the wave frequency (in Hz), and C is a constant which depends on the mass and stiffness of the rod or beam. C is given by

$$C = [4\pi^2 EI/(dA)]^{(1/4)} \qquad (8.2)$$

where E is Young's modulus of the material, I is the area moment, d is the density, and A is the cross-sectional area of the material in the beam. The value of I depends only on the cross-sectional dimensions of the rod or beam and whether it is solid or hollow. For a solid rectangular beam of width b and thickness a (in the bending direction), A = ab and $I = ba^3/12$. For a circular rod of radius R, $I = \pi R^4/4$. For an aluminum beam or rod, $E = 7 \times 10^{10}$ N/m^2 and $d = 2700$ kg/m^3. Graphite/epoxy composite materials used in modern racquets have a larger value of E (about 10×10^{10} N/m^2) and a lower of d (about 1650 kg/m^3), meaning that they are stiffer and lighter.

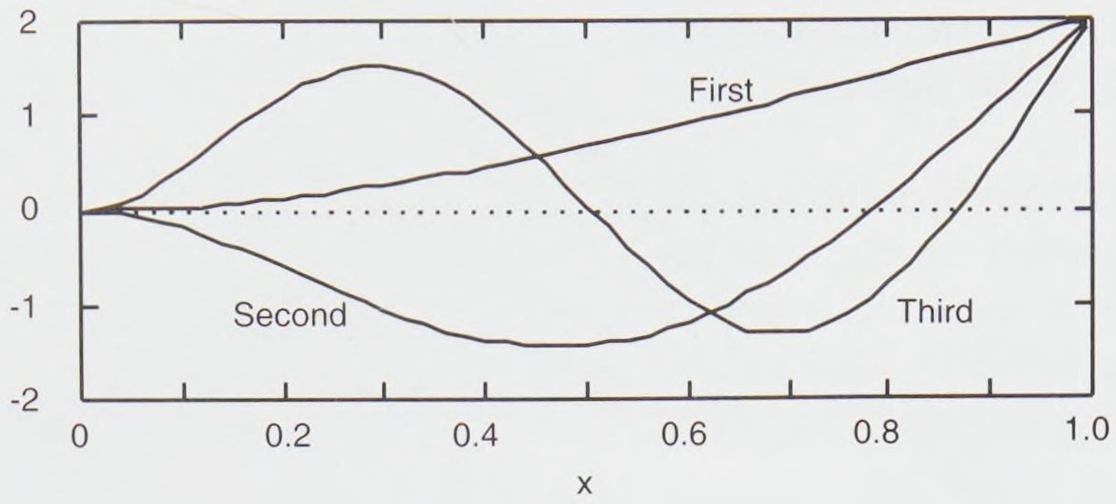


Figure 8.4

First three vibration modes of a cantilever beam. Each curve represents a picture of the beam at one instant of time. Half a cycle later, the beam bends in the opposite direction. The diving board mode is the one that doesn't cross the y = 0 axis (dashed line). The highest frequency mode crosses the axis twice.

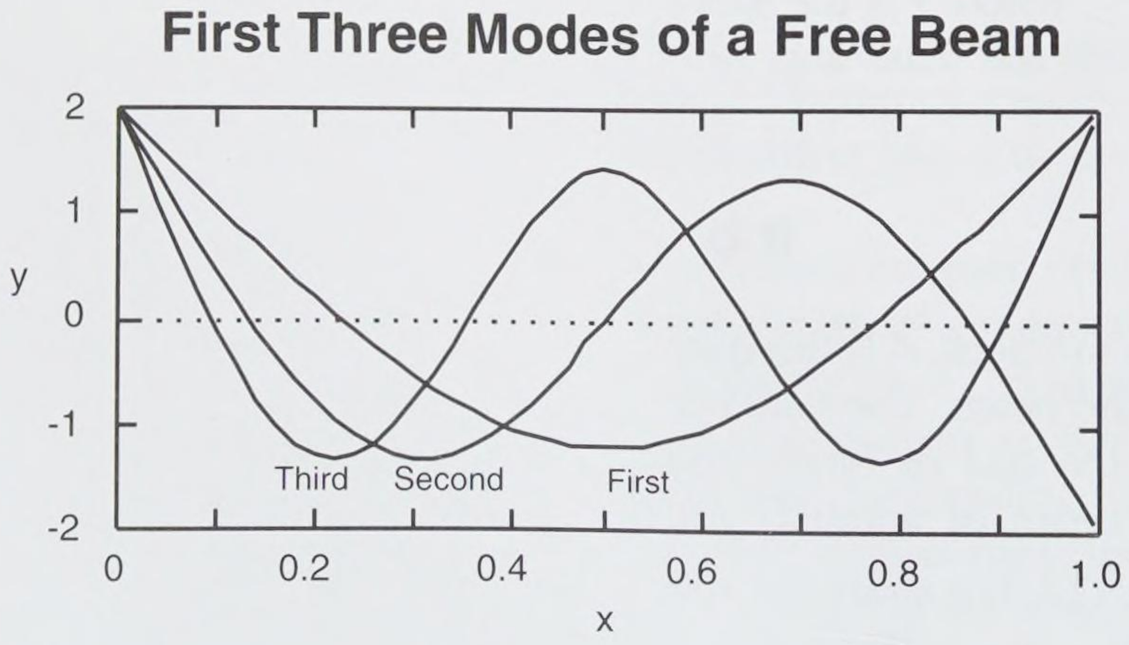


Figure 8.5

First three vibration modes of a free–free beam (both ends free). The fundamental mode has nodes at x = 0.22 and x = 0.78

The vibration frequencies of a uniform rod or beam of length L are given by

$$f = [GC/(2\pi L)]^2 \qquad (8.3)$$

where the value of G depends on how the beam is supported. The constant C can be regarded as a measure of the stiffness to weight ratio. The vibration frequency increases as C increases (stiffness increases or weight decreases) and as the beam length decreases.

Cantilever beam

If a beam is rigidly clamped at one end and completely free at the other end, as in a cantilever, then G = 1.875, 4.694, 7.853, 10.996 for the four lowest frequency modes. The first three of these modes is shown in Figure 8.4. The lowest frequency mode is called the diving board mode. This is the mode that gets excited when you clamp a ruler on the edge of a table and flick the free end, or when a diver jumps off a diving board.

A simple experiment with a ruler clamped at one end shows that the vibration frequency increases as the overhang length decreases, and it varies as $1/L^2$. Many different modes will be excited at once if you flick the free end of the ruler, but the most obvious one is the diving board mode. It is much more difficult, if not impossible, to excite this mode by holding one end of the ruler rather than clamping it, especially if it is a standard 30 cm ruler. However, one can excite the diving board mode by holding one end of a 1 m-long ruler in two hands and giving the ruler a sharp flick of the wrist so that the ruler swings quickly and then comes to a sudden stop.

Free beam

If the beam is free at both ends (or one end supported lightly by a length of string) then G = 4.730, 7.853 and 10.996 for the first three modes. These modes are shown in Figure 8.5. The frequencies are very similar to the previous case, but the diving board mode is missing. The lowest frequency mode with G = 4.730 (the fundamental mode) is the one that gets excited in a racquet when you hit a tennis ball. The handle end is not completely free, but neither is the hand rigid enough to act as a clamp. As shown in Figure 8.5,

the fundamental mode has nodes located at x = 0.22 and 0.78 (where x is the fractional distance along the beam). These are the points in the standing wave that don't vibrate at all. Furthermore, if a ball impacts at these points, then the fundamental mode does not get excited (see Figure 8.1 — no bending takes place). Other modes can get excited, but the ball sits on the strings for too long and effectively damps all the high frequency modes. In a tennis racquet, the nodes are about 15 cm from each end of the racquet.

If the fundamental vibration frequency is 150 Hz, then from the formula $f=[GC/2\pi L]^2$, the next mode has a frequency of 150 x $(7.853/4.730)^2$ = 150 x 2.76 = 413 Hz.

Impact with a Beam or Racquet

As shown by the graphs in Figures 8.2 and 8.3, the handle begins to rotate before the ball leaves the strings. One might therefore expect that a hand-held racquet would behave quite differently since motion of the handle would be resisted by the hand. In fact, the ball bounces off the strings at the same speed, regardless of whether the handle is free, or held in the hand, or clamped in a vice. The hand itself is not as rigid as a vice, and offers very little resistance to the motion of the handle during the time the ball is on the strings. One can see this effect by dropping a ball on a hand-held racquet. No matter how firmly the racquet is gripped, the wrist will always jerk backwards when the ball lands on the strings. The wrist and the racquet eventually stop moving after the impact is over (within about 0.05 sec), but the force of the hand on the handle is much smaller than the sudden large force exerted by the handle on the hand when the ball impacts the strings.

A vice will resist the handle motion completely and will prevent it moving at all. The racquet is therefore unable to rotate or to translate, but it can still bend and vibrate. Nevertheless, the ball bounces off the strings at the same speed as when the racquet is completely free. This effect is very surprising, but there is a simple explanation. When the ball impacts the strings, it causes the racquet head to bend locally near the impact point. As a result, a bending wave starts to propagate towards the tip of the racquet and also in the opposite direction down to the handle. The wave takes about 8 ms for a round trip, so it takes about 5 ms to travel down to the handle and back to the impact point. But the ball leaves the strings after 4 or 5 ms, and doesn't receive any information from the handle until it after it bounces off the strings. As far as the ball is concerned, the handle plays no role in the bounce process and it may as well be 6 feet away. It is for this reason that the grip on the handle makes no difference to the bounce speed. However, it does make a difference if the tip of the racquet is clamped in a vice. It takes only about 1 or 2 ms for a wave to reflect off the tip and get back to the impact point. As a result, the ball bounces much better off the tip if the tip is held in a vice.

Bounce Speed Off a Vibration Node

When a ball impacts at the vibration node near the middle of the strings, the frame recovers its original shape just as the ball leaves the strings, giving all its stored elastic energy back to the ball. One might expect that the ball should then come off the strings with the maximum possible speed. However, this is

only one of several ways that energy is shared between the racquet and the ball. One also needs to consider rotation and translation of the racquet to find the impact point where the ball speed is maximized. In general, the maximum power point is indeed near the middle of the strings, but the speed off the tip or throat is only slightly smaller. There is no dramatic increase in power when the ball impacts at the vibration node. The maximum power point shifts towards the tip of the racquet if the incident ball speed is smaller than the racquet head speed, and it shifts towards the throat if the incident ball speed is larger than the racquet head speed. In the latter case, the speed off the throat is a lot larger than the speed off the tip, as one can easily verify by dropping a ball on a stationary hand-held racquet.

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9

Racquet Feel

What Makes One Racquet Feel Different From Another?

By Rod Cross

Despite the fact that most racquets these days look much the same and are made from the same basic materials, every racquet feels different from the next. The reason is that no two racquets are the same in every respect. The most obvious differences are in the weight and headsize. Other differences that contribute to racquet feel include the swingweight, the twistweight, frame stiffness, stringbed stiffness, head shape, handle grip, sweetspot size, and shock absorbing properties. The most relevant property, in terms of feel, is probably the stiffness of the frame since it determines the amount of shock and vibration generated on impact with the ball. The amount of shock and vibration depends also on where the ball makes contact. If the ball contacts the middle of the strings, then there is almost no vibration in any racquet and only minimal shock, regardless of the frame stiffness. In that case, the feel of the racquet will depend mainly on its weight, swingweight, and string tension. These factors determine how easy it is to swing the racquet and how far and how fast the wrist rotates backwards when the racquet strikes the ball.

A common view is that a racquet with a stiff frame will feel stiff and will generate more shock at impact. In fact, measurements show that stiff racquets vibrate less than flexible racquets and generate less shock, not more. A flexible frame bends further than a stiff frame on impact with the ball. As the ball leaves the strings, the frame springs rapidly back to its original shape, but it may overshoot and then bend in the opposite direction. The frame will then vibrate, bending back and forth rapidly through a large distance if the frame is flexible, but only

a small distance if the frame is stiff. The high level of vibration in flexible racquets adds to the total shock or jarring sensation experienced during impact. The only exception would be an extremely flexible racquet made from something like rubber. A rubber racquet would have very little power, but it would feel really soft since the handle itself would be softer than the hand, and it would bend just beyond the end of the hand. The handle in a modern racquet is a lot stiffer than the hand and it does not bend in this way. Consequently, any sudden motion of the handle arising from an impact tends to jar.

Effects of String Tension on Jarring

Even though stiff frames reduce jarring, stiff strings normally increase jarring since they cause the handle to vibrate more and to slam into the hand at a higher speed if the player misses the sweetspot. The strings are not nearly as stiff as the frame and have very little effect on frame stiffness. The function of a soft stringbed is to reduce the impact force on the racquet, in the same way that jumping on a trampoline feels nicer than jumping up and down on concrete. You could hit a ball at about the same speed with a solid wood paddle or bat, but the strings are there to reduce the jarring effect and to reduce the drag force through the air when you swing the racquet (see Note 1).

As string plane stiffness decreases, the impact force on the frame decreases, and hence, the racquet bends less. The force on the ball is also reduced, but it acts over a longer time with the result that the ball speed is a tiny fraction larger than the speed off stiff strings. The fact that the ball sits on soft strings for a longer time also helps reduce frame vibrations. In effect, the ball helps to damp both the string and the frame vibrations. This effect is quite significant and is explained in more detail below. The most comfortable racquet is, therefore, one with a stiff frame strung at low tension. In fact, one can eliminate frame vibrations completely if the frame is stiff enough, and light enough, and if the string tension is low enough. As described below, one of the racquets tested was almost ideal in this respect. It vibrated slightly for impacts near the tip, but frame vibrations were almost undetectable for impacts in the throat area. This is not necessarily a good thing. It depends on individual taste. Some players like feeling the vibrations since it gives them some feedback on where they are striking the ball and whether they hit a good shot. But too much vibration gives "bad vibes" to most players.

Professional tennis players normally string their racquets at high tension rather than low tension, despite the fact that an impact away from the middle of the strings will feel harsh. However, professionals are generally more consistent than the average player at hitting the ball in the middle of the strings. For the professional player, shock and vibration is not such an important issue. In fact, if a player hits the ball in the middle of the strings, high string tension will reduce the shock caused by sudden rotation of the wrist.

An impact in the middle of the strings produces essentially no vibrations and no sudden movement of the handle in one direction or the other. What you feel is due almost entirely to rotation of the racquet. The whole racquet rotates about an axis through your wrist when you hit a ball in the middle of the strings, even if you "lock" your wrist. Some players grip the handle loosely and use a lot of wrist rotation when hitting a ball. Others grip the handle more firmly to prevent the wrist rotating when swinging the racquet. Wrist rotation is used by squash players and can also be used by tennis players to good effect.

Wrist rotation just before impact can be controlled by the player since racquets are reasonably light. However, wrist rotation during the impact itself cannot be controlled since the impact force is so large. The force of the ball on the strings is typically around 100 lb (45 kg or 450N) and can be as large as 300 lb in a very fast serve. Try lifting a 5 lb weight on the end of your racquet using wrist action alone. If you have a strong wrist you may be able to lift a 10 lb weight, but no one can lift a 20 lb weight, let alone 100 or 300 lb. The impact force causes the wrist to rotate backwards, but not very far since the force acts for less than one hundreth of a second. Heavy racquets, or racquets with a large swingweight, will resist this rotation better than light racquets. Racquets strung at high tension will also rotate less on impact, despite the larger impact force, since the dwell time on the strings is shorter (see Note 2 for an explanation). This should also give a player better control on the angle of the ball coming off the strings.

Simple Stiffness and Vibration Experiments

The effect of stiffness on vibration can be demonstrated by some simple experiments that you can do yourself. Take a ruler and hold, or clamp, one end firmly on the edge of a table so that the other end extends past the edge of the table. If you pluck the far end, the ruler will vibrate back and forth at a frequency of about 10 cycles per second. This mode of vibration is called the diving board mode, since one end vibrates like a diving board while the end doesn't move at all. If you decrease the length of the ruler extending past the edge of the table, then the ruler becomes stiffer (meaning harder to bend), the vibration frequency increases (you hear a higher pitch tone), and the vibration amplitude decreases (the tip moves back and forth through a smaller distance).

Now hold one end of the ruler in one hand and pluck the other end. If your hand is stiffer than the ruler, the ruler will vibrate just like it did when you clamped it on the table. However, most rulers are stiffer than the hand and it is usually impossible to get them to vibrate in this way. Racquets are even stiffer than rulers, but they can still vibrate when they are used to hit a tennis ball. If the handle of a tennis racquet is clamped firmly in a vice (but not too firmly or it will break) then it will vibrate like a diving board at low frequency, around 20 cycles per second. In that case, the handle doesn't vibrate at all. When the handle is held by hand and the racquet is used to strike a ball, the racquet vibrates at a much higher frequency, around 100 to 200 cycles per second. That is a different mode of vibration, where both ends of the racquet vibrate strongly, even though the handle is held firmly in the hand. The hand is not stiff enough to act like a rigid clamp, and both ends of the racquet vibrate simultaneously. The hand and the forearm also shake back and forth when the racquet is vibrating in this manner. The mode of vibration of a hand-held racquet can be demonstrated by holding a long, thin rod, or a length of stiff wire, or a narrow strip of cardboard in the middle and shaking it back and forth. These objects are very flexible and each end will vibrate back and forth at only one or two cycles per second. In fact, they are flexible enough to support both the low frequency diving board mode (when held at one end) and the higher frequency tennis racquet mode.

Measurements of Shock and Vibration

The language used to describe feel is not very precise. Every racquet "jars" every time it is used to hit a ball, but exactly what happens depends on the

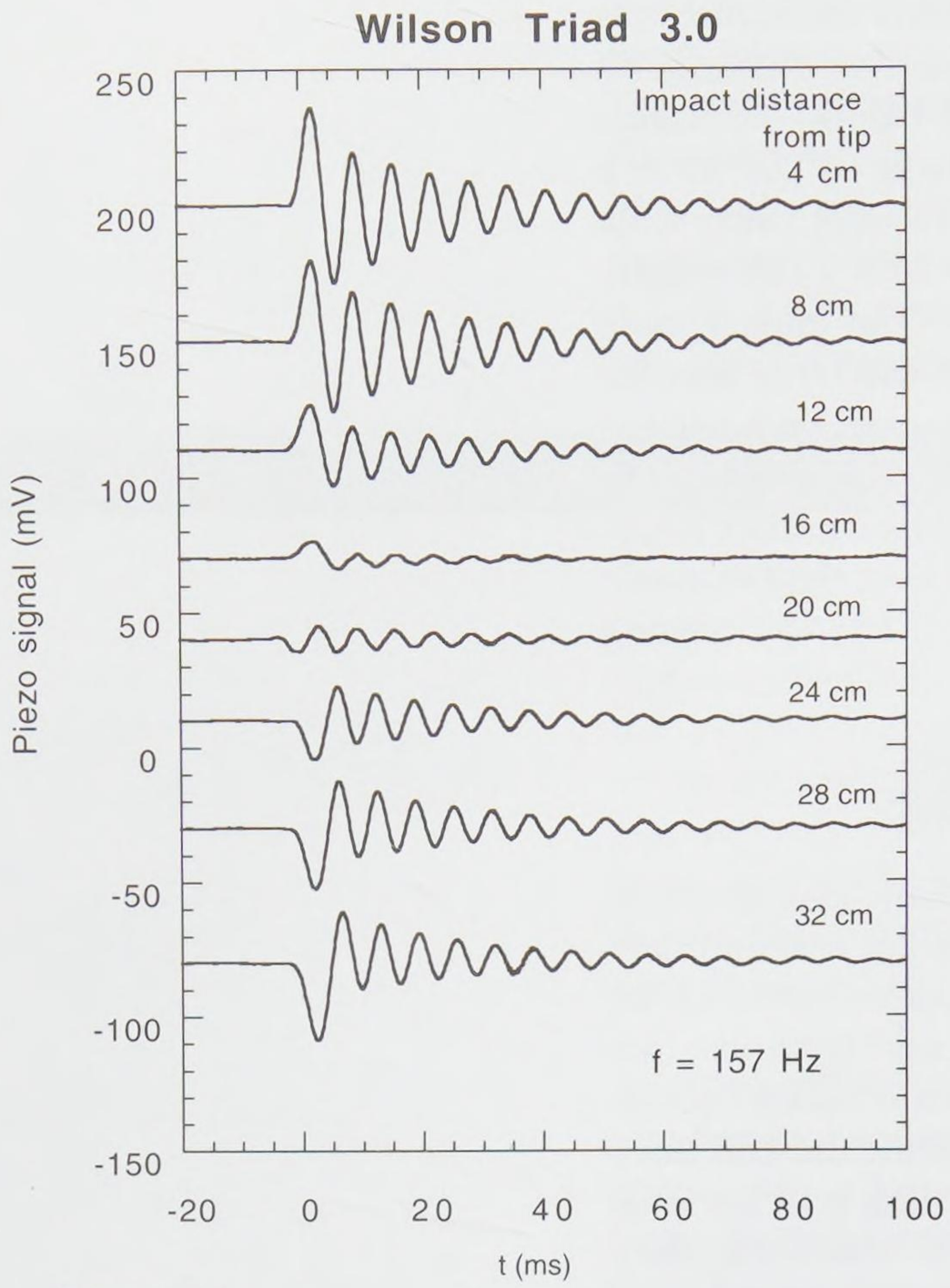


Figure 9.1

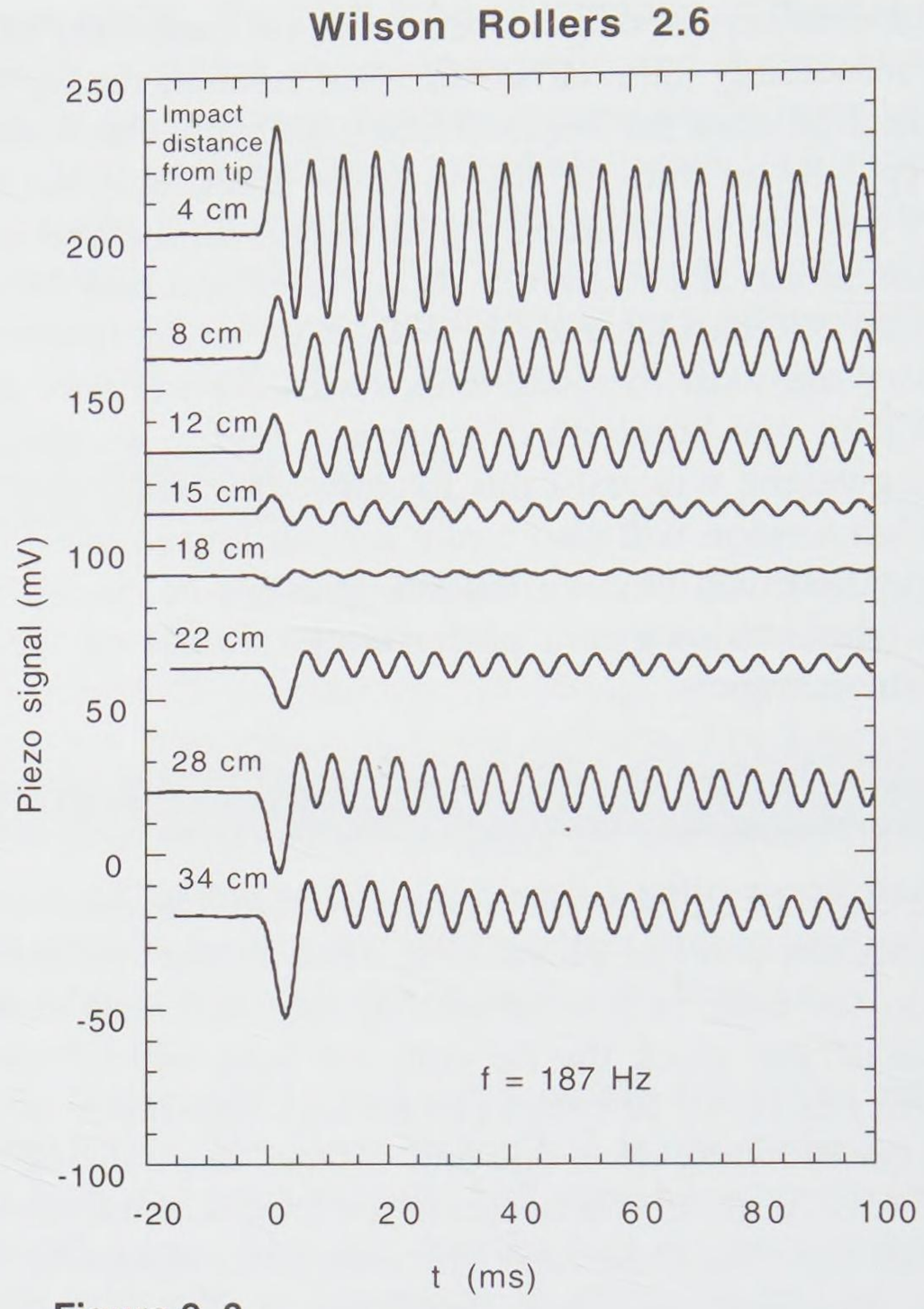


Figure 9. 3

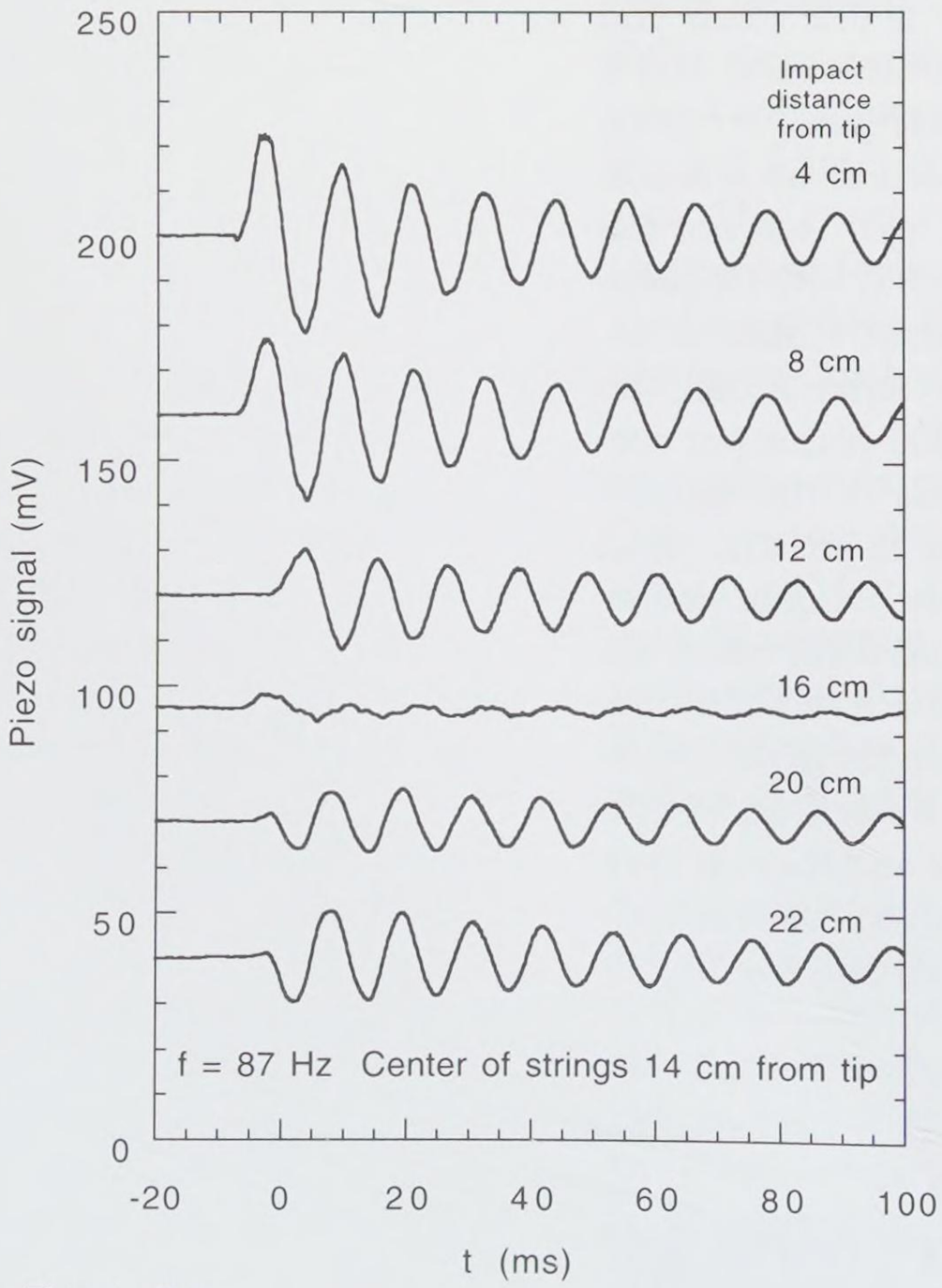


Figure 9.2

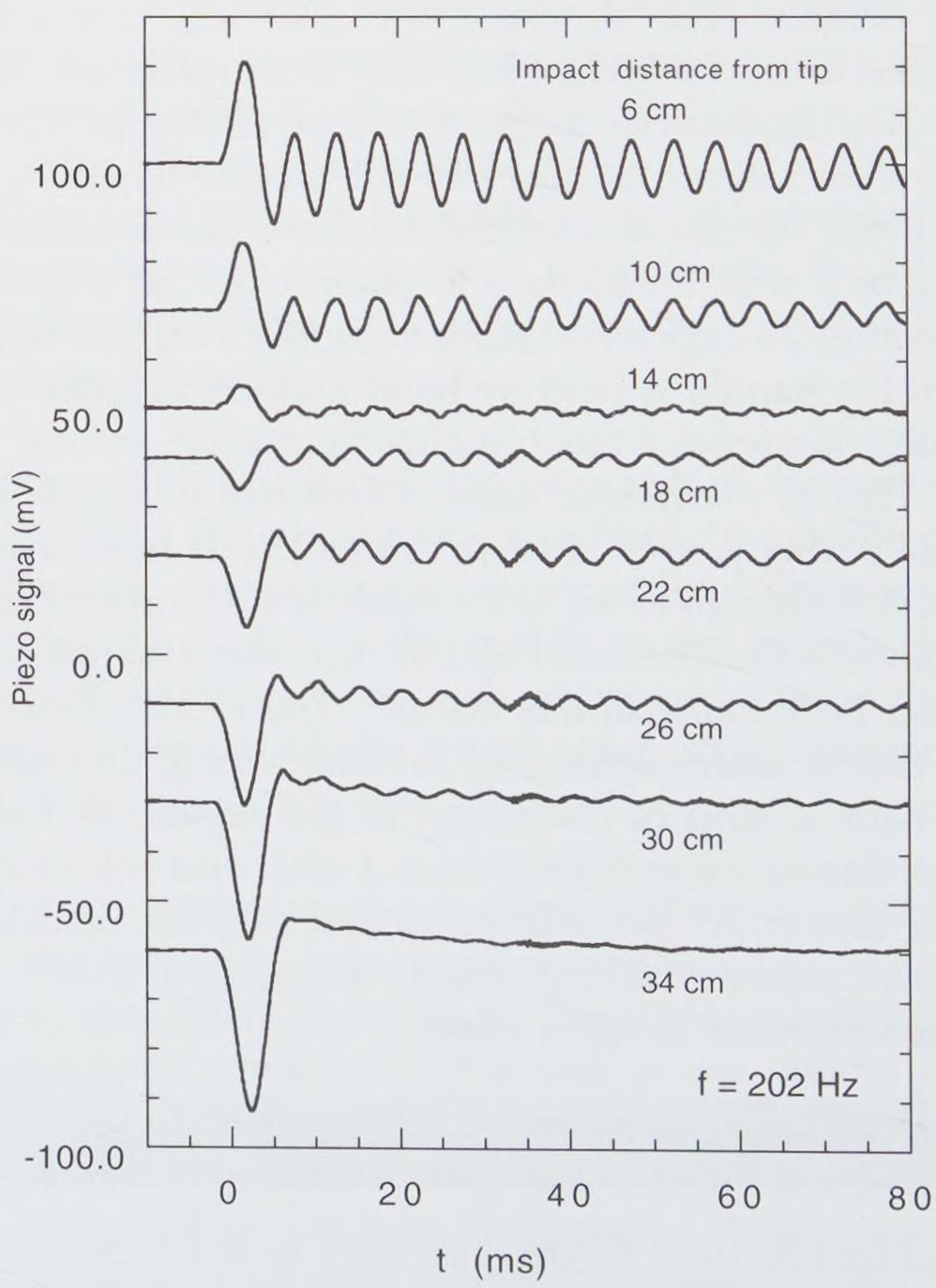


Figure 9.4

Table 9.1

Racquet	Mass	Length	Frame stiffness	Stringbed stiffness
Wilson Triad 3.0	291 gm	699 mm	70-76 DA	62 DA
Prince TT Viper	261 gm	697 mm	81 DA	60 DA
Slazenger Challenge	374 gm	685 mm	<0 DA	45 DA

circumstances. It could be a sudden motion of the handle in one direction or a sudden vibration back and forth or a combination of both and it might last for a short time or it might last for a longer time. To help sort this out, shock and vibration measurements were made on a number of different racquets using a piezo sensor attached to the handle near the butt end. The piezo generated a voltage signal proportional to the acceleration of the handle. The racquets were not held in the hand but were suspended at rest from a horizontal beam using two lengths of string. A tennis ball was also suspended from the beam as a pendulum so that the ball hit a selected spot on the strings at the same speed on every spot and on every racquet. The method is described in more detail in the side box "How To Measure Vibrations." Results for four of the racquets are shown in Figures 9.1-9.4 and the properties of each racquet are indicated in Table 9.1. The Slazenger racquet was an old wood racquet known from its feel to vibrate a lot more than the other three modern graphite racquets. The frame and stringbed stiffness was measured on a Pacific diagnostic machine (higher reading = stiffer). The wood racquet was so flexible that the frame stiffness reading dropped below zero.

Results of measuring handle motion of the Triad racquet are shown in Figure 9.1 for eight different impact points on the strings, all lying on the long axis through the handle. These results are typical of those for all racquets in that:

1. The level of vibration in the handle is largest for impacts near the tip or the throat of a racquet.

2. The vibration level in the handle is zero for one particular impact spot near the middle of the strings. This spot is called a vibration node.

3. The impact of the ball on the strings lasts about 5 ms, after which the ball bounces off the strings. During this time the racquet head accelerates backwards, but the handle itself can accelerate forwards or backwards depending on the impact point. If the ball hits the strings near the tip, then the handle accelerates forwards and the voltage signal from the piezo is large and positive. If the ball hits in the throat area, the handle accelerates backwards, and the signal from the piezo is large and negative. This can be seen by the reversal in direction of the first part of the piezo signal. There is an impact point between the tip and the throat where the handle doesn't accelerate forwards or backwards at all. This point is called the center of percussion (COP) and it is located close to the vibration node. However, every point in the handle has a different COP on the strings. For the results in Figures 9.1-9.4, the piezo was located 50 mm from the butt end of the handle and the COP is then only a few mm from the vibration node.

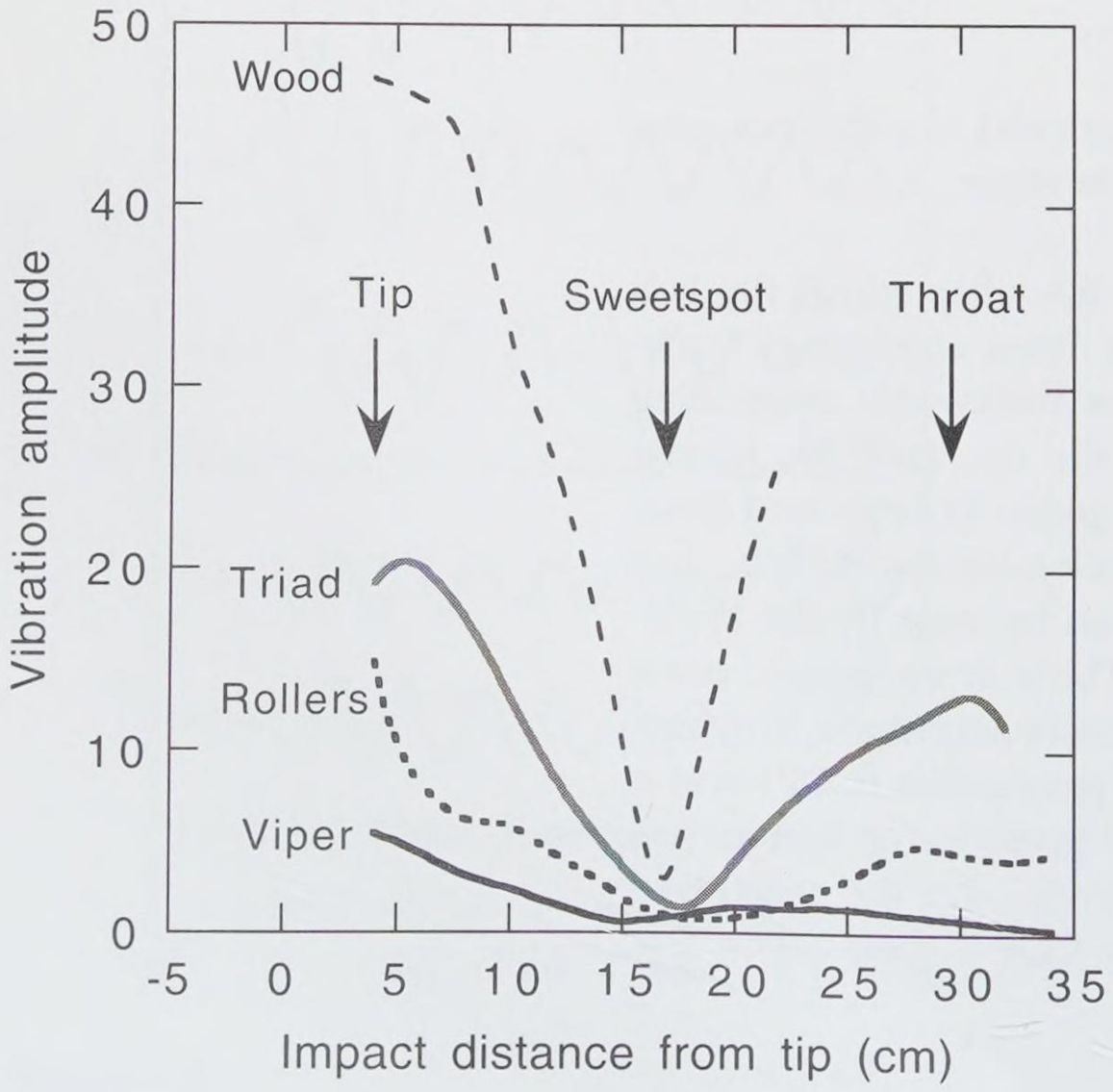
4. The region extending from the node to the COP is the sweetspot region since the total shock and vibration is minimized. An impact at the node results in zero vibration, but there is a tiny amount of shock since the handle has a small acceleration forwards. An impact at the COP generates zero shock, but there is a small level of vibration generated in the handle.

The sweetspot is located about 16 cm from the tip of all racquets. The sweetspot was located near the middle of the strings for the three graphite racquets, but it was closer to the throat in the wood racquet since the headsize was smaller. In all cases the sweetspot could be located within 5 mm. The sweetspot did not fill up the whole string plane, or even half of it, or even 10% of it. The level of vibration remained small over a distance about 1 cm either side of the sweetspot in the wood racquet and over a larger distance in the graphite racquets. In fact, the Prince Viper was very surprising, with almost no vibration anywhere past the middle of the strings, and with no vibration at all in the throat. The explanation of this provides the key to understanding racquet vibrations. Racquet manufacturers have tried all sorts of tricks to reduce frame vibrations, and the Viper demonstrates how it can actually be done.

Vibration Frequency and Amplitude

One obvious difference between the wood and the graphite racquets is the vibration frequency. The graphs show that one complete cycle of oscillation takes 11.4 ms for the wood racquet, but it takes only 6.3 ms for the Triad and 5.0 ms for the Viper. The wood racquet vibrated one cycle in 0.0114 seconds. This is equivalent to 1/0.0114 = 87 cycles per second, commonly designated as 87 Hz (Hz is short for Hertz). The Triad vibrated at 157 Hz and the Viper at 202 Hz. The vibration frequencies were measured with wood blocks on the handle to hold the piezo and to simulate the weight of the hand. If the wood blocks are removed, the racquets vibrate at slightly higher frequencies, but not a lot higher. The different vibration frequencies makes a big difference to the feel of each racquet. The handle vibrates back and forth in the hand, reversing direction 157 times a second with the Triad, but only 87 times a second with the wood racquet. But the biggest difference in feel is due to the size of the vibrations.

Figure 9.5



The piezo signals shown in Figures 9.1-9.4 represent the acceleration of the handle as a function of time. The actual distance traveled back and forth by the handle during each cycle of oscillation is shown in Figure 9.5 for each of the four racquets. The distance (technically called the vibration amplitude) is given by the acceleration divided by the vibration frequency squared. The first positive or negative peak in the piezo signals was ignored when calculating the vibration amplitude since the first peak is due mainly to sudden rotation of the handle rather than vibration. The accelerations for all four racquets are similar in magnitude, but the vibration amplitudes are a lot different. The wood racquet vibrated with much greater

How To Measure Vibrations

The arrangement used to measure motion of the handle is shown in Figure 9.6. A racquet was supported by two lengths of string tied to a metal frame at the top end and to two points on the racquet at the bottom end, one at the 3 o'clock position and one near the butt end of the handle. This is obviously not the same as swinging a racquet on the court, but it still gives useful and relevant information. One problem with swinging a racquet is that the handle must be held in the hand, just where an instrument needs to be located to measure the vibrations in the handle. Another problem is that any instrument on the handle needs to be connected with wires to a measuring system, and that interferes with the ability to swing the racquet. The way around both of these problems is to suspend the racquet freely, with a force gauge or an accelerometer located on the handle, and then allow a ball to impact on the strings. The racquet swings around and vibrates, but that is just what we need to measure. The sudden motion of the handle generates the shock that you feel as the handle suddenly slams into, or tries to pull out of, your hand. The vibration of the handle is what you feel after the initial shock is over.

The simplest and cheapest force gauge is a piezoelectric disk. This type of disk is used in piezo buzzers and in musical greeting cards and it costs about $2. Normally, one applies a voltage to the disk which causes it to vibrate and to generate a buzzing noise. A piezo disk can also be used the other way around. If the piezo is made to move or vibrate, it will generate a voltage that can be measured on an oscilloscope. One can attach a piezo disk directly to the handle of a racquet to measure movement or vibration of the handle, but the size of the voltage signal generated depends on how well the disk is attached to the handle. It also depends strongly on whether the disk remains flat or whether it bends slightly when it is attached to the handle. A piezo disk is typically about 0.3 mm thick, about 1/2 in. diameter, and is made from ceramic. Since racquet handles are not perfectly flat, and since every handle is different from the next, the best approach is to attach a flat surface firmly to the handle and then attach the flat disk to the flat surface. That way, the result does not depend on the exact shape of the handle in each racquet or on how firmly the disk is attached to the grip.

The arrangement used to attach a piezo to the handle is shown in Figure 9.7. Two identical wood blocks were shaped to fit over the handle and two steel bolts were used to clamp the blocks firmly onto thin rubber pads on the handle to obtain good contact. A 16 mm diameter piezo disk was taped firmly onto one of the wood blocks. The total weight of the blocks and the bolts was 170 gm, about the same as a human hand. In this situation, the piezo works as an accelerometer. If the handle accelerates, a voltage is generated by the piezo. If the handle is accelerated from rest to a constant speed, then the voltage drops to zero while the handle is moving at constant speed.

The arrangement used to impact a ball on the strings is also shown in Figure 9.6. The ball was mounted as a pendulum so that it could impact precisely at any selected spot and at exactly the same speed every time, regardless of which spot or which racquet was chosen. To control the impact speed, the pendulum was moved back off the strings by the same amount each time, using a mechanical stop to control the distance off the strings. When the ball was released, it swung towards the racquet and hit the strings at a fixed speed. The ball hit the strings at a low speed, much lower than the normal impact speed in tennis, but the location of the sweetspot has nothing to do with impact speed. The vibration level is larger at high racquet or ball speeds but the issue of interest is how the vibration level changes when the impact point is varied, and not how it varies when the ball speed changes.

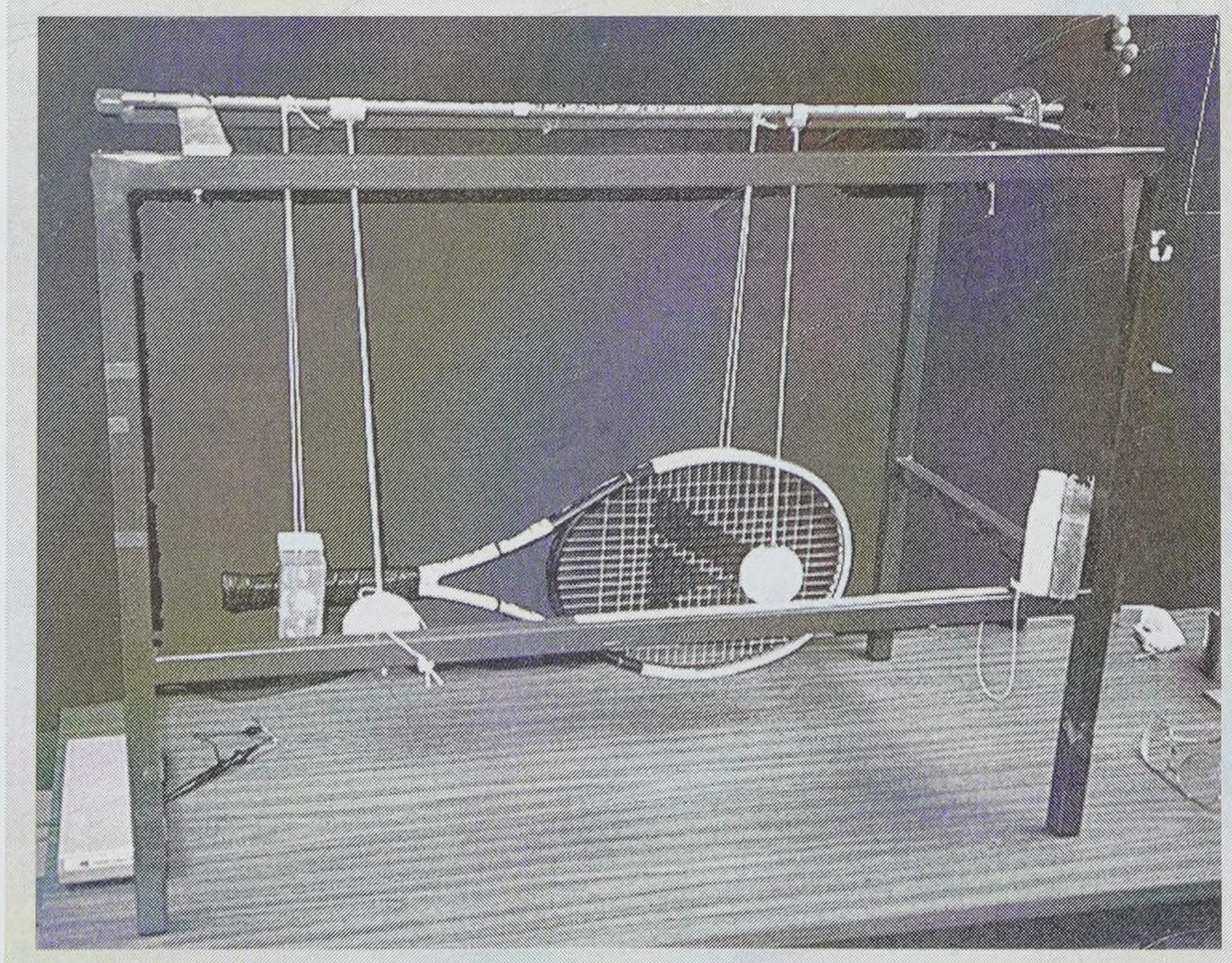


Figure 9.6

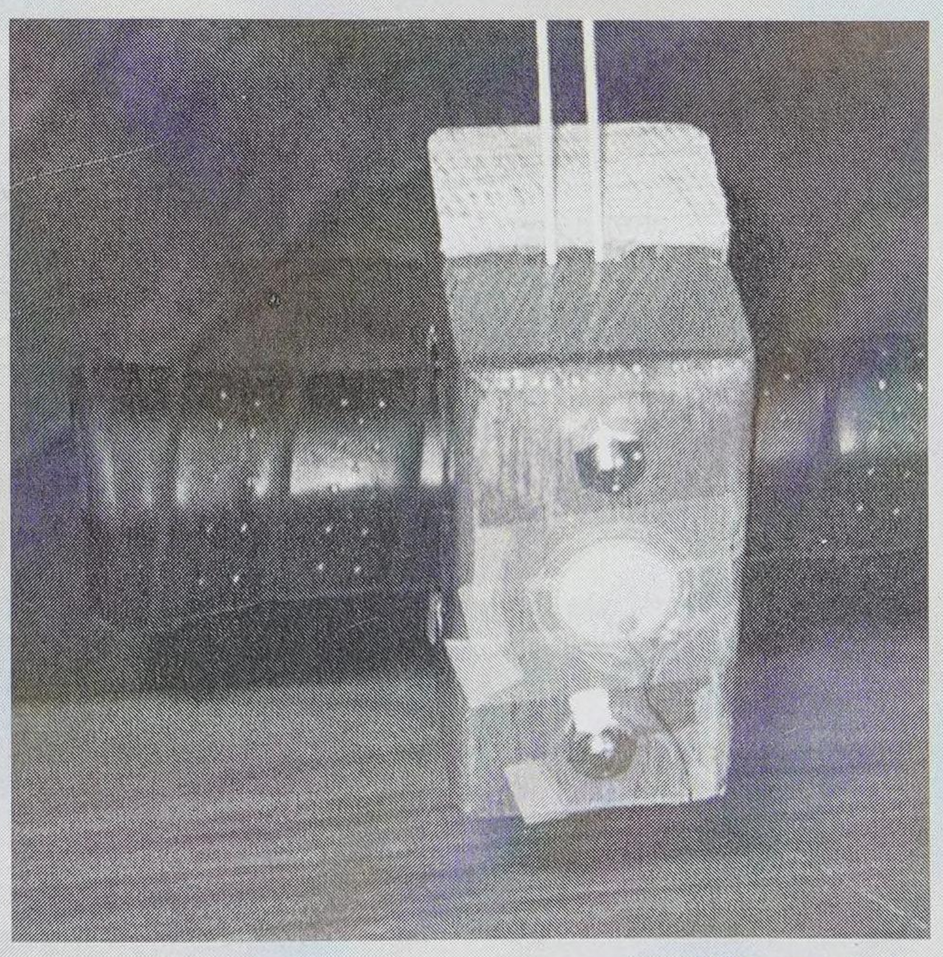


Figure 9.7

amplitude than the graphite racquets. The Triad vibrated much less than the wood racquet, but more than the Viper, at least at the beginning. The Triad has a vibration dampening rubber strip glued between the head and the V-section at the top end of the handle. As a result, the frame vibrations are damped much faster than in the other racquets.

Vibration Frequency of a Racquet

It is possible to measure the amount of vibration in a racquet, how it varies with the impact point on the strings, and how it varies from one racquet to the next. This provides a better indication of the actual properties of a racquet than the information commonly provided by the manufacturers. The manufacturers usually claim that their sweetspot is bigger and that there is less shock and vibration, but it would be much more useful if the manufacturers provided actual numbers on shock and vibration, in addition to the usual information provided on the weight, length and headsize of each racquet. In fact, only one number is needed, as we will see below. The number that is missing from the advertising brochures and magazines is the vibration frequency of the racquet.

The measurements show very clearly that as racquet stiffness increases, the vibration frequency of the racquet increases, and the vibration amplitude decreases. The amplitude drops to zero when the vibration frequency of the racquet is about 200 Hz or higher, depending on the impact duration and hence on the string tension and the impact point. A surprising result is that it is not necessary for a racquet to be infinitely stiff to stop all vibrations. It is necessary only that $f > 1/T$, where f is the vibration frequency, and T is the time that the ball is in contact with the strings before it bounces off. This time is known as the dwell time, or the impact duration. For example, if $T = 0.005$ s then f must be greater than $1/0.005 = 200$ Hz. Most racquets vibrate at around 140 Hz. Widebody racquets vibrate at around 170 Hz, but 200 Hz racquets are rare.

Most tennis players like to hit the strings of their racquet with their hand, or with another racquet, to check the string tension. Provided there is no string dampener, a nice ping tells them if the strings are tight or loose. Hardly anyone listens to the racquet itself. If you tap the frame or the strings with the end of your finger and hold the handle near your ear, you will hear the frame vibrating. It works best if you hold the handle lightly about 6 inches from the butt end. Holding it anywhere else will damp out the frame vibrations. A low pitch means the frame is heavy and flexible and a high pitch means the frame is stiff and light. More than anything else (apart from hitting the ball) listening to pitch will tell you how much vibration you can expect from the racquet. A medium pitch could mean that (a) the frame is heavy and stiff, or (b) it is light and flexible, or (c) it is medium in both weight and stiffness, but it is easy to figure out which is which just knowing the weight or the "flex" (stiffness) of the frame.

Sweetspot Softness and Feel

Racquet feel before impact is related primarily to the weight and swingweight of the racquet. The feel during and after impact is related to the sudden motion and vibration of the handle arising when a player hits the ball. When you hit a ball, what you feel is not the ball pushing on the strings, but the handle pushing against your hand. The impact of the ball on the strings lasts for less than

one hundreth of a second. This results in a sudden change in the motion of the whole racquet. The racquet head slows down more than the handle, with the result that your hand rotates backwards about an axis through the wrist. It takes about 50 ms after the impact for the racquet to stop vibrating and it also takes about 50 ms for your hand to stop rotating backwards. The initial sudden motion of the handle at impact generates a shock that depends primarily on where on the strings the ball is struck. Maximum shock occurs when the ball impacts near the tip or throat of the racquet. Minimum shock occurs at a sweetspot near the center of the strings called the center of percussion. Part of the initial shock is due to rotation of the racquet and part is due to the fact that the racquet also starts vibrating. Maximum vibration occurs for an impact near the tip of the racquet and zero vibration occurs at the vibration sweetspot. The two sweetspots are quite close together and are almost impossible to distinguish in terms of feel. They can, however, be distinguished in the laboratory by careful measurements of handle motion. In practice, an impact at the vibration sweetspot will result in a barely perceptible amount of shock and an impact at the center of percussion generates only a tiny amount of vibration.

If the handle slams into your hand at impact, the shot does not feel as nice as when the handle barely moves. Excessive vibration of the handle does not feel nice either. Minimum motion of the handle due to shock and vibration occurs when the ball strikes the sweetspot near the middle of the strings. Even then, not all racquets feel the same. Some sweetspots seem to feel different than others. The difference is mainly in the stiffness or the softness of the sweetspot. Several factors contribute to sweetspot softness. If the racquet has a soft sweetspot, the ball stays on the strings a bit longer, and the force on the hand will be smaller. This is partly due to differences in the string tension or gauge or type of string or headsize, and partly due to the stiffness of the racquet itself or the way the stiffness varies along the length of the racquet. A racquet can be stiff at one end and soft at the other, or it might be stiff at both ends and soft in the middle.

Comfort by itself is nice, but good players prefer to win points regardless of the comfort. If that means using a bone-rattler, then some players might like the way it feels simply because that's what works best for them. Alternatively, a player might like a racquet because it feels solid or powerful rather than because there is very little shock and vibration. Very stiff and light racquets give the least shock and vibration, but they are not necessarily as powerful as a heavier racquet.

Sweetspot Size

Another difference between racquets could be the size of the sweetspot. The sweetspot of a tennis racquet is not actually a tiny spot, but neither is it as big as some manufacturers would have you believe. Some manufacturers show diagrams where the sweetspot on their racquet is almost as big as the whole stringbed. To most of us, the sweetspot is a small region near the middle of the strings where the impact with the ball feels best. In this region, there is almost no shock or vibration transmitted to the hand or the arm. It feels almost as if you didn't actually hit the ball and you just swung at the air. It is not really that good, but it feels much better than a miss-hit where your arm and elbow and hand get tugged and shaken. If you hit the ball near the sweetspot, it still feels pretty good, but the further away from the sweetspot, the worse it feels, and the closer to the sweetspot, the better it feels.

Suppose you rank the spots on your racquet on a scale from 0 to 10 where 0 feels terrible and a 10 is perfect. A 9 is almost perfect and an 8 is pretty good. By definition, a 10 is a single spot. It is the only spot that feels perfect. But for all practical purposes, you would probably not be able to pick the difference between say a 9.5 and a 10. In that case, the spot is slightly bigger than a single point. In fact, the area that feels really good is probably about the size of a tennis ball. All points inside this area would be around a 9.5 or higher.

It seems that some manufacturers include all points that are about a 3 or above, since their sweetspot is so large. The only way to find out for sure is to measure the size of the sweetspot. One way to do that would be to mark a ball with colored chalk so you can tell where on the strings you hit the ball. That would tell you how big your sweetspot is for that racquet, but a shot that feels nice to you might feel less than nice to someone else. A better method is to measure the actual shock and vibration in the handle when the ball impacts the strings at various spots.

Shock and Feel

For any given impact point on the strings, the acceleration of the handle was about the same for the three graphite racquets, since the ball speed was the same, and since the weight and the swingweight of each racquet was about the same. The wood racquet accelerated more slowly since it was heavier and the swingweight was larger. The sudden motion of the handle due to the ball striking the strings results in a sudden shock that is felt by the hand and arm. The nature of this sudden motion is best illustrated by the results in Figure 9.4 for the Viper racquet. The vibration amplitude is much reduced for this racquet, but the impact shock occuring in the first 5 ms is still clearly visible. For the other racquets the effects of impact shock and vibration tend to merge together and probably feel about the same, resulting in a jarring feel that is part shock and part vibration.

The amount of shock depends on the impact point on the strings and it increases with the ball speed, but it also depends on the mass and stiffness of the racquet. Light racquets accelerate faster and will slam into the hand faster, but since they are light, the shock experienced by the hand is likely to be smaller than that felt by a heavier but slower racquet. To exaggerate the point, the shock felt by a heavy truck hitting you at low speed would be much greater than the shock felt by a small insect hitting you at high speed.

The shock of an impact depends on the following factors:

1. A ball hitting a light racquet will usually bounce at a lower speed than a ball bouncing off a heavy racquet if swing speed is the same or the racquet is stationary. As a result, the momentum transferred to a light racquet is less than the momentum transferred to a heavy racquet. Your hand absorbs this momentum by bringing the racquet to rest, and it will therefore absorb a greater momentum from a heavy racquet.

2. The shock depends on how big a force is exerted on your hand, and that depends on how long the force acts. For example, suppose you catch a baseball in your bare hands. You can lessen the shock by allowing your hands to move backwards as you catch the ball. That way you can stop the ball by exerting a smaller force for a longer time. If you don't move your hands backwards

then the ball will slam into your hands with a bigger force and the shock will be greater even though your hands absorb exactly the same momentum in both cases.

3. The shock felt through the handle of a racquet might depend as much on the softness and thickness of the grip as on the weight or stiffness of the racquet itself because it affects the duration of the force acting on your hand.

4. Soft strings in a stiff frame tend to give a soft feel, while stiff strings in a flexible frame tend to give a stiff feel. Experienced players can usually pick whether it is the frame or the strings that are stiff. Stiff strings will give a short impact duration, while a stiff frame will vibrate less and at a higher frequency than a flexible frame.

5. Some racquets are stiff in the head and soft in the handle, others are the other way around and some are soft in the middle. Imagine hitting a ball with a racquet made from stiff graphite at one end and soft rubber at the other. If the racquet has a stiff head and a flexible handle, the shock transmitted to the hand will be quite small.

Recoil Speed of the Handle

The speed at which the handle slams into the hand at impact can be calculated from the results shown in Figures 9.1-9.4. The piezo signals represent the acceleration of the handle during and after the impact, starting with the handle at rest. The racquet was not swung towards the ball, but the other way around in this experiment. The racquet was suspended horizontally from a beam with two lengths of string and the ball was swung onto the strings as a pendulum. This way, the impact speed of the ball could be kept constant for every impact point, and the impact point itself could be accurately controlled. In effect, this is what an insect would see if it was sitting on the handle of a racquet that is swung towards the ball. The insect thinks that it is at rest and sees the ball coming in and bouncing off, but the insect itself jerks forwards or backwards depending on where the ball hits the strings.

Speed is acceleration multiplied by time, provided the acceleration remains constant. In Figures 9.1-9.4 the acceleration is not constant, but the handle speed can be found by calculating the area between the acceleration curves and the time axis. This was done on a computer and the results are shown in Figure 9.8. The results plotted in Figure 9.8 give the maximum handle speed for each impact point on the strings. The handle speed increases to a maximum just as the ball leaves the strings, and then

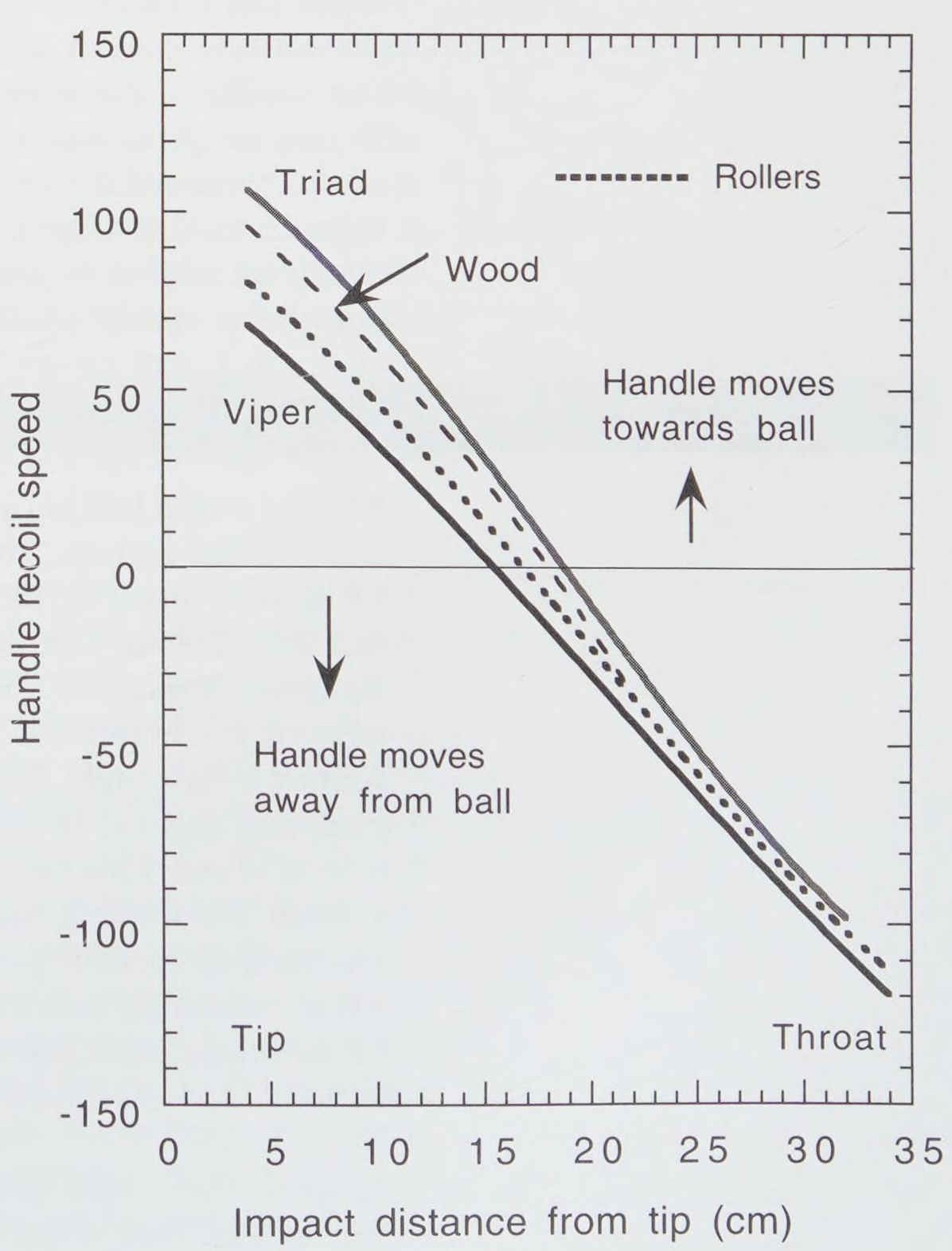


Figure 9.8

decreases and increases rapidly with time due to vibration of the handle. Vibration of the handle acts to increase the speed of the handle just as the ball leaves the strings. At this instant (or shortly afterwards) the handle has its maximum rotation speed and it also has its maximum vibration speed, both causing the handle to move in the same direction at the maximum possible speed. It would be nice if the handle rotated one way and vibrated the other way, but this would just delay the time at which the handle reached its maximum speed. If rotation and vibration are in opposite directions during the first half cycle of the vibration, then they are in the same direction during the next half cycle.

For an impact near the tip of the racquet, the handle swings around towards the incoming ball and the handle speed is taken to be positive. For an impact near the throat, the handle recoils away from the incoming ball and the handle speed is taken to be negative. The impact point where the handle speed remains zero during the impact is the center of percussion. Figure 9.8 shows that:

(a) the handle speed is greatest for impacts near the tip and throat of the racquet,

(b) the handle speed is about the same for all four racquets, and

(c) the center of percussion (where the handle speed is zero) is located about 16 cm from the tip for all four racquets, but is a bit closer to the tip for the Viper and a bit closer to the throat for the Triad.

The handle of the heavy wood racquet actually rotates at the slowest speed of all four racquets, but it also vibrates the most. As a result, the combined speed of the handle due to rotation, translation, and vibration is just as large, if not larger, than the other racquets. The Triad was modified before these measurements by adding an extra 7 gm grip to the handle. As a result, the center of percussion shifted slightly towards the throat.

Impact Duration, Force, and Feel

When a tennis ball bounces on a tennis court it spends only about 4 ms in contact with the surface. The time depends on the stiffness of the ball, but it doesn't depend much on ball speed. A high speed ball might spend 3.5 ms in contact with the surface, compared with about 4.5 ms for a very slow ball. The decrease in time for a faster ball is due to the fact that a ball gets a bit stiffer when it is compressed a long way. A soft ball will remain in contact with the surface a bit longer. Similarly, the contact time on the strings of a racquet is extended slightly, to about 5 or 6 ms, due to the softness of the strings. The time is about 5 ms on a stiff stringbed and about 6 ms on a soft stringbed. As a result, the force acting on a stiff stringbed is larger than on a soft stringbed. The racquet head is pushed backwards by the impact and the whole racquet rotates about an axis through the wrist. The rate of rotation is relatively slow for an impact on a soft stringbed, but the racquet will rotate further during the impact.

Suppose that a ball bounces at the same speed off stiff strings as it does off soft strings. That is not quite correct, but the difference is only a few percent. If the bounce speed is the same, then the change in momentum of the ball is the same, and the momentum given to the racquet is the same. If the racquet

mass is also the same then the speed of the racquet at the end of the impact will also be the same since momentum = mass x speed. On soft strings, the force on the ball or the racquet is relatively small, but it acts for a long time. The soft-string racquet reaches the same speed as the stiff-string racquet, but it took longer to get there so it traveled a greater distance. The hand absorbs the momentum of the racquet in much the same way as it does when you catch a ball. The shock of the impact will therefore be less using a racquet with soft strings.

Effect of Frame and String Stiffness on Frame Vibrations

The results for the Viper racquet are especially interesting since there are almost no vibrations at all for an impact in the throat region, and since the vibration amplitude is relatively small for an impact everywhere on the stringbed. It is easy to understand that there are no vibrations at a vibration node in any racquet, regardless of how stiff or flexible it is, but the Viper has a small vibration level everywhere. Vibrations result from the fact that a racquet bends suddenly at the impact point, especially if the ball impacts near the tip or throat of the racquet. The only exception is an infinitely stiff racquet which will not bend at all, and hence, there will be no vibrations. The Viper seems to be almost infinitely stiff in the throat region and less stiff near the tip. It is true that this racquet is the stiffest of the four tested, but it is only slightly stiffer than the Rollers. So why should the vibrations disappear almost completely for an impact at the throat?

The answer is related to the vibration frequency of the racquet and the duration of the impact. If a golf ball is dropped on the strings of the Viper in the throat region, then the racquet frame vibrates just like any other racquet. The vibration amplitude is smaller in the Viper than when a golf ball is dropped on the other racquets, but at least there is a relatively large vibration, and it is almost as big as the initial shock caused by sudden motion of the handle. The difference between a golf ball and a tennis ball is that a golf ball is lighter and stiffer. As a result, a golf ball bounces off the strings sooner than a tennis ball. Furthermore, it bounces off the strings before it has any significant effect on damping the vibrations. A tennis ball sits on the strings long enough to act as a vibration dampener.

A racquet vibrates at a particular frequency because it takes a certain time for the initial bending wave to travel along the frame, bounce off the ends and return back to the starting point. The speed of the wave depends on the stiffness and lightness of the frame. If the wave travels at a speed of 140 m/s along a 70 cm long racquet it will bounce up and down the racquet 100 times per second and the racquet will vibrate at 100 Hz. If the wave bounces up and down 200 times per second, then the racquet vibrates at 200 Hz. If the ball is pushing on the strings, then it acts in the same way as putting your finger on a vibrating guitar string. It damps the vibrations. The longer your finger is on the string, the more it damps the vibrations. A wave that travels up and down the racquet 100 times per second takes 1/100th of a second = 10 ms for the round trip or 5 ms from one end to the other. If the ball bounces off the strings after 5 ms, it doesn't get in the way of the wave, so the wave keeps propagating up and down the racquet and the racquet keeps vibrating. But it takes only 2.5 ms for a one-way trip when the racquet vibrates at 200 Hz and a tennis ball DOES get in the way. It damps the frame vibrations strongly. A golf ball sits on the strings for a shorter time and it doesn't damp the vibrations as strongly. A

similar effect occurs with a guitar string or a piano string. A string vibrates strongly if it is plucked or given a quick blow, but it won't vibrate strongly if you push it sideways and release your finger slowly. The finger or impact hammer must be removed from the string very quickly, otherwise it damps the vibrations.

The vibration frequency of a racquet frame is typically about 140 Hz, but it is higher in stiff racquets and lower in flexible racquets. The impact between a ball and a racquet lasts for about 5 ms (0.005 seconds). The impact duration is longer at low string tension, shorter at high string tension, and increases as the impact point moves from the tip to the throat of the racquet. If F = vibration frequency and T = impact duration, then the impact will cause the racquet to vibrate if F is less than 1/T, but the racquet will not vibrate if F is larger than 1/T. Most racquets do vibrate since 1/T is typically about 1/0.005 = 200 Hz. It is not necessary for a racquet to be infinitely stiff for the vibrations to disappear. All racquets have a sweetspot near the middle of the strings where no vibrations are excited, but frame vibrations are excited at all other impact points, especially near the tip and throat of a racquet.

The above explanation accounts for the fact that there are no vibrations in the Viper for a throat impact, but what about an impact at the tip? The answer is that the impact duration at the tip is shorter than the impact duration at the throat. In effect, the tip is lighter than the throat since the tip accelerates away from the ball faster than the throat due to the greater rate of rotation of a racquet for an impact at the tip. As a result, the ball sits on the strings for a shorter time and the damping effect is weaker. A similar effect occurs if the string tension is increased. A ball will sit on the strings for a shorter time if the tension is increased, with the result that the racquet will vibrate more for any impact away from the sweetspot.

The above explanation also accounts for the fact that higher frequency modes of a racquet are never seen when a racquet strikes a tennis ball. Like a guitar string, a racquet can vibrate at many different frequencies, but in practice the only mode of any importance is the fundamental (lowest frequency) mode at around 100 to 200 Hz. In order to observe the high frequency modes, it is necessary to impact the strings with a small steel ball so that the impact duration is much smaller than the vibration period. Even higher frequency modes, up to 10 kHz or higher, can be seen if the frame itself is struck with a metal object so that the impact duration can be reduced to 100 microseconds or less. Detailed calculations of impact duration and vibration amplitude for a tennis racquet can be found in my article in Sports Engineering (2000), 3, 111-122.

String Vibrations

There is one small point that appears to be inconsistent with the arguments presented above. If an impact of duration 5 ms can excite vibrations only up to 200 Hz, then how come the strings can be heard to vibrate at around 500 Hz? The ping from the strings arises from the fact that the strings vibrate like guitar strings, at a frequency of about 400 Hz at low string tension and up to 600 Hz at high string tension. If the ball can damp the frame vibrations then it should do an even better job at damping the string vibrations. In fact, the ball does help to damp the string vibrations but it doesn't sit on all the strings so some of them are not well damped. A string dampener located in the throat also damps the vibrations but it takes about 100 ms to damp them complete-

ly. The ball sits on the strings for only about 5 ms. However the main point is that while most of the energy in a 5 ms impact is concentrated in exciting vibrations at frequencies below 200 Hz, there is a tiny amount of energy available to excite vibrations at frequencies above 200 Hz. The strings are so much lighter than the frame that it takes only a tiny amount of energy to start them vibrating. They sound louder than the frame not because they have more vibration energy but because (a) they push slightly more air around and generate sound waves more efficiently than the racquet frame and (b) the ear is about 20 times more sensitive to vibrations at frequencies around 500 Hz than at frequencies around 100 Hz. It is impossible to hear anything at frequencies below about 20 Hz.

Measurements of the Force on the Hand

The force acting on the hand for each of the four racquets was measured using a small piezo disk located on the handle under the base of the first finger. In this case, the piezo signal is proportional to the force on the piezo, not its acceleration. A piezo will generate a large voltage even if it is at rest on a table, provided a force is applied to both sides of the disk. The disk was only 9 mm in diameter and 0.3 mm thick and it was pressed onto the handle using a firm grip. The two thin wire leads soldered to the disk were anchored with tape and the voltage generated when a ball was dropped on the strings was measured with an oscilloscope and also by a data acquisition system connected to a PC. The results are shown in Figures 9.9, 9.10 and 9.11. Some variation in the size of each signal was due to variations in the flatness of the handle, but the variation was no more than 30%, and it

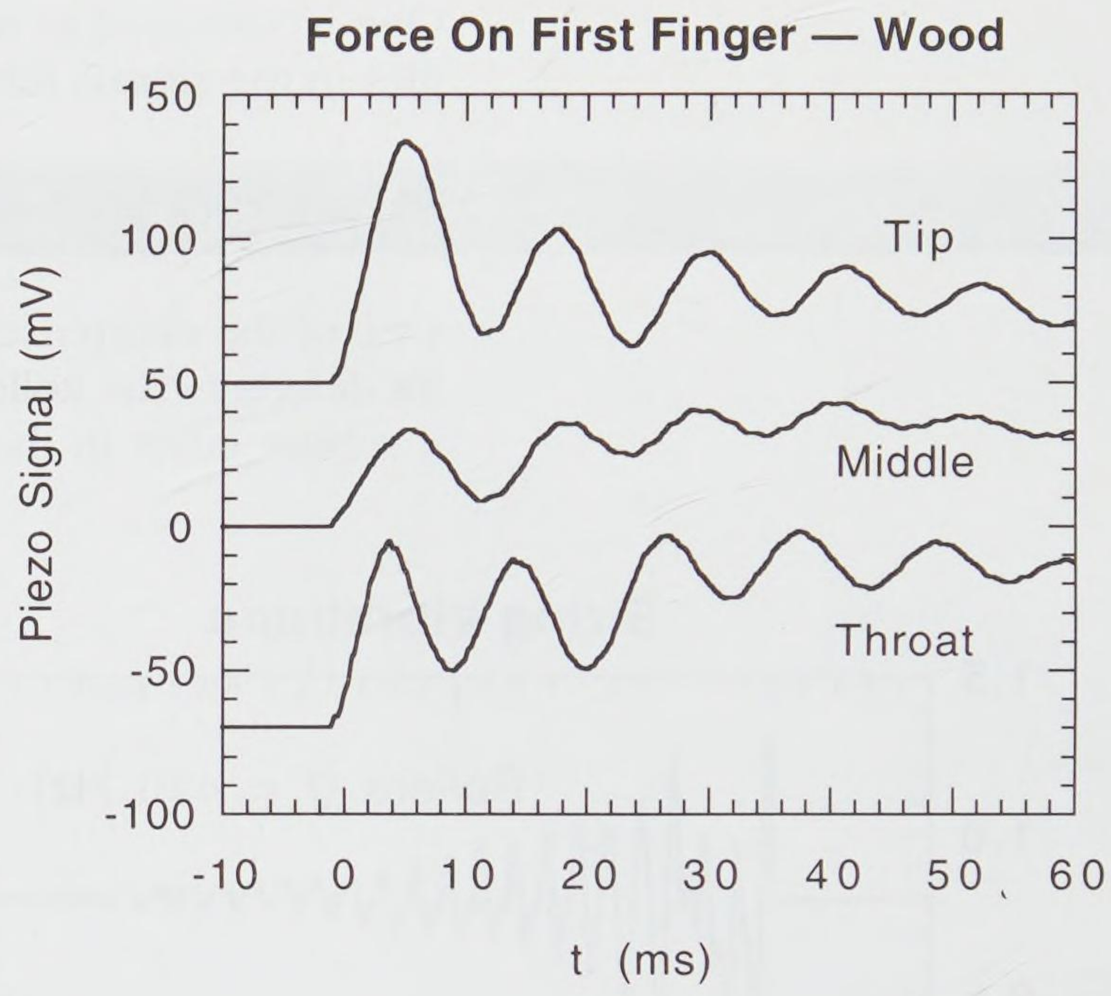


Figure 9.9

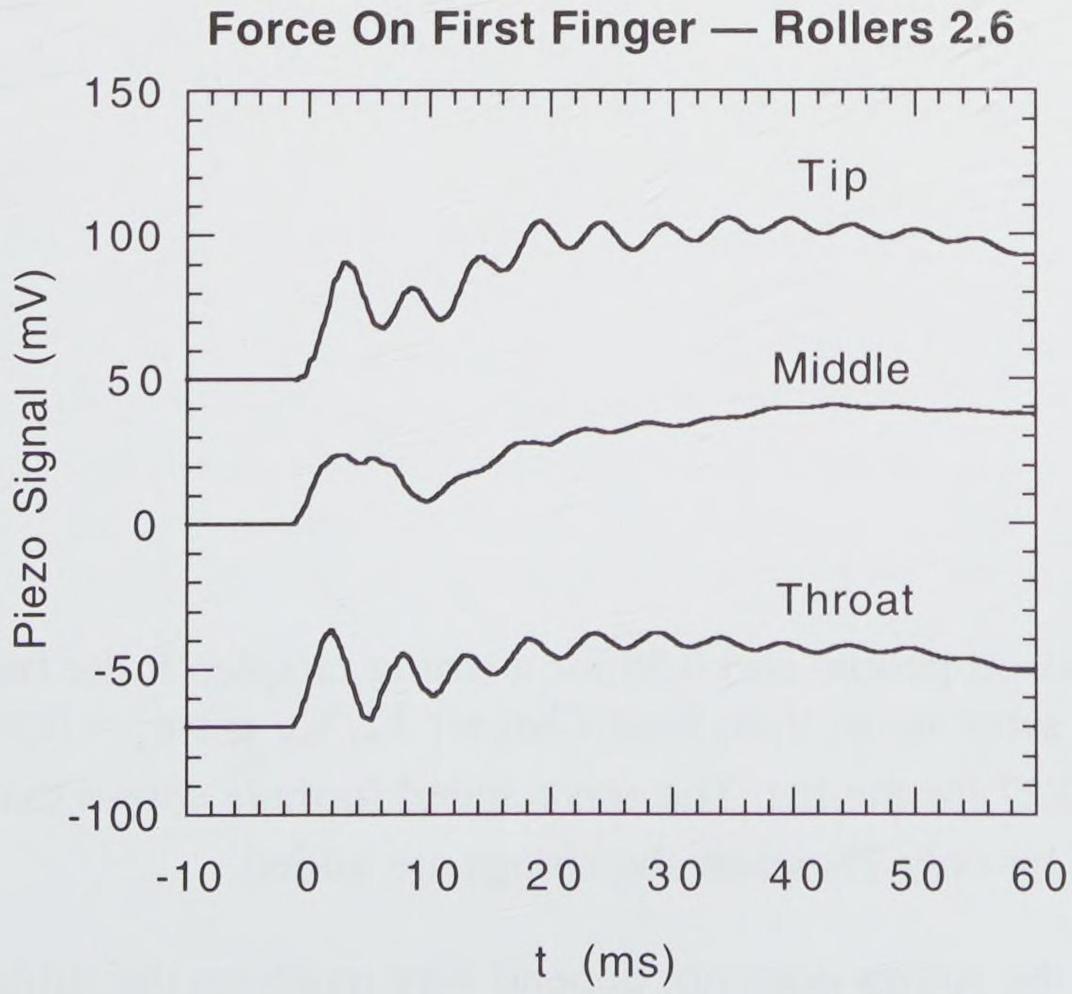


Figure 9. 10

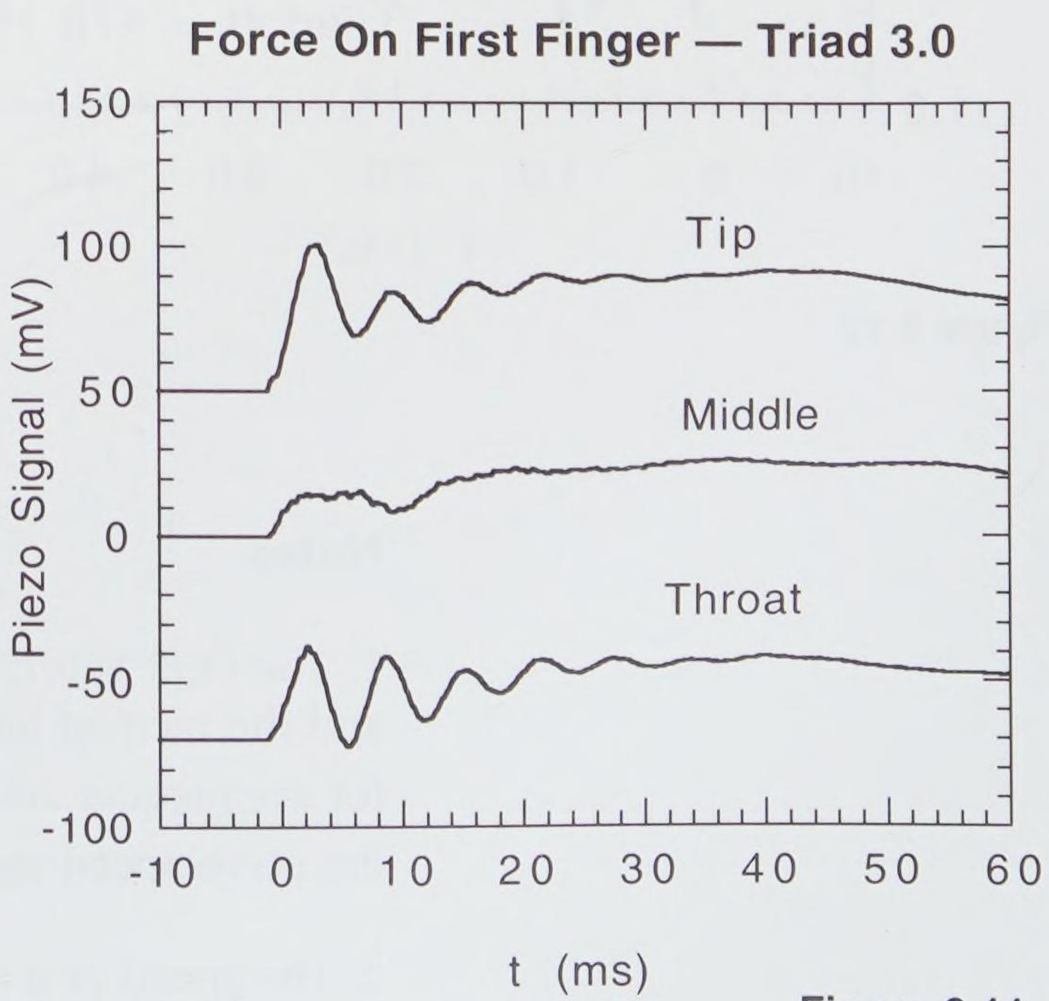


Figure 9.11

had no effect on the shape of each signal. The disk was removed and replaced many times to check this. The signals were obtained by dropping a tennis ball from a height of 40 cm onto the tip, middle or throat region of the strings. This is a very low speed impact, but the signals had the same shape at higher speeds — they were just bigger.

Vibration of the handle is minimized for an impact in the middle of the strings for the three graphite racquets. For an impact near the tip or throat, the vibration level is much larger, especially for the wood racquet. However, there is very little difference between the two graphite racquets. There are minor differences, but there is no particular feature of the force signals that one could identify as being responsible for a difference in the feel of the two racquets. One would need to measure lots more racquets to identify which of the wriggles in the signals felt nice and which caused jarring.

String Vibrations and Feel

One of the differences between the Rollers and the Triad is that they sound a lot different. The Rollers has two throat sections. The main strings pass through a rubber insert in the top throat section. As a result, the strings are strongly damped, even without a string dampener. The addition of a string dampener stops the strings vibrating quickly, as shown in Figure 9.12. For comparison, vibrations of the Triad strings with the same dampener inserted is shown in Figure 9.12. An impact on the Rollers generates a dull thud, while an impact on the Triad generates a short ping. The string vibrations can also be felt through the handle, but only when the string dampeners are removed. Vibrations due to the strings are much weaker than the frame vibrations but they last for a longer time when the dampeners are removed.

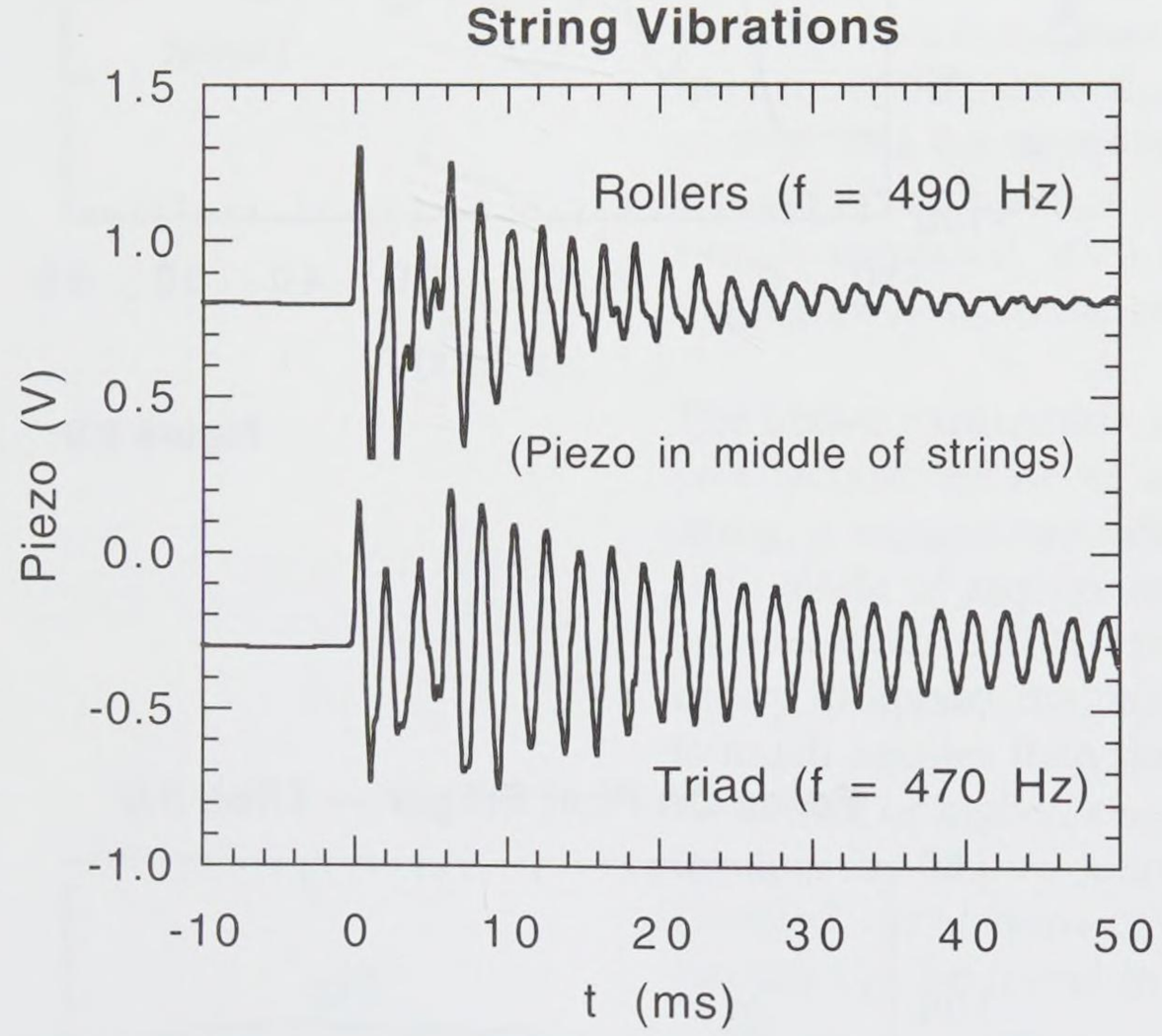


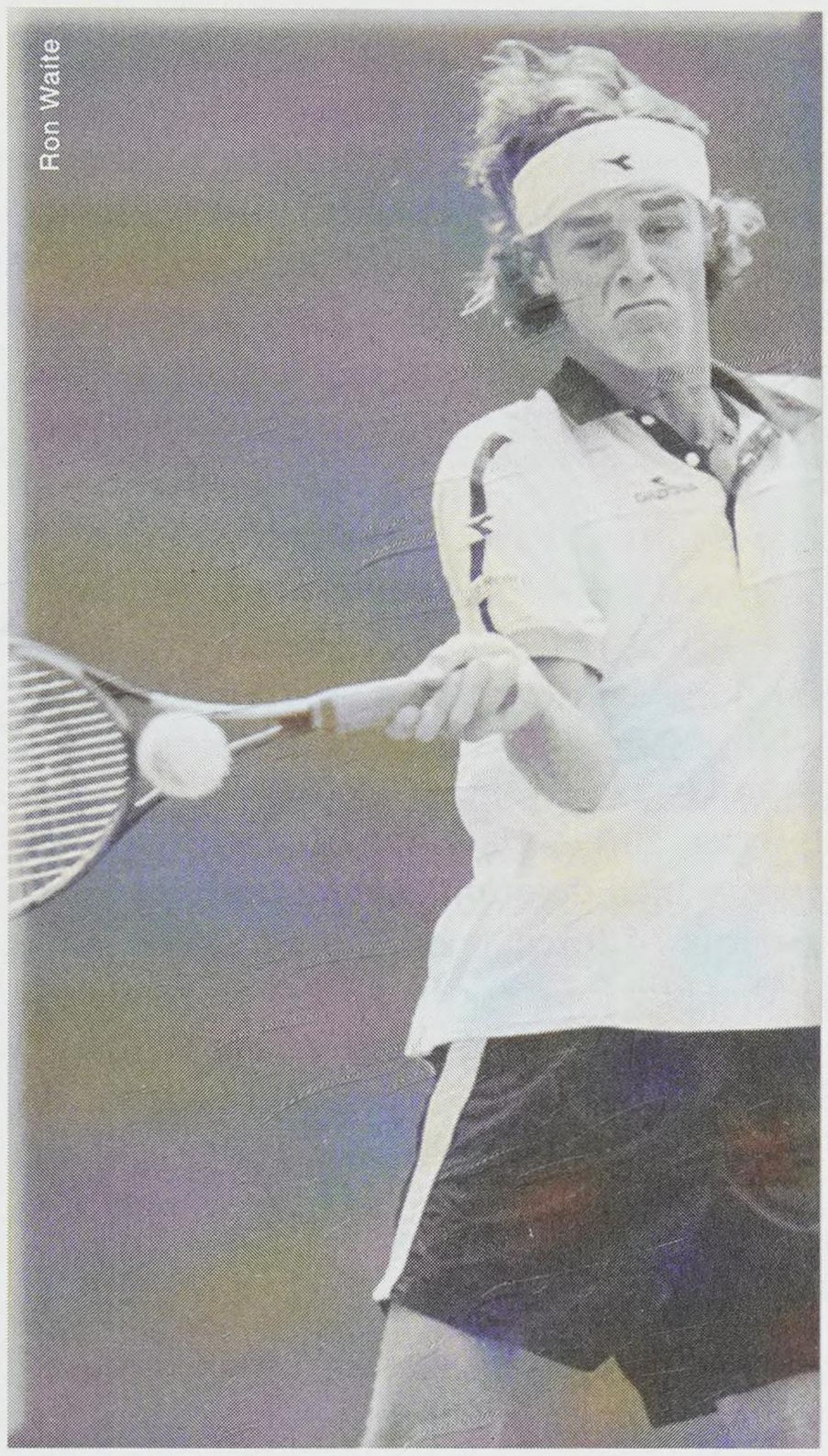
Figure 9.12

Notes

1. The COR is 0.75 for a wood paddle and 0.88 for a strung racquet. If the bat and the racquet have the same mass, then from Chapter 12, we get $e_A = 0.36$ for the racquet and $e_A = 0.27$ for the bat. The serve speed formula shows that the serve speed increases by only 7% when the strings are added.

2. The speed of a ball off the strings does not depend very much on the string tension. The ball speed depends mainly on how fast the racquet is swung. The

increase in momentum of the ball is accompanied by an equal and opposite decrease in momentum of the racquet. The change in momentum = (force on the strings) x (impact duration) and change in angular momentum = torque x (impact duration). The force is large and the impact duration is small with stiff strings and vice-versa with soft strings. The change in momentum and the decrease in the racquet head speed is essentially the same regardless of whether the strings are stiff or soft. However, the racquet head speed drops faster if the strings are stiff, and the racquet rotates through a relatively small angle during the impact. Conversely, if the strings are soft, the force on the racquet is small and it takes a longer time to reduce its speed. As a result, the racquet rotates through a relatively large angle during the impact. In each case the final and average speed of the racquet is the same, so the distance traveled during the impact is proportional to the impact duration.



Gustavo Kuerten, Brazil

10

Force and Momentum

An Introduction to Racquet-Ball Collisions

By Crawford Lindsey

Introduction

Tennis has become a power game. Everyone wants to know how fast they are hitting the ball. This chapter and the next are first steps in determining just that. There is just one caveat, however. We will assume that all collisions described in Chapters 10 and 11 are occurring on a line through the center of mass of the racquet and ball. In other words, we are pretending the ball is hitting the racquet at the balance point where the mass involved in the collision is the same as the racquet's mass. This makes the explanations simpler. Chapter 12 will then examine how to apply these concepts to a hit anywhere on the racquet. In that chapter we will define "effective mass." It is the effective mass that would then be used in all the formulas in this chapter.

What happens when a ball hits a racquet? Some, or all, of the following events, among others, may occur, depending on where you hit the ball on the racquet:

- strings stretch
- ball deforms
- racquet bends longitudinally
- racquet head bends in and out
- racquet hoop twists like wringing a towel
- racquet shaft twists like a screw

- strings vibrate
- racquet vibrates
- arm vibrates
- ball vibrates
- wrist bends backward
- string tension increases
- racquet slows down
- racquet rotates longitudinally backward head over heels
- racquet rotates around long axis towards 3 or 9 o'clock
- ball decelerates
- ball stops
- ball changes direction
- ball accelerates
- ball leaves

These events, which occur in a few milliseconds (4-8 ms depending on several factors), represent a flow of energy and forces — those mysterious and magical "somethings" that pervade the universe and that are inextricably bound up in hitting a tennis ball. This single event, which determines the course of a tennis match, is variously described in terms of force, momentum, energy, and coefficient of restitution. These are the big four concepts that explain the dynamics involved in purposefully colliding a racquet and a ball into each other in a tennis shot. Of course, Sir Isaac Newton glimpsed all of this in his falling apple that led to his laws of motion.

Newton's First Law of Tennis

A tennis ball in motion tends to stay in motion;
a tennis ball at rest tends to stay at rest.

Let's get one thing clear from the outset — the ball is not a force by itself. The ball is not traveling "with a force" of 30 mph or of anything else. Once in flight, no force is causing the ball to move along its journey. In fact no forces are acting on the ball at all, except air resistance and gravity, which we will ignore since they have nothing to do with the ball/racquet collision, which is the essence of tennis. If you don't think the world is magical, just consider that no forces at all are necessary to keep this ball speeding toward its fateful collision. It just goes. Galileo and Isaac Newton figured this out in the 1600s. Newton formulated three laws of motion, which, for the most part, describe virtually every physical event we witness, but more importantly, every tennis event. Newton's First Law is that any object will keep on doing what it is doing until something stops it from doing so. That "something" is a force. You've probably heard it paraphrased as "an object in motion tends to stay in motion, and an object at rest tends to stay at rest." This "tendency" is also known as inertia and it is measured by its mass. The more massive an object is, the harder it is to change what it is doing or not doing. There is no privileged status for an object at rest or in motion. They are both equivalent natural states of being, requiring nothing to maintain their states. To emphasize this equivalency, we sometimes refer to a resting object as having inertial mass and to an object moving force-free as having inertial motion. In each case, it is the same inertia (mass) that a force must overcome to change the ball's velocity.

By itself, a racquet isn't a force either. It, like the ball, is just a mass of stuff. There are stationary masses and there are moving masses. And whenever two

masses come in contact with each other, they create a force. Each mass exerts a force on the other. There is no force until there is an interaction. The two colliding objects "decide" what the force between them is going to be. Two separate sets of two objects traveling at the same speed with the same masses will create different size forces depending on the properties of the colliding objects such as stiffness. This goes for two different racquets with equal mass hitting balls also. The force created at impact with a ball will depend on the properties of the racquet like string tension and racquet flex, among others.

Newton's Second Law of Tennis

The force required to change a tennis ball's speed or direction is equal to the ball's mass times its acceleration.

Strange but true — a force can never be seen. Only the effects of the force can be seen. A force shows itself by the changes in the state of motion of the tennis ball. A force has acted if the ball changes its direction or speed. Speed in any given direction is known as velocity, so a force has acted if the ball changes its velocity (direction or magnitude). Changing a ball's velocity is the entire life's purpose of a tennis racquet. Changing velocity from 30 mph to 50 mph or from 50 mph to 30 mph means that you have accelerated (or decelerated) the ball. Newton realized that a force becomes manifest as the acceleration of a mass. His famous Second Law of Tennis thus became

$$F = ma \qquad (10.1)$$

where,
F = the force,
m = mass of the object, and
a = acceleration.

Notice that if the mass of the racquet or ball is doubled, the force necessary to achieve the same acceleration is doubled. Or said differently, if the mass is doubled and the force remains constant, the acceleration will be halved. Since all balls have about the same mass, changing the mass is only relevant to the racquet.

Remember, this force isn't seen, only inferred. The force acting on the ball was equal to the ball's mass multiplied by how much it was accelerated. The force acting on the racquet was the racquet's mass multiplied by its acceleration. As you can see, this force, the mover of all things, is hard to pin down. At this point you might ask, "How are we to measure this force? And what about acceleration? How are we to determine our final ball velocity from this equation?" We will see below that we don't actually have to measure forces to figure out our ball velocities. But we do have to understand the steps in between which build on each other to attain our goal of determining these velocities. Newton's Third Law of Tennis provides us with the next step.

Newton's Third Law of Tennis

The force of the ball on the racquet is equal and opposite to the force of the racquet on the ball.

The Third Law states that whenever two objects like a racquet and ball inter-

act and create a force between them, the force exerted by the ball on the racquet is equal and opposite to the force exerted by the racquet on the ball. Both the ball and the racquet experience the same magnitude force but in opposite directions. But we usually say the racquet hits the ball, and not vice versa. This makes sense and it is obvious that it is the ball's behavior that we are trying to influence. But sometimes our language used in such descriptions gets in the way of our understanding. Our usual descriptions of collisions are shaped by three things — which of the objects is moving before the collision, which object's behavior appears to be most changed, and what is our intent for performing the collision. When a ball is thrown against a wall, we say, "the ball hit the wall," rather than "the wall hit the ball." This is true, even though both the wall and the ball experience the same force. It is also true even though it is the motion of the ball that is changed, not the wall's, even though the wall was the recipient of the "hit." The reason our language describes it this way is because the ball was "doing" the action.

When a racquet is swung at a tennis ball, we say, "The racquet hit the ball," even if both ball and racquet are moving. In this case, the object being "hit" — the ball — changes its speed and direction the most. Also, we are hitting the ball with a purpose, so we say, "I hit the ball."

These scenarios show how language can corrupt and influence our understanding of an event. If you hit the ball, the force is not in the racquet. If the ball hits the wall, the force is not in the ball. The force is generated between the ball and racquet at impact and they both experience the same force. This is the essence of Newton's Third Law. When a racquet and a ball collide (or any two objects), the force of the racquet on the ball is the same but in the opposite direction to the force of the ball on the racquet.

But if they both experience the same force, why is it only the ball that goes flying? The answer lies in Newton's Second Law — F = ma. The force on the ball and the racquet are the same, so the mass of the racquet multiplied by its acceleration due to the force will be equal to the ball's mass multiplied by its acceleration.

$F = \text{force}$	by the First Law	
$F = ma$	by the Second Law	
$\mathbf{F_r = F_b}$	**by the Third Law**	**(10.2a)**
$\mathbf{m_r a_r = m_b a_b}$	**by the Second and Third Law**	**(10.2b)**

where the subscript r is the racquet, and the subscript b is the ball.

The same force on the lesser mass ball will cause more acceleration of the ball than the racquet. Basically, the ball's mass is always the same (about 57 grams), but the mass of the racquet is totally up to you. So depending on the mass of your racquet, the force of the collision will accelerate the racquet different amounts, and the player will feel the results.

It's obvious when you analyze it, but it is also very wondrous, strange and beautiful. The bottom line is this: whenever two objects interact, be it a racquet and a ball, strings and the frame, or the handle and your hand, a force is created and all parties to the event experience the same force. The visible consequences of that interaction will depend on the relative mass and stiffness of the interacting objects.

Again, I hear you saying, "Interesting, but I still can't determine how fast I'm hitting." True, but this very important fact that the force is the same on both the racquet and ball allows us to set up an equality — Fr = Fb — that is a key step in our quest. The next step comes from defining and calculating momentum, to which we now turn.

Momentum

During an impact, the force on the ball and the racquet are the same. Nature is apparently very fair. It dispenses its resources with great equity. There's another example of this equanimity that lies behind Newton's laws. Whenever the racquet and ball collide, the change in the momentum of the ball is equal and opposite to the change in the momentum of the racquet. That's odd. We know for a fact that the ball's behavior doesn't look equal to the racquet's behavior in any way. They may experience the same force, but they don't act the same way. So, what is this mysterious momentum?

We've seen that mass is defined as the amount of matter there is. It is harder (requires more force) to change the motion (accelerate) of a more massive object because there is more matter to move. Once the inertia of the mass is overcome, the object becomes a moving mass. There are two variables here: the mass and its velocity. Moving mass is mathematically represented as mv (mass multiplied by velocity) and momentum is represented as p. So momentum equals mass times velocity, or, p = mv.

Momentum represents how much stuff is moving and how fast. So, given two racquets traveling at the same speed, but with different masses, the heavier racquet will pack a bigger punch since it has more momentum. Likewise, given two racquets of the same mass but traveling at different velocities, the faster racquet will pack a greater wallop.

The easiest way to conceptualize momentum is to consider what it takes to stop a moving object like a tennis ball. We learned from Newton's First Law that changing an object's velocity requires a force. It takes a force acting for a period of time to bring an object to a stop. It takes more force acting for the same amount of time to stop a tennis ball traveling at 27 m/s (approximately 60 mph) than one travelling at 13.5 m/s (approximately 30 mph). A tennis ball has a mass of approximately 57 grams or .057 kilograms. The momentum (quantity of motion) is mv. This calculates as follows: 27 m/s x .057 kg equals 1.35 kg·m/s for the faster ball, and 13.5 m/s times .057 kg equals .77 kg·m/s, for the slower ball. The faster ball has twice the momentum and requires twice as much force acting for the same amount of time to stop it. Or, as we will see, it requires the same force acting for twice as much time.

Impulse = Change in Momentum Theorem

This is intuitively obvious, but it also follows directly Newton's Second Law. We can rewrite this law as follows:

By definition, acceleration is the change in velocity over time:

$a = (v_2 - v_1)/ t$

where v_2 = final velocity, and

v_1 = initial velocity.

So, the following rewriting of Newton's Second Law brings us straight to the Impulse Momentum Theorem. We start with the Second Law:

$F = ma$

Substitute acceleration, a, with the equivalent expression $(v_2 - v_1)/ t$

$F = m(v_2 - v_1)/t$

Multiply each side by t to get rid of the fraction:

$$\mathbf{Ft = m(v_2 - v_1)} \qquad \textbf{(10.3a)}$$

Scientists use the symbol Δ for changes in anything between an initial and final state. So the above equation can be written in the following shorthand:

$$\mathbf{Ft = m\Delta v} \qquad \textbf{(10.3b)}$$

where Δv means "change in velocity." (We will go back and forth between these equivalent equations depending on our purpose.)

This says that a force acting for a given amount of time is necessary to change the momentum of an object. Or similarly, a force applied to a mass will accelerate it by changing its velocity over a certain amount of time.

This is an incredibly handy relationship (if only for the fact that velocity has finally shown up in our calculations, and we now are getting closer to determining how fast we are hitting the ball). The Ft part of the equation is known as the impulse and the mv is the momentum. So, in a collision of a ball and racquet, each object experiences an impulse equal in magnitude to the change in its momentum:

$$\mathbf{F_b t_b = m_b \Delta v_b}$$

Impulse on the ball = Ball's change in momentum

$$\mathbf{F_r t_r = m_r \Delta v_r}$$

Impulse on the racquet = racquet's change in momentum

Notice what else this relationship tells us. It says to change the ball's momentum, any combination of Ft that equals mΔv will do. A small force for a longer time can change the velocity the same amount as a larger force for a shorter time, provided the amount Ft is the same in each case. You can immediately see the consequences of dwell time (time of ball on strings) on force here. If the ball stays on the strings of the racquet for a longer time (by using softer string or lowering tension), then the force produced by the collision that is necessary to accelerate the ball a given amount will be less; it just will last longer. And as we will see, the force on the player will also be less. (Note: during a collision the force starts at zero, rises to a maximum, and goes back to zero. Therefore, we are assuming F = average force during the collision.)

We also see the following relationships in the impulse equation:

1. Force and time are inversely related. To produce the same change in the ball's velocity, if the force is increased, the time that force acts must decrease. If the time is increased, the force must decrease.
2. If we divide each side by m to get everything that influences a change in velocity grouped together, we get

$$\mathbf{Ft/m = \Delta v}$$

This says that for a given force, the change in velocity is inversely related to mass. The same force acting for the same time on a heavier ball will change the velocity less. But if a lighter ball is used, the same impulse (Ft) will cause a greater change in velocity.

In a collision, according to Newton's Third Law, the racquet and ball exert equal and opposite forces on each other. The observable consequences of those forces are quite different, however. The force on the smaller mass ball sends it across the net, while the same force on the much more massive racquet has little visually obvious effect. However, these consequences provide the basis for very powerful predictions, as we will see.

Now we are ready to derive our next conceptual step. If the force on the ball and racquet are the same but in opposite directions (represented by the minus sign below), then

$$F_b = -F_r$$

And if the ball and the racquet collide, then the ball touches the racquet the same amount of time the racquet touches the ball, so,

$$t_b = t_r$$

Therefore, the quantity Ft, the impulse, must be the same but opposite for the ball and the racquet, since t is equal for each and F the same but opposite:

$$F_b t_b = - F_r t_r$$

And if the impulse is the same but opposite, then the change in momentum must be the same but opposite for each (Equation 10.3a and 10.3b), since the impulse felt by an object equals the change in momentum of the object. So,

$$\mathbf{m_b \Delta v_b = - m_r \Delta v_r.} \qquad \mathbf{(10.4a)}$$

Change in ball's momentum = Opposite of change in racquet's momentum

Expanding Δv_b and Δv_r to $(v_{b2} - v_{b1})$ and $(v_{r2} - v_{r1})$, the equation looks like this:

$$\mathbf{m_b v_{b2} - m_b v_{b1} = - (m_r v_{r2} - m_r v_{r1})}$$

and multiplying by the -1 on the right side, we get,

$$\mathbf{m_b v_{b2} - m_b v_{b1} = m_r v_{r1} - m_r v_{r2}.} \qquad \mathbf{(10.4b)}$$

During an impact, the momentum gained by the ball equals the momentum lost by the racquet. Let's look at an example.

Example: Calculating final velocities with given information. Let's say that we are going to hit a 57 gm tennis ball and launch it at 27 m/s (about 60 mph). It is coming at us at 13.5 m/s (about 30 mph). The racquet mass is 300 gm and is swinging at 9 m/s (about 20 mph). The time of impact is .005 s. The change in ball velocity will be $(v_{b2} - v_{b1})$ = 27 - (-13.5) = 40.5 m/s. In other words, the ball's velocity actually goes from minus 13.5 m/s to 0 and then back up to 27 m/s. So the total change in velocity will be 40.5 m/s. The racquet's velocity does not change direction. The racquet slows down but continues in the same direction. Therefore, the change in velocity of the racquet will be $(v_{r2} - v_{r1})$. From this information, we want to know six things:

Question 1. What is the final velocity of the racquet?
Question 2.. What is the change in velocity of the racquet?
Question 3. What is the final momentum of the racquet?
Question 4. What is the change in the momentum of the racquet?
Question 5. What is the impulse during collision to change the racquet's momentum by the amount in 4 above?
Question 6. What is the force required to change the racquet's momentum by the amount determined in 4 above?

Figure 10.1 summarizes what we know starting out.

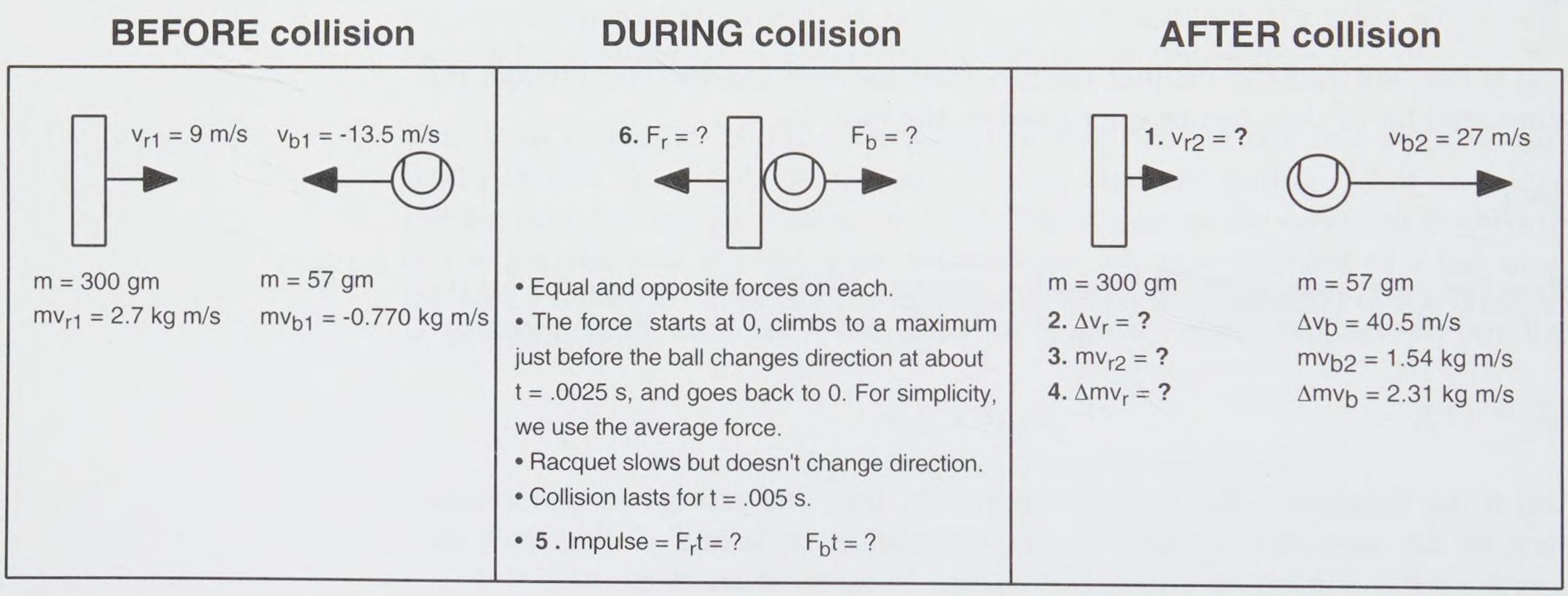


Figure 10.1
What we know and the six unknowns that we will determine.

Notice that the ball starts out with a negative velocity and momentum. This simply indicates that it is moving in the opposite direction than that of the racquet.

From equation 10.4b, we know that the change in the ball's momentum equals the opposite of the change in the racquet's momentum,

$$m_b v_{b2} - m_b v_{b1} = -(m_r v_{r2} - m_r v_{r1}) = m_r v_{r1} - m_r v_{r2}$$

and, from above, we know everything except v_{r2}. So let's plug them into the equation:

$1.54 \text{ kg·m/s} + 0.77 \text{ kg·m/s} = 2.7 \text{ kg·m/s} - 0.3v_{r2}$

Now we solve for v_{r2}

$0.3v_{r2} = 2.7 \text{ kg·m/s} - 1.54 \text{ kg·m/s} - 0.77 \text{ kg·m/s}$

$0.3v_{r2} = 0.39 \text{ kg·m/s}$

Answer 1: Final racquet velocity = v_{r2} = 1.3 m/s.

Answer 2: The change in the racquet's velocity is thus $v_{r2} - v_{r1} = 1.3 - 9 = -7.7$ m/s.

Answer 3: The racquet's final momentum is $m_r v_{r2} = .3 \times 1.3 = .39$ kg·m/s.

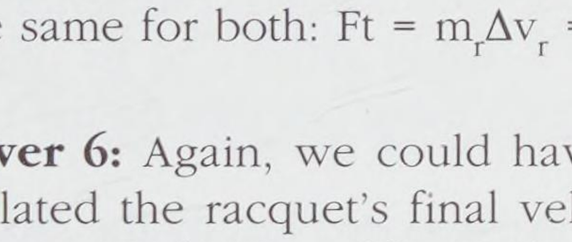
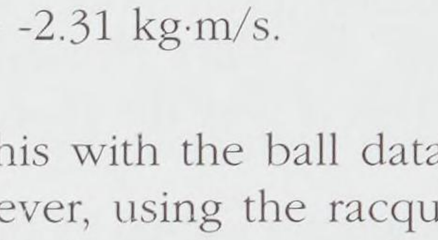
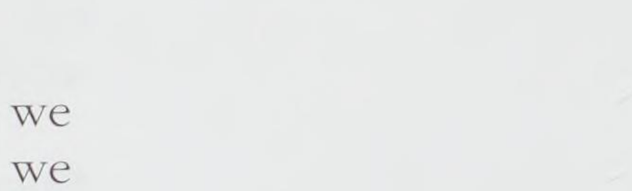
Answer 4: The change in the racquet's momentum is $m_r v_{r2} - m_r v_{r1} = .39 - 2.7 = -2.31$ kg·m/s. That is equal but opposite to the ball's change in momentum, just like the law of conservation of momentum tells us.

Answer 5: We actually knew the impulse (Ft) before finding v_{r2}. Because Ft = $m\Delta v$, and since we knew $m_b\Delta v_b$, we knew Ft. And because $m_b\Delta v_b = m_r\Delta v_r$, it is the same for both: $Ft = m_r\Delta v_r = .3 \times -7.7 = -2.31$ kg·m/s.

Answer 6: Again, we could have figured this with the ball data before we calculated the racquet's final velocity. However, using the racquet data, we get Ft = -2.31/.005 = -462 N. The force of 462 Newtons is equal to about 104 pounds (1 N = .2248 lb).

The results of our calculations are summarized in Figure 10.2.

Figure 10.2

BEFORE collision	DURING collision	AFTER collision
		
m = 300 gm mv_{r1} = 2.7 kg m/s	m = 57 gm mv_{b1} = -0.770 kg m/s	
	• Equal and opposite forces on each. • The force starts at 0, climbs to a maximum just before the ball changes direction at about t = .0025 s, and goes back to 0. For simplicity, we use the average force. • Racquet slows but doesn't change direction. • Collision lasts for t = .005 s. • **5.** $F_r t$ = -2.31 kg m/s $F_b t$ = 2.31 kg m/s	
		m = 300 gm **2.** Δv_r = -7.7 m/s **3.** mv_{r2} = .39 kg m/s **4.** Δmv_r = -2.31 kg m/s
		m = 57 gm Δv_b = 40.5 m/s mv_{b2} = 1.54 kg m/s Δmv_b = 2.31 kg m/s

It is interesting that the impulse can be any combination of force multiplied by time that, in this case, equals -2.31 for the racquet and 2.31 for the ball. If the time were .004 seconds, instead of .005 s we used above, the force would be 577.5 Newtons, and if the time were .006 seconds, the force would be 385 Newtons. If the time were .008 seconds, the force would be 288.8 Newtons. You can see that small changes in time make a big difference in the magnitude of the force. Impact time will be increased by more flexible racquets, softer strings, and lower string tensions.

We arrived at final velocities for the ball and racquet here, but only because we presented one of them, the ball, as a given at the beginning of the example, just to see how the relationships work. We still have yet to figure out the after collisions velocity based only on what we know before the collision. We now turn to The Law of Conservation of Momentum which sets up our equation to determine after collision velocities.

Law of Conservation of Racquet and Ball Momentum

In a collision, one object loses momentum and the other gains momentum. And since the change in the momentum of each is the same, the amount of momentum lost by one is the amount gained by the other. The momentum lost by the racquet was gained by the ball, and the momentum gained by the ball was lost by the racquet. In the above example, the change in the momentum of the ball was 2.31 kg·m/s, and the change in the racquet was -2.31 kg·m/s. The ball changed its momentum 124 percent and changed direction while the racquet changed its momentum 71 percent by slowing down but continuing in the same direction. Even though the ball and racquet changed momentum by equal amounts, the ball's momentum change is larger relative to its original momentum. That is why the change was so much more obvious on the ball.

Since the momentum gained by the ball is the same as that lost by the racquet, the total combined momentum of the racquet and ball is the same before and after the collision. This is true of every collision and is known as the Law of Conservation of Momentum. The conservation equation was lurking in equations 10.4a and 10.4b. Let's pick up where we left off in equation 10.4b with the change in the ball's momentum equalling the opposite of the change in the racquet's momentum:

$$m_b v_{b2} - m_b v_{b1} = m_r v_{r1} - m_r v_{r2}.$$

If we simply rearrange this so that initial states of the ball and racquet are on one side of the equation, and the final states on the other side, we get the Law of Conservation of Momentum for balls and racquets:

$$\mathbf{m_r v_{r1} + m_b v_{b1} = m_r v_{r2} + m_b v_{b2}} \qquad \mathbf{(10.5)}$$

Total Racquet and Ball Momentum Before = Total Momentum After Collision.

where,

m_r = mass of the racquet,

m_b = mass of the ball,

v_{r1} = initial velocity of the racquet,

v_{b1} = initial velocity of the ball,

v_{r2} = final velocity of the racquet, and

v_{b2} = final velocity of the ball.

Conclusion

The beauty of analyzing collisions in terms of conservation of momentum is twofold: you don't have to know the forces or the time of impact in order to describe changes in motion, and momentum is conserved in all collisions (unlike energy). You just set the sum of the momentums before the collision equal to the sum of the momentums after the collision and solve for your unknown velocities. For example, if we want to know the ball's velocity after impact, we solve equation 10.5 for v_{b2}:

$$m_r v_{r1} + m_b v_{b1} = m_r v_{r2} + m_b v_{b2}$$

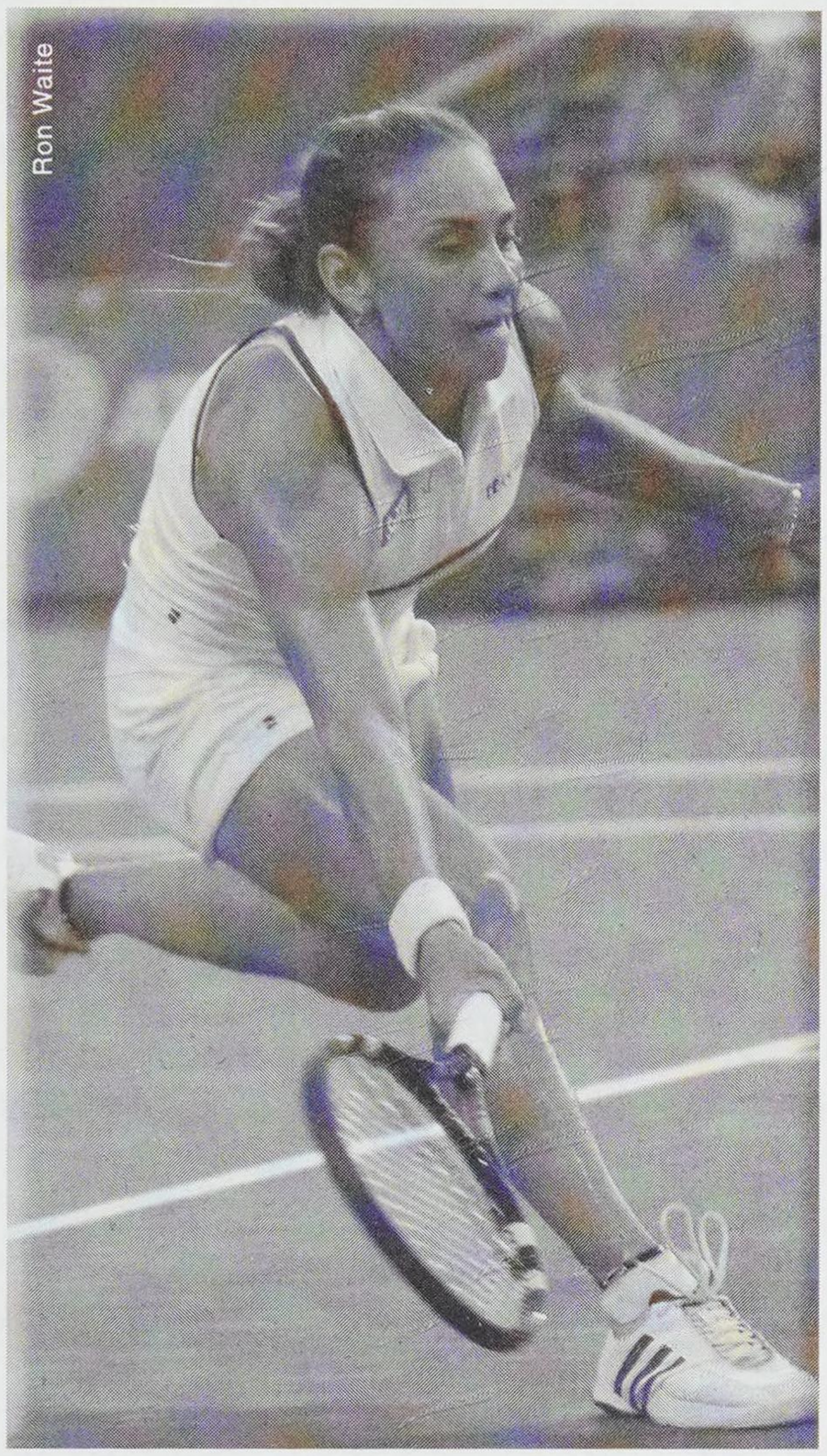
$$m_b v_{b2} = m_r v_{r1} + m_b v_{b1} - m_r v_{r2}$$

$$v_{b2} = \frac{m_r v_{r1} + m_b v_{b1} - m_r v_{r2}}{m_b}$$

There we have a formula for the final ball speed. But even if we use that, we have a problem. The problem, of course, is that we have two unknown velocities, v_{r2} and v_{b2}, so we can't solve for one without knowing the value of the other. So, as close as we are to calculating our final velocities, we will need a second independent equation to solve for one of the unknowns, and then plug it back into our momentum equation. We will find that equation and more in our discussion of energy in Chapter 11. There we will demonstrate two different ways to determine ball velocities — one using conservation of of momentum combined with the "coefficient of restitution," and another using only the "apparent coefficient of restitution." The latter solution will be our "physics Rosetta Stone," leading to what we call "the most important formula in the book"

$$v = (1 + e_A)V$$

which was introduced in Chapter 1 as the "serve speed formula."



Anna Smashnova, Israel

11

Energy and Coefficient of Restitution

Energy and Energy Return in Tennis Collisions

By Crawford Lindsey

Introduction

Describing a racquet and ball impact in terms of momentum, as we did in Chapter 10, is one way to look at a tennis hit. But there is another way — describing the interaction in terms of energy, as we will do in this chapter.

How many times have you heard players talk about the ball's energy, the energy return of the racquet, the racquet or ball losing energy, or that a light racquet can swing faster and has more energy to hit the ball faster? We use the term "energy" everyday, but like force, it is one of those invisible creations of the mind that are everywhere and nowhere at the same time.

So what is energy? Fortunately, it is an outgrowth of everything we discussed in Chapter 10. To recap, we said that a ball will continue its state of motion (whether it is at rest or moving) unless a force acts upon it to change its velocity. To change the direction or speed is to accelerate an object. This statement is summarized by $F = ma$. Since acceleration is change in velocity over time, we were able to rework the formula into the Impulse-Change in Momentum formula which is stated as $Ft = m\Delta v$. In words, that translates to "impulse equals the change in momentum." Momentum is something every moving object has and is the quantity mv, and the total momentum of a colliding racquet and ball is always the same before and after the collision (Equation 10.5). And

we saw that this equality could be very useful to make predictions of what the outcome of a racquet and ball collision would be, based only on knowledge of their masses and velocities before impact. There, at the end of Chapter 10, we hit an impasse. Since we have two unknowns (both the final ball and racquet velocities), we need two independent equations, so we can solve for one of the unknowns in one and plug the results into the other. And this is where we find out that every moving object has another quantity — $\frac{1}{2}mv^2$ — known as kinetic energy. Thus, by exploring the kinetic energy and energy return of racquets and balls, we will discover our second relationship needed to combine with the conservation of momentum equation. The resulting formula will give us the power to predict the outcomes of racquet and ball collisions. Just as we derived momentum conceptually, we will do so for kinetic energy.

Kinetic Energy

The notion of energy also grows out of Newton's Second Tennis Law: F = ma. In Chapter 10, by transforming that formula, we arrived at further insights into the laws and relationships of the tennis universe like the nature of momentum, the Impulse-Change in Momentum Theorem, and the Law of Conservation of Momentum. Further transformation will lead us right to energy.

To continue, consider what it feels like when you apply a force to your racquet. If you swing it (push or pull), you move it while you are pushing. The force is applied over a distance. If it is a very heavy racquet, you sweat, huff and puff, and then wipe your brow saying, "Wow, that was a lot of work." I guess that's what the first tennis playing physicists must have said anyway, because a force applied over a distance is known as "work" and is represented as:

$$W = Fd \qquad (11.1)$$

But where did that come from? How does that follow from Newton's laws?

Once again it is all in that simple formula F = ma, which describes a force F that is applied to accelerate a mass m along a straight line from an initial speed v_1 to a final speed v_2. In Chapter 10, through a simple transformation, we saw how F = ma is equivalent to $Ft = m(v_2 - v_1)$. We just have to keep playing with the terms a little to come up with energy. Let's continue our transformations of that formula:

$$Ft = m(v_2 - v_1) = mv_2 - mv_1$$

From here, we just make some substitutions. We know that time (t) equals distance divided by the average velocity: $t = d/(v_2 + v_1)/2 = 2d/(v_2 + v_1)$. So we substitute that for t in the impulse-momentum equation;

$$2Fd/(v_2 + v_1) = mv_2 - mv_1$$

Then multiply both sides by $(v_2 + v_1)$ to isolate 2Fd:

$$2Fd = (mv_2 - mv_1)(v_2 + v_1)$$

Divide each side by 2 to isolate Fd:

$Fd = \frac{1}{2}(mv_2 - mv_1)(v_2 + v_1)$

Multiply out the right side:

$$\mathbf{Fd = \frac{1}{2}\,mv_2^2 - \frac{1}{2}\,mv_1^2} \qquad \textbf{(11.2)}$$

Work = Change in kinetic energy

Stated this way, the above formula is known as the Work-Energy Theorem. The equation highlights the following fact: a force changes the motion of a racquet or ball, and energy is a measure of that change. Just like force, energy isn't seen by itself, but only inferred from observing changes in velocities.

With equation 11.2, our term for kinetic energy ($\frac{1}{2}mv^2$) has turned up. So, we see that the formulas for our concepts of force, momentum, energy, acceleration, and velocity are all lurking in the simple formula F = ma. This is an important observation if you like feeling that everything is connected to everything else and that everything is derived both conceptually and mathematically from some common starting point. It seems that everything that happens to our racquets, strings and balls will be derived from this one simple equation that says that the world is made of things and to change the state of motion of those things, you have to apply a force.

We're not quite there yet, however. I don't know about you, but I'm still not seeing what Christian Huygens (1629-1695) must have seen in this formula that made him exclaim one sleepless night, "Aha, there's energy — it's $\frac{1}{2}mv^2$!" Yes, I see the terms that we call kinetic energy, but we have the benefit of hindsight and know what we are looking for. What if you were just playing with your formulas one day, and you came across this transmutation? How would you interpret it into a meaningful concept such as energy?

Huygens wasn't happy with the idea, up to that time, that momentum was considered the essence of a tennis hit. Although he appreciated how it could help determine final velocities, and therefore was very useful, he found it troubling that if a ball and racquet approached each other with equal momentums, the total momentum was thus zero both before and after the hit, even though the collision broke a string, hurt his hand, went out of the court and broke his neighbor's window. That zero momentum system sure had a lot of non-zero consequences! Likewise, it concerned him that the total momentum of a racquet and ball before the collision could be less than the momentum of either one of them separately. Huygens realized that by squaring the racquet's and ball's velocities, he got rid of the directional and cancelling property of momentum equations. Even though the total momentum of the racquet and ball system might be zero, there was still an inherent capacity to do work in both objects. Here, in this energy that was independent of direction, was the real fuel behind flying tennis balls.

So, back to formula 11.2, we notice that if the initial velocity were 0 (as in serving a ball), then the second term goes away and we are left with $Fd = \frac{1}{2}mv_2^2$. So if you apply a force over a distance to a stationary mass, it will be associated with this mysterious $\frac{1}{2}mv^2$. And this quantity remains after the force is no longer applied. It is a descriptive quantity that is now a "possession" of this mass. Applying a force to the ball over a certain distance adds energy to the ball in the amount of Fd. The ball has an amount of kinetic energy of $\frac{1}{2}mv_1^2$

before the collision and a kinetic energy of $\frac{1}{2}mv_2^2$ after the collision. You can determine how much work was done and how much energy was added to the ball by using the Work-Energy equation:

Work = Change in Energy = $Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

The unit of work and energy is the Joule (J), which is a N·m or $kg \cdot m^2/s^2$.

Law of Conservation of Racquet and Ball Energy

Huygens also noticed that in many experiments with billiard balls that the sum of all balls' $\frac{1}{2}mv^2$ was equal before and after the collisions. Whatever this term represented, it was conserved every time. Whenever there is some quantity in a system of objects that is the same before and after innumerable transformations to that system, you have to suspect that that "quantity of something" is very fundamental. In this case it was, and it came to be known as energy, and more specifically, kinetic energy when moving objects are involved — energy of motion.

The net result is the Law of Conservation of Energy, which states that the total energy before the collision will be equal to that after the collision. The word "total" indicates that energy comes in many forms (mechanical, chemical, electrical) and sub-forms (kinetic, potential, elastic, heat), and it can change from one form into another. So, in a totally elastic collision (i.e., where there is no energy loss due to transformation of form) between the ball and racquet, the conservation of energy equation looks like this:

$$\frac{1}{2}m_r v_{r1}^2 + \frac{1}{2}m_b v_{b1}^2 = \frac{1}{2}m_r v_{r2}^2 + \frac{1}{2}m_b v_{b2}^2 \qquad \textbf{(11.3)}$$

Total kinetic energy before collision = Total kinetic energy after collision

This too, like the conservation of momentum, is a handy equation, because it does not involve any difficult-to-measure terms like force or time of impact, but we are obviously going to have to find a way around the assumption of total elasticity, since a racquet-ball collision certainly is not totally elastic.

But first, we still need to determine WHAT is this thing called "energy"? We know what a racquet's mass is; we know its position, velocity and acceleration. We know and see its motion and momentum. These are all things that we see, touch, feel, conceptualize and understand in our world. These are everyday entities. But energy? Energy is a total abstraction. In the case of momentum (mv), we see a racquet moving at a particular velocity. For kinetic energy ($\frac{1}{2}mv^2$) we don't see a half of a racquet moving at its velocity multiplied by its velocity. We just see the whole racquet moving at a certain speed. Up to this point in our exploration of the physics of tennis collisions, everything has evolved nicely from both a reworking of F = ma and from ordinary perceptual observations of racquets hitting balls. With energy, we seem to leave our touch-and-feel world and enter a world of make-believe. Yet, energy is the Holy Grail of physics and tennis. So, let's examine the difference between momentum and energy in order to become completely familiar with this thing called energy.

Difference between Racquet Energy and Momentum

Kinetic energy and momentum look similar in that both their equations contain mass and velocity. They are not, however. Here's what we know:

- Momentum = mv; energy = $\frac{1}{2}mv^2$.
- Momentum calculations depend on the relative directions of the ball and racquet before and after the collision, energy does not.
- Momentum is directly proportional to velocity; energy is directly proportional to the square of velocity. If you double the racquet's velocity, you will double its momentum, but you will quadruple its energy (since $(2v)^2$ equals $4v^2$).
- Both momentum and energy are directly proportional to mass.
- Momentum is ALWAYS conserved in every racquet-ball collision; energy is always conserved, but it often changes form. Kinetic energy is usually not conserved, and that is what we are interested in. (We will solve this in the next section.)

Given these differences, is it more important to have more momentum or more energy when you hit the ball? Let's look at an example. Figure 11.1 shows the

Momentum and Energy in Elastic Collisions

Figure 11.1

These are collisions with no energy loss using racquets with the same momentum but different masses, velocities and energies. The racquet with the greatest amount of energy hits the ball the fastest. It is also the racquet with the greatest change in velocity, momentum, and energy. In other words, there was a price to pay for the extra ball velocity — shock.

BEFORE collision		AFTER collision	
v = 12.5 m/s	v = -15 m/s	v = 5.64 m/s	**v = 33.14 m/s**
m = 400 gm	m = 57 gm	m = 400 gm	m = 57 gm
mv = 5 kg m/s	mv = -.855 kg m/s	mv = 2.26 kg m/s	mv = 1.9 kg m/s
$.5mv^2$ = 31.25 J	$.5mv^2$ = 6.4 J	$.5mv^2$ = 6.4 J	$.5mv^2$ = 31.3 J
total momentum = 4.15 kg m/s total energy = 37.7 J		total momentum = 4.15 kg m/s total energy = 37.7 J	
v = 16.7 m/s	v = -15 m/s	v = 6.57 m/s	**v = 38.27 m/s**
m = 300 gm	m = 57 gm	m = 300 gm	m = 57 gm
mv = 5 kg m/s	mv = -.855 kg m/s	mv = 1.97 kg m/s	mv = 2.18 kg m/s
$.5mv^2$ = 42 J	$.5mv^2$ = 6.4 J	$.5mv^2$ = 6.5 J	$.5mv^2$ = 41.8 J
total momentum = 4.15 kg m/s total energy = 48.3 J		total momentum = 4.15 kg m/s total energy = 48.3 J	
v = 25 m/s	v = -15 m/s	v = 7.26 m/s	**v = 47.26 m/s**
m = 200 gm	m = 57 gm	m = 200 gm	m = 57 gm
mv = 5 kg m/s	mv = -.855 kg m/s	mv = 1.45 kg m/s	mv = 2.7 kg m/s
$.5mv^2$ = 63 J	$.5mv^2$ = 6.4 J	$.5mv^2$ = 5.3 J	$.5mv^2$ = 63.7 J
total momentum = 4.15 kg m/s total energy = 68.9 J		total momentum = 4.15 kg m/s total energy = 68.9 J	

final velocities of balls launched by racquets having equal momentums but different velocities, masses, and energies. We are assuming there is no energy lost in the collision. (In order to illustrate the difference between momentum and energy, we have cheated by using the formulas we are trying to discover so we could calculate the final ball and racquet velocities for our illustration.) It is clear that the more energy in the racquet, the greater the final ball velocity. Does that mean you should purchase the lightest racquet possible and swing it as fast as you can? The answer is no for two reasons. First, given today's racquets, you can swing as fast as you want anyway. Second, most players have a swing speed they are comfortable with in every situation. The speed of the swing is really a given according to what feels best to the player, not how fast he is capable of swinging a particular racquet. That being the case, the problem becomes, given a swing speed, what racquet will produce more ball speed. Framed that way, the answer is the heaviest racquet you feel comfortable swinging at that speed. If the speed of the swing stays constant, both the momentum and the energy of the racquet will increase with an increase in racquet mass (Figure 11.2).

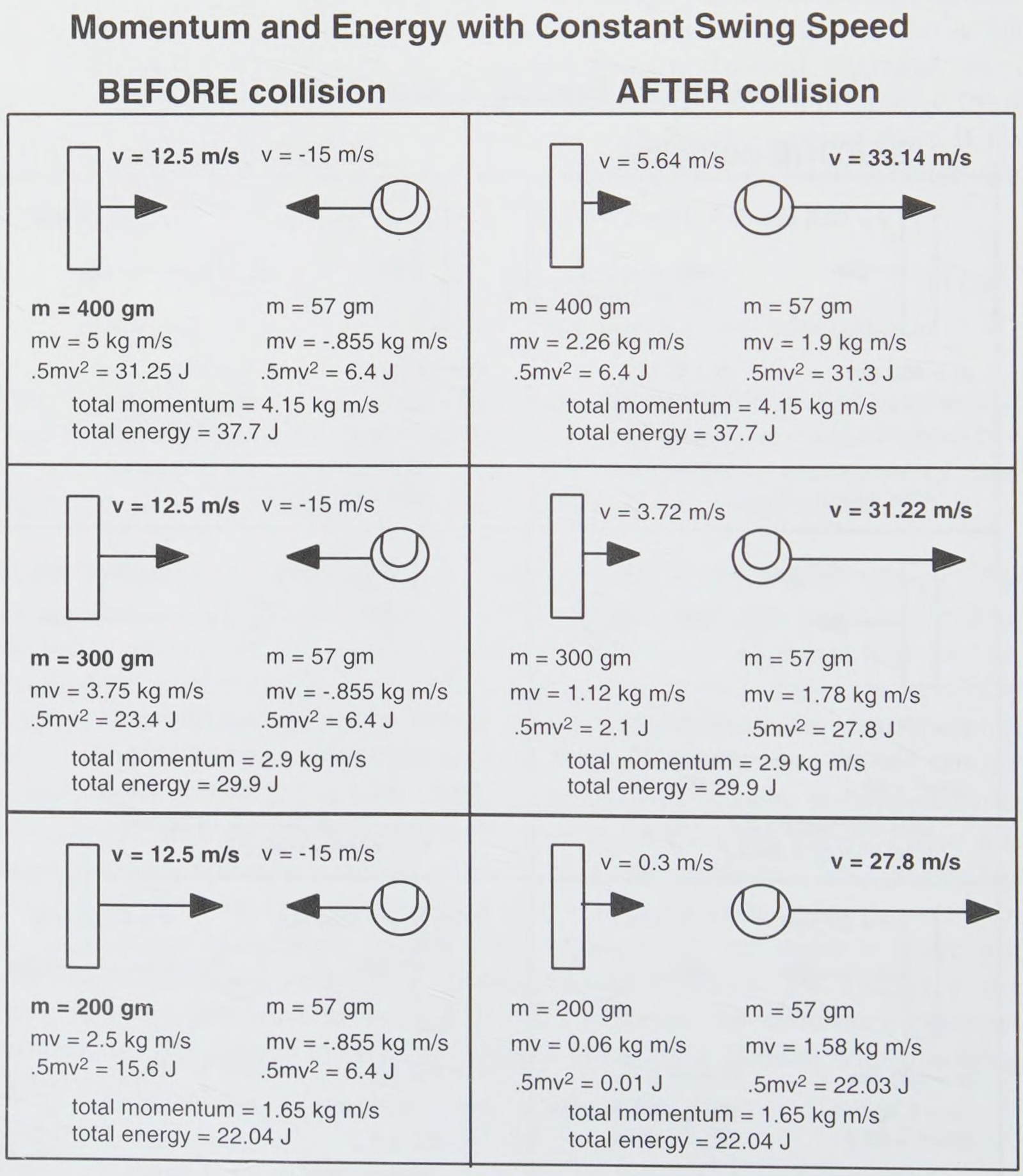


Figure 11.2

Most players have a "grooved" swing speed, so lowering the weight of the racquet does not necessarily mean that the swing speed will increase nor that it is desireable to do so. The collisions here illustrate what happens to the racquet and ball given racquets with the same starting velocity but different masses. As weight goes down, ball speed goes down. And look at the racquet — it is almost brought to a complete stop with the 200 gm racquet. There is thus more shock and less velocity with a lighter racquet, given the assumption that swing speed is constant.

To keep the final ball speed at 33.1 m/s going from the 400 gm racquet to the 300 gm racquet, swing speed would have to increase from 12.5 m/s to 13.6 m/s (a 9% increase). The trade-off to get a 9% increase in swing speed just to maintain ball speed may be less control.

In the examples above, we assumed that no kinetic energy of the racquet or ball was lost or converted during the collision (and remember, also, that impact is at the balance point). This is an unrealistic assumption. But it is a necessary assumption in order to use the conservation of energy relationship without having to also track and calculate all the energy transformations during impact. So even though energy is the most important component in a collision, it is the most difficult to keep track of.

Energy Return and Coefficient of Restitution (COR)

There is another way, however, that combines the energy and energy loss concept into a simple, easy to use formula. In Figure 11.1, in each case, the velocity of the racquet minus the velocity of the ball is the same, but opposite, before and after the collision. In other words, the relative velocities are the same, but opposite, before and after the impact. This is true in all completely elastic collisions where no energy is lost. To illustrate let's look at the first collision in Figure 11.1. The relative velocity before the collision is 12.5 m/s + 15 m/s = 27.5 m/s. After the collision the relative velocity is 33.14 m/s - 5.64 m/s = 27.5 m/s.

But what about the real life where energy is lost in the racquet and ball? By "lost" we don't mean that it disappears. It means that it has been converted to another form like heat or been used doing things that are not useful in propelling the ball, like bending the racquet. Given these losses, as it turns out, whenever any two objects collide, the amount of energy lost depends on the nature of the two colliding objects, where they hit each other, and their relative speeds. The amount of energy lost can be derived from one very important relationship — the relative velocities before and after a collision for any two given objects, will always be proportional to some constant. This constant of proportionality is known as the coefficient of restitution, also referred to as COR, or as e. COR is defined as the negative of the ratio of the relative racquet and ball speeds after collision divided by the relative speeds before:

$$\mathbf{COR = e = -\frac{(v_{r2} - v_{b2})}{(v_{r1} - v_{b1})}} \qquad \mathbf{(11.4a)}$$

And if we rearrange the terms, we get:

$$\mathbf{(v_{r2} - v_{b2}) = -e\,(v_{r1} - v_{b1})} \qquad \mathbf{(11.4b)}$$

This shows that the relative speed of the racquet and ball after the collision is equal to the relative speed before, multiplied by the constant of proportionality — the COR.

The minus sign in (11.4a) is used to make sure that e is a positive number. Defined this way, it turns out that e always lies between 0 and 1. If the numerator in (11.4a) is positive then the denominator will be negative and hence e is positive. Keeping track of the signs can be a bit tricky. We first need to decide which direction is positive. The usual convention is that an object traveling left to right has a positive velocity and an object traveling right to left has a negative velocity. Since we are interested in the head-on collision between a ball and a racquet, one will have a positive velocity before the collision and the other will have a negative velocity. If the racquet travels left to right before and after the collision, then the ball will have a negative velocity before the

collision and a positive velocity after the collision. In that case, the v_{b1} term in (11.4a) is negative but all the other terms are positive.

It is often easier to keep track of the signs if we change our definition of the v terms so that they represent the speed of the ball and the racquet rather than the velocity. Speed values are always positive, regardless of the direction. In that case, the formula for the COR for a head-on collision can be written as

$$\mathbf{COR = e = \frac{(v_{b2} - v_{r2})}{(v_{b1} + v_{r1})}} \qquad \mathbf{(11.5)}$$

You will often see COR written both ways — as Equation 11.4a or 11.5. But if you use Equation 11.5, you must plug in the speeds of the ball and racquet, independent of their direction (i.e., you don't plug in a negative ball speed). The tradeoff is that though this equation is easier to work with because it gets rid of all the negative signs, it also hides the original relationships of relative velocities that define COR, so it is more difficult to remember why you are using the equation.

In the examples in Figure 11.1, e = 1.0. COR is always between 0 and 1.0 — 1.0 being a completely elastic collision with no energy loss, and 0 would be a completely inelastic collision where the objects stick together after the collision. COR is about 0.88 for a tennis ball hitting the center of the strings. However, the COR of a tennis ball dropping onto a concrete floor is 0.75. It is different on clay, grass, and hard courts, and even at different locations on the strings. It is not proper to talk about the COR of the ball without referencing what it is colliding with, and in the case of the racquet, where it is impacting. The COR will be different in each case. It also depends on which ball you are using.

COR is easily determined for a ball and racquet collision. Simply place a racquet on the floor, step on its handle, and drop a ball on the center of the strings. In this case, racquet velocity before and after the impact is zero, so equation 11.5 becomes

$$e = v_{b2} / v_{b1}.$$

How are we to measure these speeds? There's a trick — COR can also be obtained by measuring the bounce height of the ball from the racquet, given any specific drop height (see sidebar: COR and Bounce Height):

$$\mathbf{e = v_{b2} / v_{b1} = \sqrt{(h_b/h_d)}} \qquad \mathbf{(11.6)}$$

where,
h_b = height ball bounces,
h_d = drop height of ball,
v_{b2} = speed of ball leaving the strings
v_{b1} = speed of ball at impact with strings
v_{r2} and v_{r1} are 0.

COR and Bounce Height

How did we determine that COR was equal to the square root of the bounce height? It is as follows:

Starting conditions:
h_d = drop height
h_b = bounce height
$a = g = 9.8\ m/s^2$ = acceleration due to gravity
v_d = speed at impact of dropped ball
v_b = speed of ball after the bounce

We know from equation 11.2 that $Fd = 1/2mv^2$, and we know that F=ma. We also know, that d = h, and for free falling bodies, we use a = g. Combining we get,

$mgh = mv^2/2$.

Solve for v^2:
$mv^2 = 2mgh$
$v^2 = 2mgh/m$
$v^2 = 2gh$
Free fall speed formula = $v = \sqrt{(2gh)}$

So the speed at impact = $v_d = \sqrt{(2gh_d)}$.

The bounce speed = $v_b = \sqrt{(2gh_b)}$. v_b is actually in the opposite direction and thus negative, but we are considering only the speeds here and plugging them into Equation 11.5, as explained in the main text.

The COR from Equation 11.5 is

$$e = \frac{v_b - v_{r2}}{v_d + v_{r1}}$$

but v_{r2} and v_{r1} are 0 in this case of dropping the ball on a "clamped" racquet, so we have,

$$e = \frac{v_b}{v_d}$$

Substituting from above, we get

$$e = \frac{v_b}{v_d} = \frac{\sqrt{(2gh_b)}}{\sqrt{(2gh_d)}} = \frac{\sqrt{h_b}}{\sqrt{h_d}} = \sqrt{\frac{h_b}{h_d}}$$

And there you have it. The COR is equal to the square root of the ratio of the bounce height to the drop height.

So, if you drop a ball from 100 inches onto the center of the strings of a racquet secured by your foot on the floor, it will bounce about 77 inches. COR is then calculated as follows:

$$e = \sqrt{(h_b / h_d)} = \sqrt{(77/100)} = 0.88$$

The COR is 0.88, and the ball will bounce with 77% of its initial energy. This is important and often misunderstood. The ratio of the bounce height to drop height is the ratio of the bounce and impact energy of the ball. Remember, $mgh = \frac{1}{2}mv^2$ = the potential energy of an object at a certain height. So, $mgh_b/mgh_d = h_b/h_d$ = ratio of bounce energy to impact energy. The COR is not a ratio of energy. It is a ratio of relative speeds which must be squared to relate back to energy.

Final Ball and Racquet Velocites — The Hard Way

We now have everything we need to derive a general formula that determines ball and racquet velocities after a collision. We have two unknown variables, v_{r2} and v_{b2} and two independent equations — the conservation of momentum (Equation 10.4b) and coefficient of restitution (Equation 11.4a). You will remember that we use the COR equation (11.4a) instead of the conservation of kinetic energy equation (11.3) for two reasons. First, because the conservation equation only works if we assume a totally elastic collision with no energy losses. Second, the COR equation incorporates the energy equation and energy losses in the sense that if energy is conserved, then COR = 1, but if energy is lost, then COR is less than 1. The derivation goes like this (we are starting with Equation 11.4a here so we are using velocity, not speed, in the derivation):

$$e = -(v_{r2} - v_{b2}) / (v_{r1} - v_{b1})$$
$$-e(v_{r1} - v_{b1}) = v_{r2} - v_{b2}$$
$$v_{r2} = v_{b2} - e(v_{r1} - v_{b1})$$
$$v_{r2} = v_{b2} - ev_{r1} + ev_{b1}$$

Now we substitute the equation for final racquet velocity into the conservation of momentum equation as follows:

$$m_r v_{r1} + m_b v_{b1} = m_r v_{r2} + m_b v_{b2}$$

substitute for v_{r2}

$$m_r v_{r1} + m_b v_{b1} = m_r(v_{b2} - ev_{r1} + ev_{b1}) + m_b v_{b2}$$

$$m_r v_{r1} + m_b v_{b1} = m_r v_{b2} - em_r v_{r1} + em_r v_{b1} + m_b v_{b2}$$

$$m_r v_{r1} + m_b v_{b1} = v_{b2}(m_r + m_b) - em_r v_{r1} + em_r v_{b1}$$

$$m_r v_{r1} + m_b v_{b1} + em_r v_{r1} - em_r v_{b1} = v_{b2}(m_r + m_b)$$

$$v_{b2} = \frac{(m_r v_{r1} + m_b v_{b1} + em_r v_{r1} - em_r v_{b1})}{(m_r + m_b)}$$

Final Ball Velocity Formula:

$$v_{b2} = \frac{em_r(v_{r1} - v_{b1}) + m_r v_{r1} + m_b v_{b1}}{m_r + m_b} \quad (11.7)$$

The final racquet velocity is derived the same way and is:

Final Racquet Velocity Formula:

$$v_{r2} = \frac{m_r v_{r1} + m_b v_{b1} - em_b(v_{r1} - v_{b1})}{m_r + m_b} \quad (11.8)$$

How To Use the Speed Equations

As you can see, we have arrived at general formulas for determining ball and racquet velocities after a collision by using only pre-collision given data. If you plug in the data from Figure 11.1 into Equations 11.7 and 11.8, and if you set e = 1.0 (i.e., perfectly elastic collision with no energy loss), then you get the values on the after collision side of Figure 11.1. For instance, if we plug in the numbers into final ball speed formula into the collision shown in Figure 11.3 (same as the first collision in Figure 11.1) with e = 1.0, we get the following:

$$v_{b2} = \frac{1(.400)[12.5 - (-15)] + (.400)(12.5) + (.057)(-15)}{(.400 + .057)} = \frac{15.145}{.457} = 33.14 \text{ m/s}$$

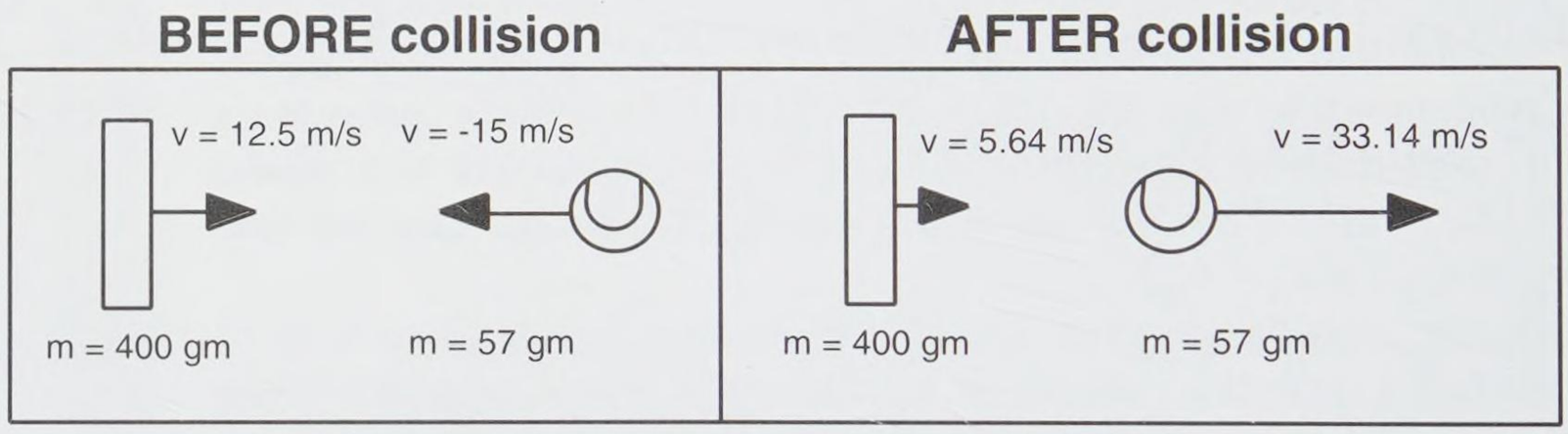


Figure 11.3

You can then plug the numbers into Equation 11.8 to get final racquet velocity. However, with v_{b2} now in hand, it would be easier just to plug v_{b2} into Equation 11.5 and solve for v_{r2}:

$$COR = e = \frac{(v_{b2} - v_{r2})}{(v_{b1} + v_{r1})}$$

$$v_{r2} = v_{b2} - e(v_{b1} + v_{r1}) = 33.14 - 1(27.5) = 5.64 \text{ m/s}$$

If we substitute all the before collision numbers from Figure 11.1 into Equations 11.7 an 11.8, but use e = 0.88, then we get the results in Figure 11.4. Both Figures 11.1 and 11.4 have the same masses and initial velocities, but they have different outcomes because Figure 11.4 has energy losses represented by the 0.88 COR.

But working with those big ugly formulas is a lot of work. There must be an easier way. There is, using the apparent coefficient of restitution, also known as ACOR or e_A, to which we now turn our attention.

Momentum and Energy in Inelastic Collisions (COR = e = .88)

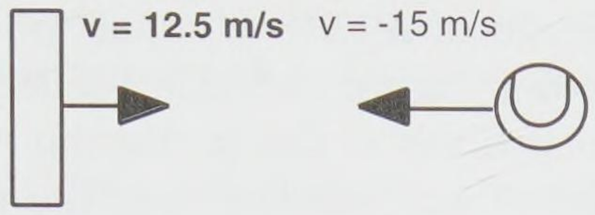
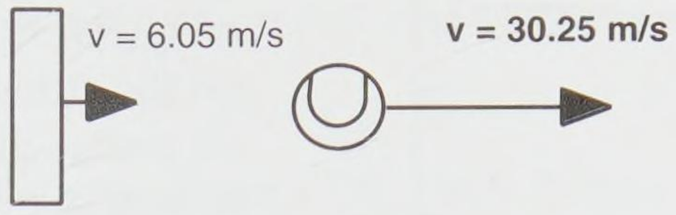
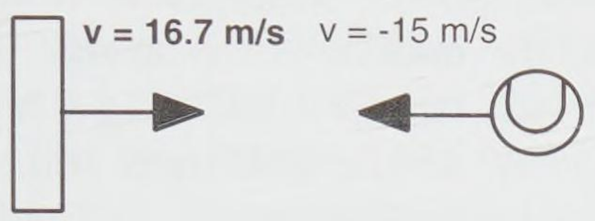
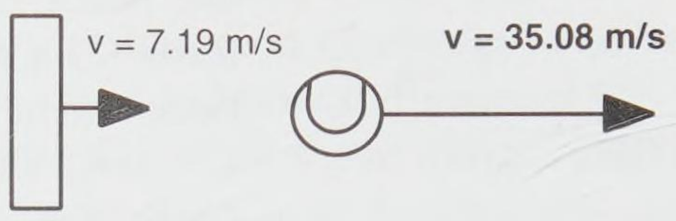
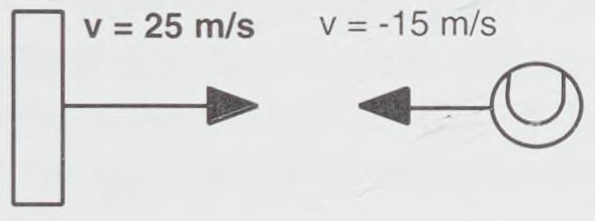
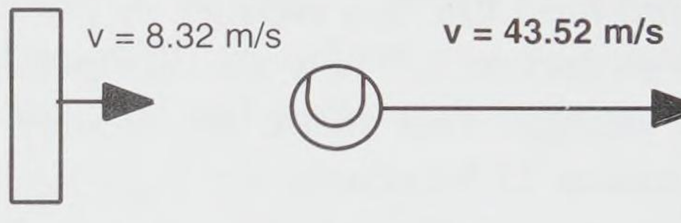
BEFORE collision	AFTER collision
 **m = 400 gm** m = 57 gm mv = 5 kg m/s mv = -.855 kg m/s $.5mv^2$ = 31.25 J $.5mv^2$ = 6.4 J total momentum = 4.15 kg m/s total energy = 37.7 J	 m = 400 gm m = 57 gm mv = 2.42 kg m/s mv = 1.72 kg m/s $.5mv^2$ = 7.32 J $.5mv^2$ = 26.1 J total momentum = 4.15 kg m/s total energy = 33.4 J
 **m = 300 gm** m = 57 gm mv = 5 kg m/s mv = -.855 kg m/s $.5mv^2$ = 42 J $.5mv^2$ = 6.4 J total momentum = 4.15 kg m/s total energy = 48.3 J	 m = 300 gm m = 57 gm mv = 2.16 kg m/s mv = 2 kg m/s $.5mv^2$ = 7.74 J $.5mv^2$ = 35.07 J total momentum = 4.15 kg m/s total energy = 42.8 J
 **m = 200 gm** m = 57 gm mv = 5 kg m/s mv = -.855 kg m/s $.5mv^2$ = 63 J $.5mv^2$ = 6.4 J total momentum = 4.15 kg m/s total energy = 68.9 J	 m = 200 gm m = 57 gm mv = 1.66 kg m/s mv = 2.48 kg m/s $.5mv^2$ = 6.92 J $.5mv^2$ = 54 J total momentum = 4.15 kg m/s total energy = 60.9 J

Figure 11.4

These are the same collisions as in Figure 11.1 except that the COR = .88 here and COR =1 in Figure 11.1. In all three collisions, compared to Figure 11.1, the racquet's final velocity and energy are greater and the ball's final velocity and energy are less.

Apparent Coefficient of Restitution (ACOR)

Even though measurements of the COR allow us to work out the ball speed for any given racquet and racquet speed, the COR is normally measured only for the relatively simple case where the racquet is clamped to a heavy frame. This measurement is simplified by the fact that the racquet speed is zero before and after the collision, and also by the fact that the racquet can't vibrate. If we unclamp the racquet and hold it or swing it by hand, then we would need to measure the racquet speeds before and after the collision, as well as the ball speeds before and after the collision, in order to measure the COR. The COR measured this way would be the same as the COR on a clamped racquet only if the racquet strikes the ball in the middle of the strings, since that is the only spot where there is no energy loss due to vibrations of the racquet.

A much simpler way to characterize the performance of a racquet is to hold the racquet at rest by hand and fire a ball at it, at right angles to the string plane. If the ball is incident at speed v_{b1} and bounces at speed v_{b2}, then we define the ratio of these speeds by the formula

$$\mathbf{ACOR = e_A = v_{b2} / v_{b1}} \qquad \mathbf{(11.9)}$$

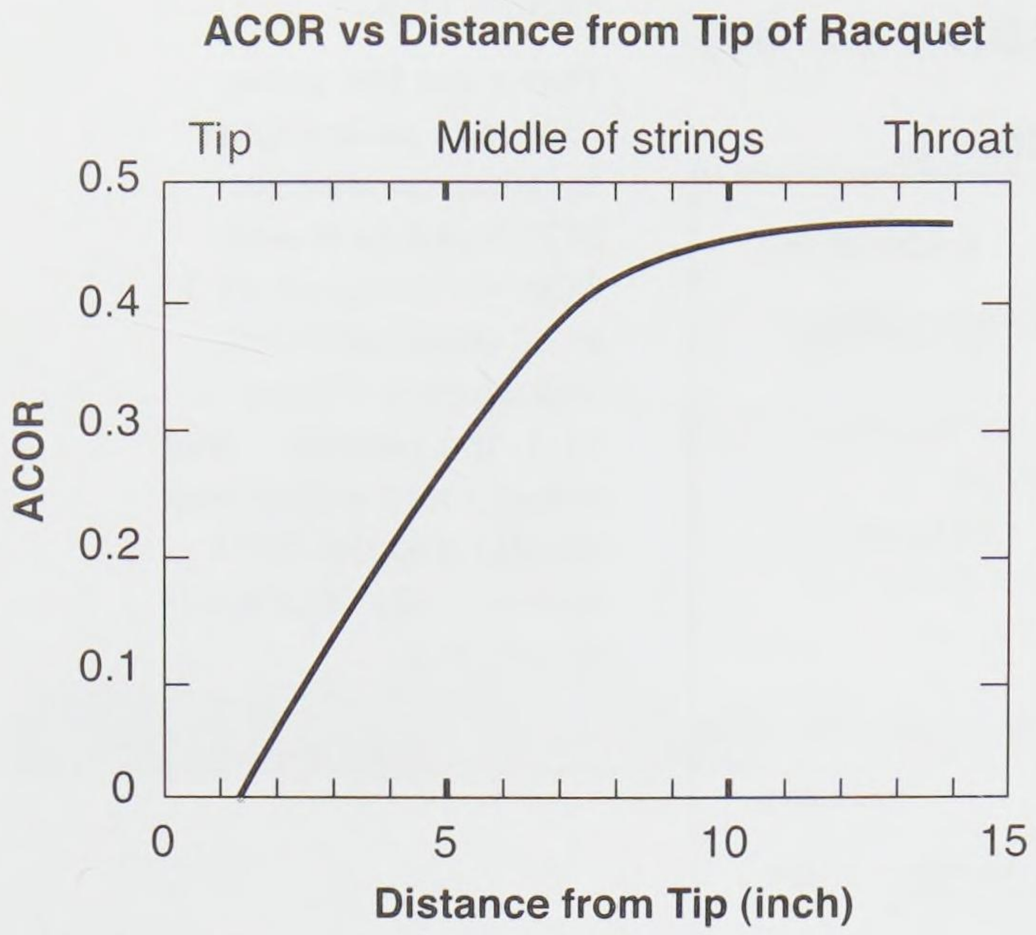


Figure 11.5

The value of e_A is typically about 0.1 near the tip of the racquet, about 0.4 in the middle of the strings, and about 0.5 near the throat. These values increase slightly with heavy racquets and decrease slightly with light racquets. A typical graph of ACOR vs impact position on the strings is shown in Figure 11.5. All racquets and all strings give a similar result to this in that the ACOR is largest near the throat and smallest near the tip. If you think about this for a second, it means that racquet power is largest near the throat and smallest near the tip. If the racquet is at rest when the ball hits it, then the power is a maximum near the throat. However, if you swing the racquet to hit the ball, then the point of maximum power shifts away from the throat to a point near the middle of the strings. That's because the middle of the strings is traveling at a higher speed than the throat so the ball then comes off faster from the middle of the strings.

The above method of determining ACOR by firing balls at a stationary hand-held racquet, requires you to be able to measure the speed of the ball both on the way in and on the way out. That requires instrumentation. You can also determine ACOR with a bounce procedure like we used to determine COR. We found the COR of a racquet by putting it on the floor and stepping on the handle to secure it. Then we dropped a ball from 100 inches onto the middle of the strings. This made the before and after speed of the racquet zero, so Equation 11.5 became $e = v_{b2} / v_{b1}$, or written differently, $v_{b2} = ev_{b1}$.

We can do the same thing for a hand-held racquet (Figure 11.6). Everything is the same except we are holding the racquet instead of clamping it down. We again drop the ball from 100 inches (though any height will do) in the middle of the strings to prevent frame vibrations (though we don't have to). This time the ball bounces to about 16 inches. If we use the bounce height rule that we used to determine COR, we get

$$e_A = \sqrt{(h_b/h_d)} = \sqrt{(16/100)} = 0.40.$$

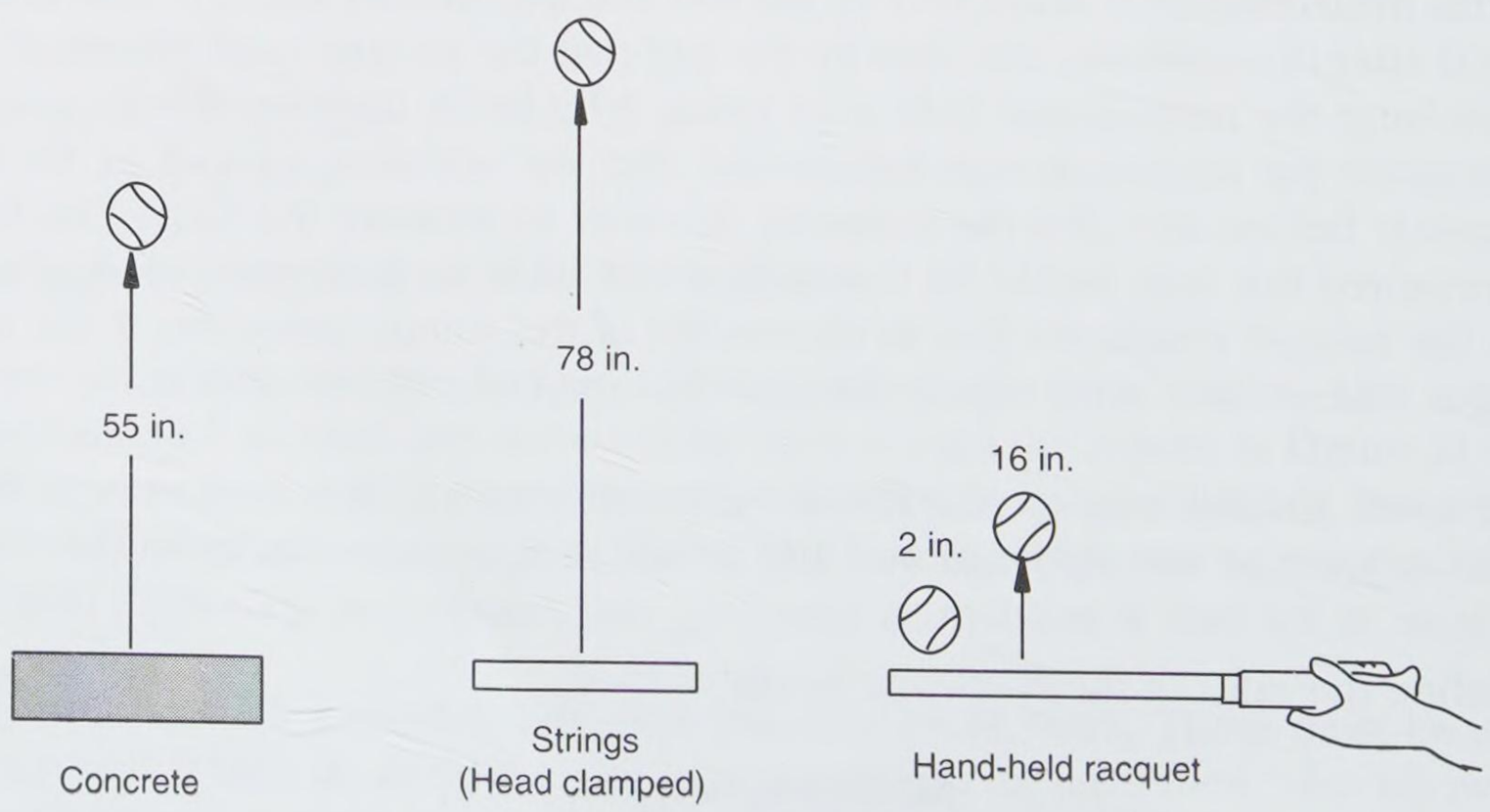


Figure 11.6

Typical bounce heights for a tennis ball dropped from a height of 100 inches.

That is much less than COR. The reason is that with a hand-held racquet, the racquet will recoil at impact. How much and how fast it will recoil depends primarily on its weight and swingweight. You can drop the ball onto any position on the stringbed and measure the ACOR for that location. The bounce height will reflect the differing amount of translation, recoil and vibration that accompanies each impact location.

The ACOR is relatively easy to measure compared to the COR, since it does not involve any simultaneous measurement of the racquet speed. Furthermore, as we will see, a measurement of the ACOR tells us everything we need to know to work out the ball speed, for any given racquet speed at any given impact point (not just at the center of mass), since the ACOR value is measured under realistic conditions where the racquet can recoil and vibrate as much as it wants to. Knowing the ACOR, we can work out the ball speed without using conservation of momentum, without using conservation of energy, and without even knowing the COR. We don't even need to know the mass of the racquet. The ball speed is the result of the interplay of all these things. They are necessary to explain why the ball speed is what it is, or to theoretically calculate it. But if we measure the ball speed off a hand-held racquet, then we already know what the ball speed is. There is no need to calculate something if we have already measured it! And because that ball speed is the result of the interplay between the different energy sharings, effective masses, translations, recoils, and vibrations that occur at different impact points, the formulas derived from the ACOR relation are predictive for all impacts at all locations at all impact point velocities. With ACOR, then, we can relieve our restrictions that impacts are at the center of mass. (Chapter 12 will show us how to relieve them for all our other calculations, as well.)

Using ACOR for calculating final velocities greatly simplifies the math. It's not as easy as you're thinking, however. You can't just take any collision and multiply e_A times v_{b1} and get the final ball velocity, unless the racquet has no initial velocity, such as our hand-held case. ACOR itself is only defined for a bounce off a racquet at rest, but from that data, we will be able to derive formulas for final ball speed given any initial ball and racquet speeds.

I can hear some readers complaining that a measurement of ball speed off a stationary racquet has nothing to do with the ball speed off a racquet that is swung vigorously towards the ball. What has a measurement off a racquet at rest got to do with say a high speed serve? Surely these two events are completely different so the ball speed will be completely different. True, the ball speeds ARE completely different, but the events are NOT completely different. In fact, the events and the ball speeds are exactly the same if we observe them in the correct frame of reference.

Changing the frame of reference is something that physicists often do to save the hassle of working things out the hard way. Suppose you are in a train travelling at 100 mph and you drop a ball vertically onto the floor. What you see is that the ball drops in a straight line at right angles to the floor. Someone outside the train looking in sees the ball travelling horizontally at 100 mph and curving downwards towards the floor. Both of you see exactly the same event, but the ball speed and the trajectory are totally different depending on whether you are inside or outside the train. Exactly the same sort of argument can be used to calculate the speed of a ball served from a high speed racquet if we already know the bounce speed off a stationary racquet. So, let's do it.

Final Ball Speed Formulas — The Easy Way

1. Serve Speed Formula.

In order to calculate the serve speed from the ACOR data, it is necessary to figure out what happens when things are the other way around. The ACOR is measured when the ball is incident on a racquet at rest. What we are really interested in is what happens when the ball starts off at rest and the racquet slams into it. That's what happens when you serve the ball. The ball is not quite at rest when you serve it, but it's almost at rest. It won't make any significant difference to the calculation if we assume that the ball is actually at rest, suspended in mid-air, when the racquet slams into it.

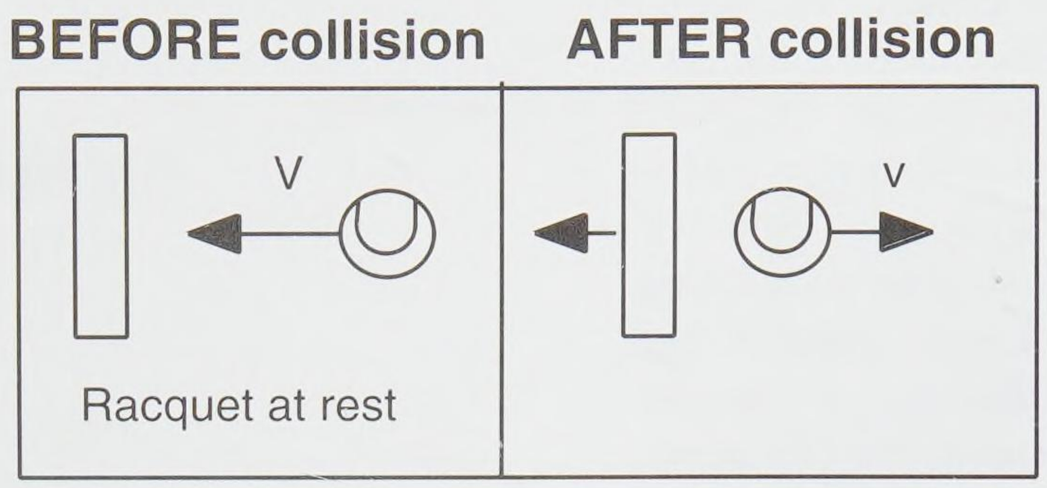


Figure 11.7

There is a very simple way to figure this out. First, we go back to the ACOR measurement. Suppose that the racquet is at rest and you fire a ball to hit the racquet at speed V. The ball will bounce off at speed v. Suppose also that the ball approaches at the right and it bounces off towards the right, as shown in Figure 11.7. In this case, ACOR = v/V.

Since ACOR is only defined for a bounce off a stationary racquet, we must turn Figure 11.7 into a serve the same way the train turned the vertically dropping ball into a horizontally flying ball in the example above. We can do this by changing the frame of reference by adding a fixed velocity to all the speeds shown in Figure 11.7. If we add velocity V to each element of Figure 11.7, we get the results shown in Figure 11.8 The racquet is now moving at speed V to the right, the initial ball velcoity is 0 (adding V in the opposite direction cancelled its previous velocity), and the ball is moving speed v + V to the right. We have added V to the racquet after the collision too, but we didn't know its velocity before the collision in Figure 11.7, so all we know is that V will be enough to move the racquet to the right.

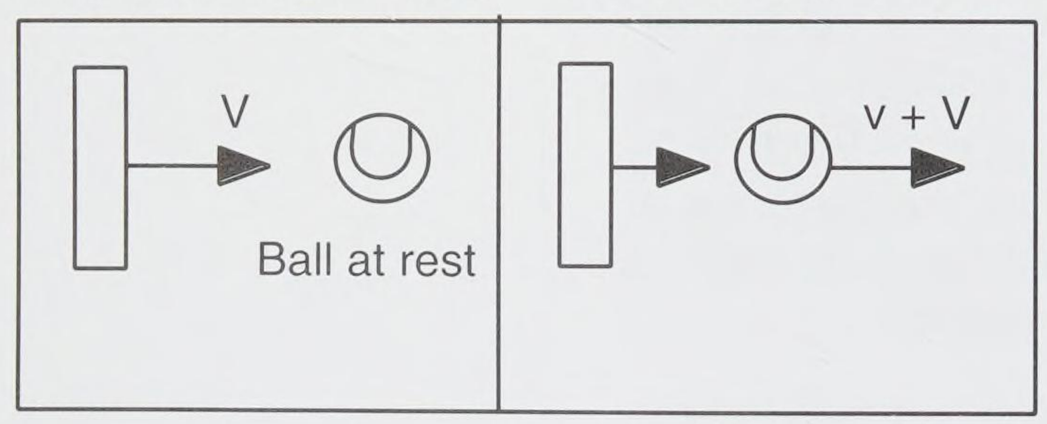


Figure 11.8

From the definition $e_A = v/V$, the bounce speed v in Figure 11.7 is given by $v = e_A V$. From Figure 11.8, we see that the serve speed $v_s = v + V$. Consequently, $v_s = e_A V + V = (1 + e_A)V$, so we get THE MOST IMPORTANT EQUATION IN THE BOOK):

Serve Speed Formula

$$v_s = (1 + e_A)V \qquad (11.10)$$

Nothing could be simpler. To determine the serve speed, all you have to know is ACOR and the racquet speed at the impact point.

Let's look at a specific example as shown in Figure 11.9 and Figure 11.10. Suppose that the racquet is at rest and you fire a ball to hit the racquet at 80 mph. If ACOR = 0.4, then the ball will bounce off at 0.4 x 80 = 32 mph. Suppose also that the ball approaches at 80 mph from the right and it bounces

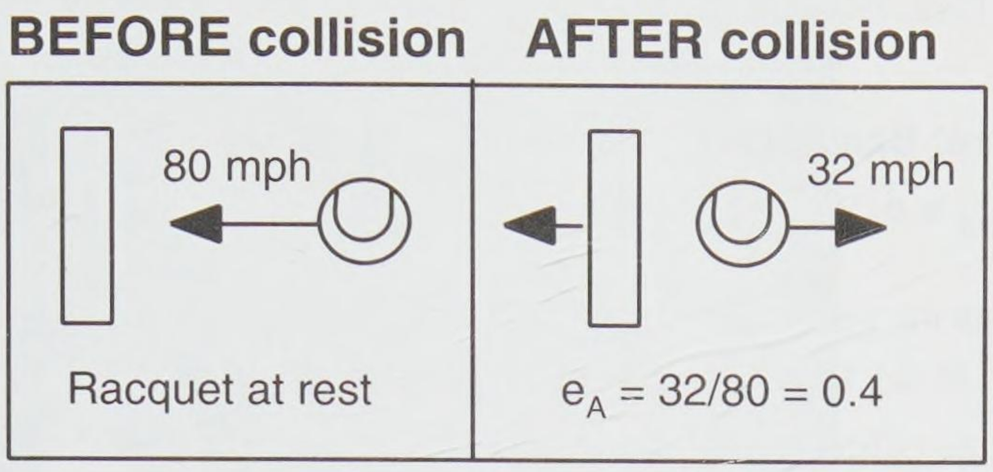


Figure 11.9

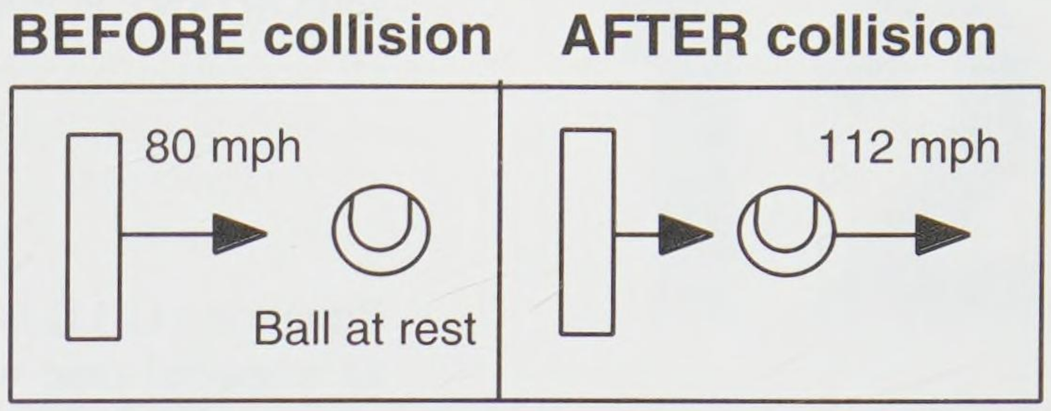


Figure 11.10

off at 32 mph towards the right. Now add 80 mph to the right to ALL these figures, as shown in Figure 11.10. Then the racquet is moving at 80 mph to the right, the ball is initially at rest, and the ball bounces off at 32 + 80 = 112 mph to the right. If the initial conditions were plugged into the serve speed formula, we get

$v_s = (1 + 0.40)80 = 112$ mph

2. General Ball Speed Formula.

The general formula follows the same logic. By "general" we mean that it encompasses any value of the initial ball speed and racquet speed, including the serve, though the serve is a special case because the ball is initally at rest and has its own simpler formula. To begin, refer to Figure 11.11 This looks similar to Figure 11.7, but we have labeled the velocities differently because we are going to have more terms to keep track of.

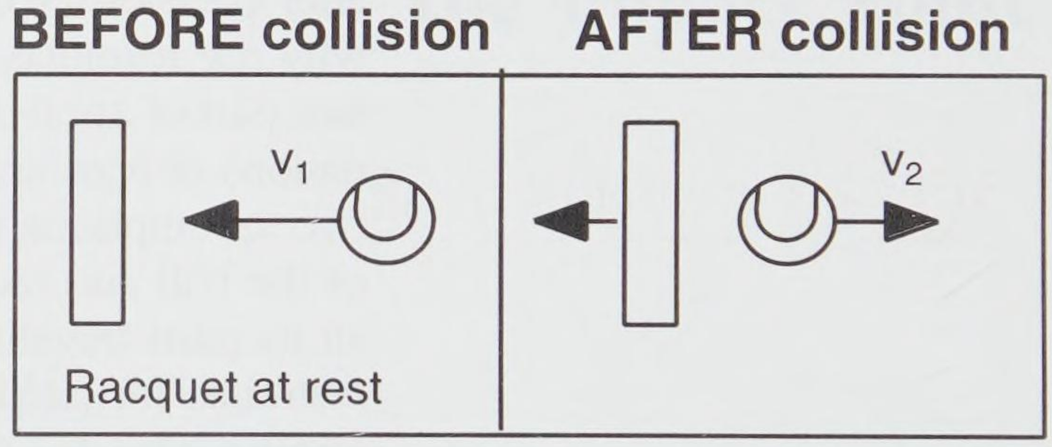


Figure 11.11

Again, we change the frame of reference, from the laboratory frame, where the racquet starts off at rest, to the court frame, where the racquet is swung towards the ball. To do that, we just add V to all the velocities shown in Figure 11.11 , where V is the racquet speed in the court frame of reference (Figure 11.12).

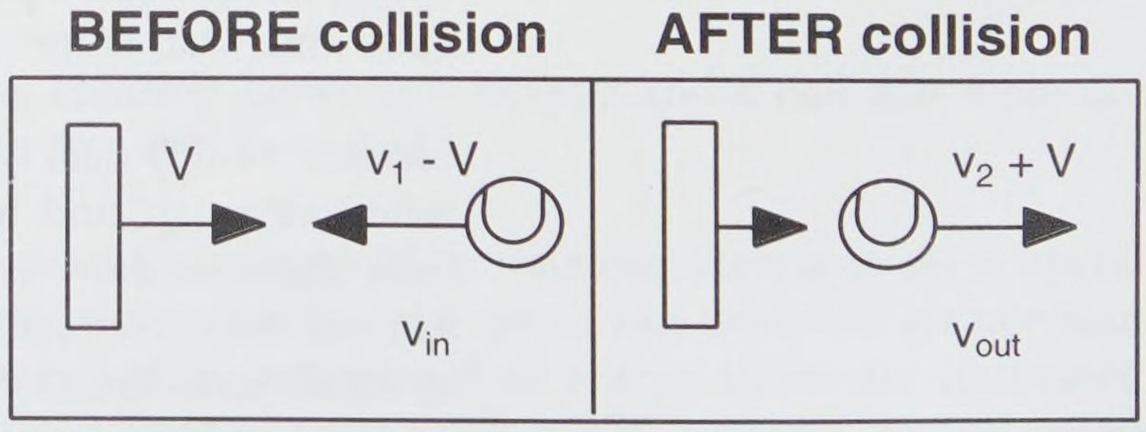


Figure 11.12

Again we do a little math:

Givens:

$e_A = v_2/v_1$ (measured in the lab frame)

$v_{in} = v_1 - V$ = the incident ball speed in the court frame

$v_{out} = v_2 + V$ (from Figure 11.12)

Calculation:

$v_2 = e_A v_1$ (from above)

$v_1 = v_{in} + V$ (from above)

so, $v_{out} = e_A v_1 + V = e_A (v_{in} + V) + V = e_A v_{in} + (1 + e_A)V$

And there we have our general ball speed formula — THE SECOND MOST IMPORTANT FORMULA IN THE BOOK:

General Ball Speed Formula

$$v_{out} = e_A v_{in} + (1 + e_A)V \qquad (11.11)$$

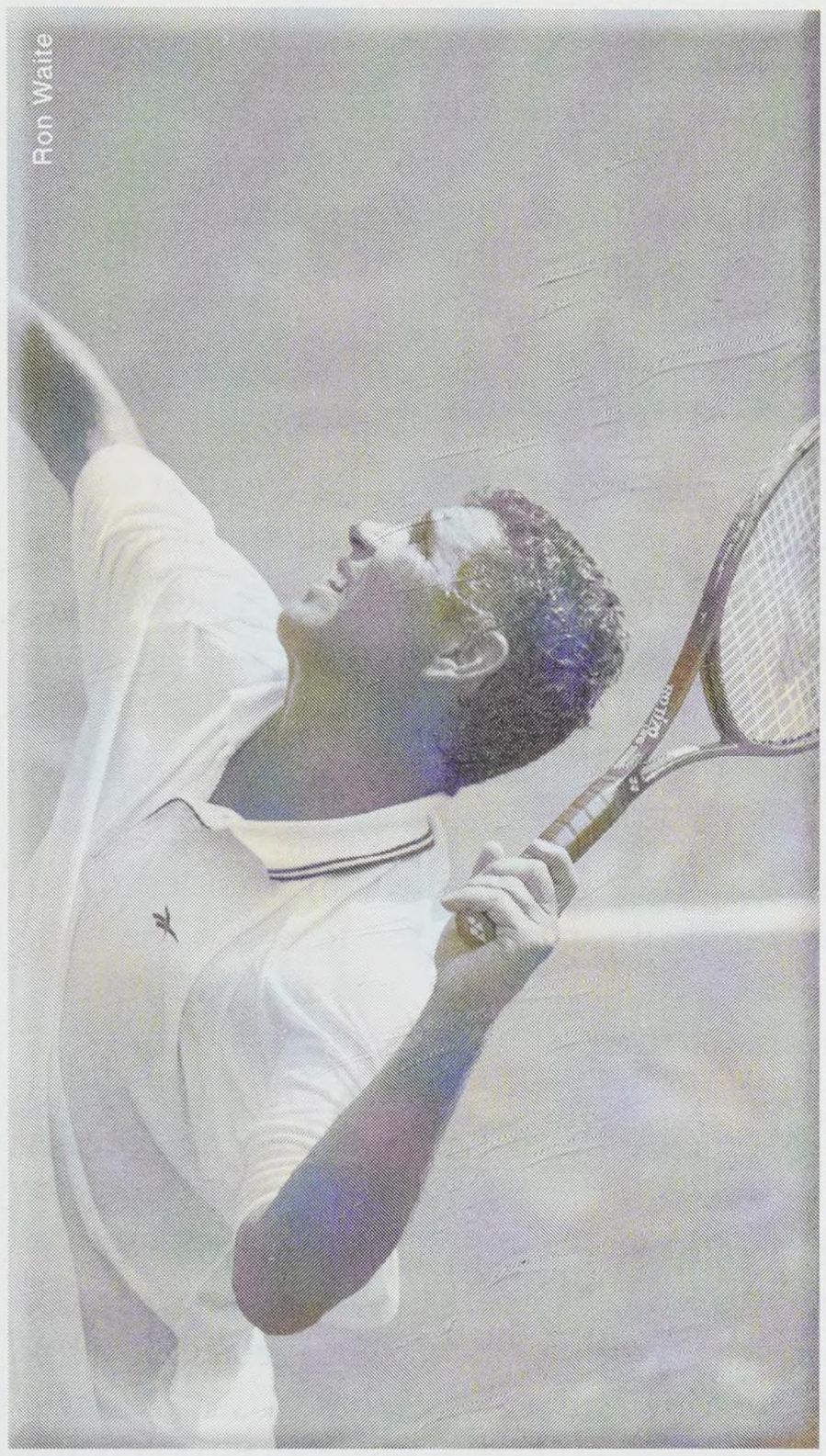
Equation (11.11) is actually more general than (11.10) since we recover (11.10) as a special case when $v_{in} = 0$. In that respect, (11.11) is more important than (11.10), but we do not use it as often in this book since we have focussed more attention on the serve than on groundstrokes. The serve is the power shot in tennis, the biggest weapon in a modern player's game, and the stroke that stretches our understanding of the underlying physics (and the strings) the furthest.

Equations 11.10 and 11.11 look a lot friendlier than Equations 11.7 and 11.8. It all begins with measuring e_A with a simple bounce test, and then plug into the equation any values you want in order to determine the results of any hypothetical collision.

Conclusion

We have come a long way to arrive at some easy to use formulas to determine after collision racquet and ball velocities, but in the process, we have learned why the formulas are what they are. Understanding is often the most important part of application. But our journey isn't quite finished. Many of our explanations of how and why things happen as they do have, so far, been based on two assumptions — that collisions are on a line through the center of masses of the ball and racquet, and that the racquet is traveling in a straight line with all its parts traveling at the same speed. Though ACOR released us from these restraints for making predictions, there are still are two missing ingredients to explain why ACOR is what it is. Since you usually don't hit the center of mass and the racquet is typically swinging in an arc around some axis, different impact points of the racquet act as if they weigh more or less than other points, and different impact points travel at different speeds that others. So, what is the mass and what is the velocity of the impact point?

Chapter 12 will deal with this question. There we will learn how to determine "effective mass" and how to calculate the velocity of the impact point. It will then show us how to incorporate these into our formulas. Once you have learned these concepts, you will always use effective mass and impact velocity in all your equations.



Andres Gomez, Ecuador

12

Collisions

Effective Mass and Velocity at the Impact Point

By Rod Cross

In this chapter we will examine several aspects of the collision between a racquet and a ball that were not considered in the previous two chapters. In particular we will show that:

• If you swing a racquet at a certain speed and keep increasing the mass of the racquet, then your serve speed will keep increasing. That's not surprising. The surprising part is that the ball speed can never be greater than twice the racquet speed, even if the racquet is as heavy as a truck and even if all energy losses are eliminated from the ball. There is a limit at which any further increase in racquet mass will actually decrease the ball speed since you will not be able to swing a truck as fast as a racquet.

• The function of the racquet strings is to store elastic energy during a collision with the ball and then give almost all of it back to the ball. When a ball is dropped onto concrete, all of the kinetic energy in the ball is converted to elastic energy in the ball since the ball comes to a complete stop on the concrete before it bounces upwards. When a racquet is swung at a ball, only a small part of the kinetic energy of the racquet is converted into elastic energy in the strings and the ball, since the racquet does NOT come to a complete stop. As a result, energy losses in the ball and the strings have only a small effect on the speed of the ball.

• When a ball strikes a stationary racquet anywhere on the strings, the racquet recoils and it also rotates. The combined recoil and rotation speed at the point of impact is greater than that for an impact at the racquet's center of mass. As a result, the racquet behaves at the impact point as if its mass is less than its actual mass. The effective mass of a racquet near the tip is about the same as the mass of a tennis ball. In the center of the strings, the effective racquet mass is about half the actual mass.

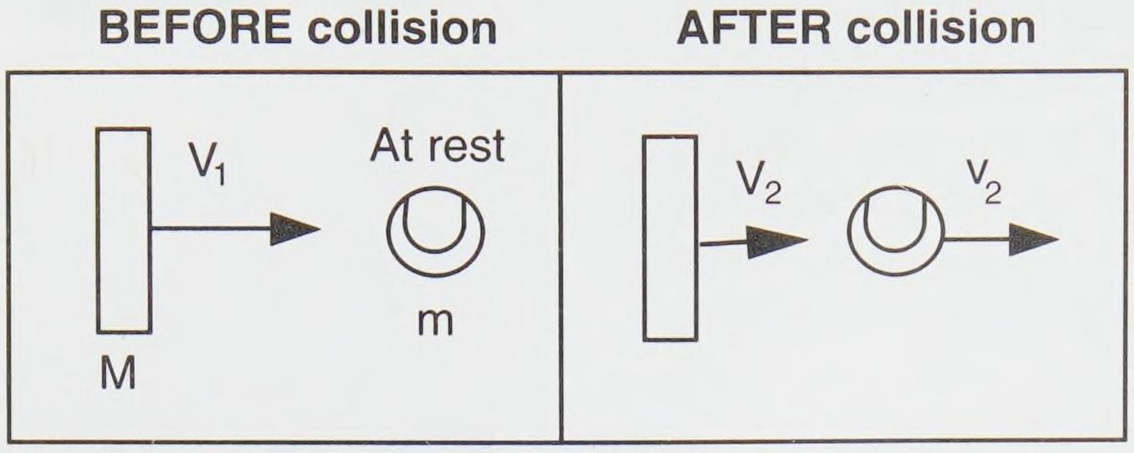


Figure 12.1

Effect of Racquet Mass on Serve Speed

Suppose that a racquet of mass M traveling at speed V_1 strikes a ball of mass m at rest, as shown in Figure 12.1. That is the usual situation when a ball is being served. After the collision, the racquet continues in the same straight line at speed V_2 and the ball is served at speed v_2. The only additional information needed to calculate the serve speed is the value of the COR for this collision, but that can easily be measured as described in Chapter 11. There is no need to know anything about the force on the ball, or the force exerted by the player on the racquet, or the time the ball spends on the strings or the diameter or stiffness of the ball, or the stiffness of the racquet. All those things are important in themselves but they are not needed to calculate the serve speed. All that is needed is the equation describing conservation of momentum (as described in Chapter 10) and the value of the COR (described in Chapter 11). We also need to know the correct value of the racquet mass, since it depends on the impact point on the strings. The correct value is known as the "effective mass". It is typically about half the actual mass of the racquet but it can be calculated as described at the end of this chapter.

The total momentum after the collision is equal to the momentum of the racquet before the collision, so

$$\mathbf{MV_1 = MV_2 + mv_2} \qquad \textbf{(12.1)}$$

If there is any energy loss during the collision, the relative speed after the collision will be less than the relative speed before the collision. The ratio of the relative speeds is given by

$$\mathbf{e = (v_2 - V_2) / V_1} \qquad \textbf{(12.2)}$$

where e is the coefficient of restitution (COR), typically about 0.88 for a racquet and ball collision. The speed of the racquet after the collision is therefore given from Equation 12.2 by

$$\mathbf{V_2 = v_2 - eV_1} \qquad \textbf{(12.3)}$$

If we substitute for V_2 in Equation 12.1 and solve for v_2 we get

$$MV_1 = M(v_2 - eV_1) + mv_2$$
$$MV_1 = Mv_2 - eMV_1 + mv_2$$
$$MV_1 = v_2(M + m) - eMV_1$$
$$MV_1 + eMV_1 = v_2(M + m)$$
$$MV_1(1 + e) = v_2(M + m)$$

so,

$$v_2 = \frac{(1 + e)\,MV_1}{(M + m)} \qquad (12.4)$$

That gives us the speed of the ball. If we now substitute for v_2 in Equation 12.3, we find the racquet speed after the collision:

$$V_2 = v_2 - eV_1$$

$$V_2 = \frac{MV_1(1 + e)}{M + m} - eV_1$$

$$V_2(M + m) = MV_1 + eMV_1 - eMV_1 - emV_1$$

$$V_2(M + m) = V_1(M - em)$$

so,

$$V_2 = \frac{(M - em)V_1}{(M + m)} \qquad (12.5)$$

Equations 12.4 and 12.5 are the same as Equations 11.7 and 11.8 for the case when the ball is initially at rest. There is a lot of interesting information here. The first thing to notice from Equation 12.4 is that v_2 is proportional to V_1. If you double the racquet speed, then the ball speed is doubled. If you could increase the racquet speed 100 times, the ball speed would increase 100 times.

The second thing to notice is that the ball speed increases when e increases, but it is not zero when e is zero. The maximum possible value of e is 1.0, which corresponds to a hypothetical situation where there is no energy loss in the ball or the racquet. The minimum possible value of e is zero which corresponds to a hypothetical situation where all the elastic energy stored during the collision is lost. Elastic energy is stored when the ball compresses and when the racquet strings stretch. If all that energy is lost, then the ball will still be served at relatively high speed, but it is only half the speed that would result if none of the elastic energy was lost. When e = 0 we see from Equation 12.3 that $V_2 = v_2$, which means that the ball sticks to the racquet and both move at the same speed after the collision.

Now consider what happens when M is increased. When M = 0 we see from Equation 12.4 that $v_2 = 0$. Obviously, a racquet of zero mass packs no punch at all. Now suppose that M = m and e = 1. Then, again using Equation 12.4, we find that $v_2 = V_1$ and $V_2 = 0$ (from Equation 12.5). In this case, the racquet stops dead and the ball takes off with the same speed that the racquet had before it hit the ball. This is what happens when one billiard ball strikes another head-on and when the second ball is initially at rest.

Now let M = 2m and e = 1. Then $v_2 = 4V_1/3$, so the ball comes off the racquet faster than the initial speed of the racquet. That's what every tennis player likes to see. So let's try a racquet with M = 10m. Then $v_2 = 20V_1/11 = 1.82V_1$ if e = 1. That's nearly twice as fast as the initial racquet speed. That's even better. How about M = infinity? Then we can forget about m in the bottom of Equation 12.4, and we get $v_2 = 2V_1$ if e = 1. That's as fast as we can possibly get. How come? One could reasonably expect that if M = infinity then the racquet has an

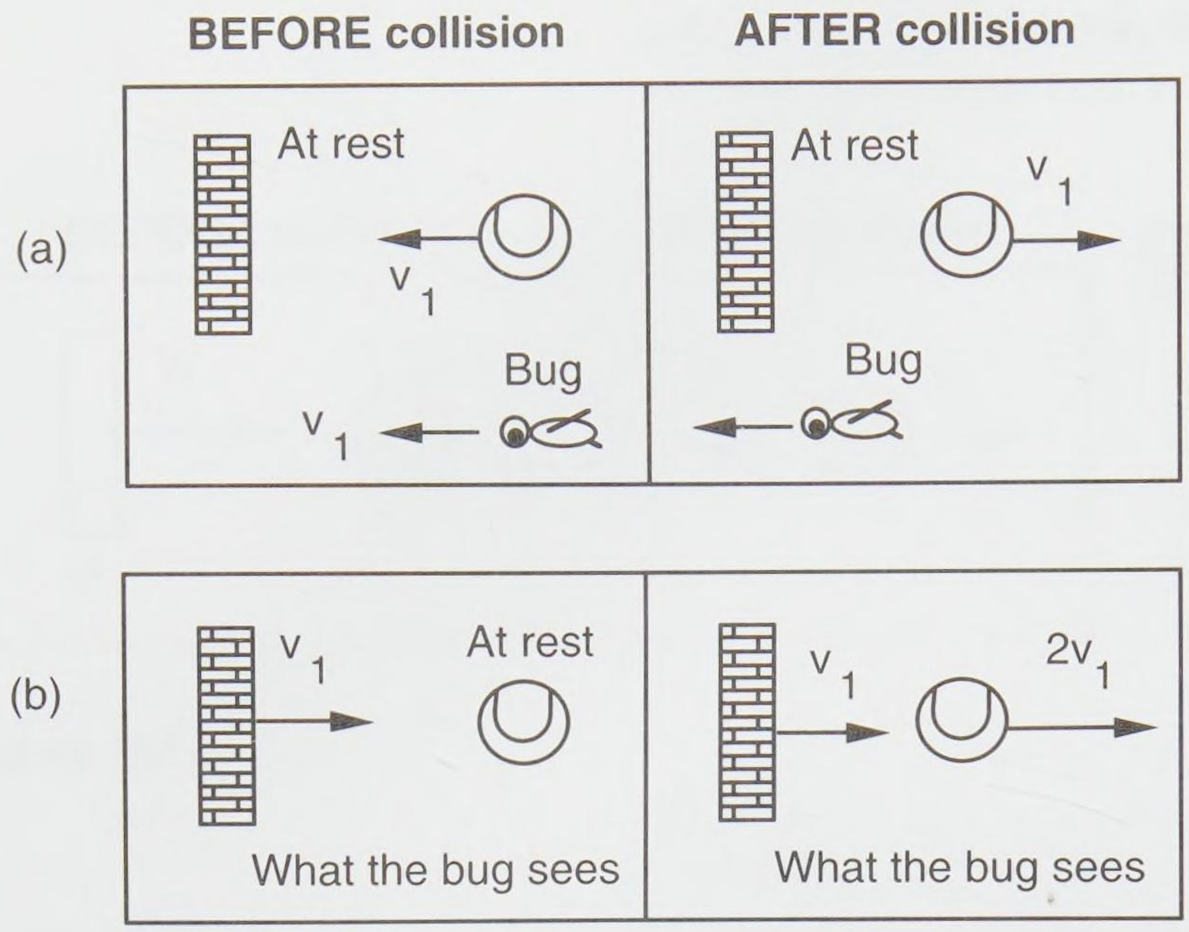


Figure 12.2

Serving the ball with a brick wall.

infinite amount of energy and momentum so the ball should be served with infinite speed (or maybe at the speed of light).

The easiest way to see what's happening is to change the frame of reference (as was explained in Chapter 11), as shown in Figure 12.2. A ball that strikes a brick wall at speed v_1 will bounce off the wall with speed v_1 if e = 1. A bug flying at speed v_1 beside the ball sees something else. The bug's view is obtained by adding v_1 to the speed of the wall and the ball. The bug sees the brick wall approaching the ball at speed v_1. It sees the ball is initially at rest, and it sees the ball being served at speed $2v_1$. In the reference frame where the wall is at rest, all that happens is that the ball bounces off the wall with the same speed as it hit the wall. In the frame where the ball is initially at rest, the wall collides with the ball and squashes it. The ball acquires the same speed as the wall, but in addition, it propels itself off the wall at v_1, so it is served at twice the speed of the advancing wall.

Elastic Energy Stored During a Collision

In the previous section, in Equation 12.4, we saw that the ball is served at speed

$$v_2 = (1 + e)\, MV_1 / (M + m)$$

when it is struck by a racquet of mass M moving at speed V_1. An interesting question is how much elastic energy gets stored in the racquet and the ball during the collision. When racquet and string manufacturers claim that their racquet or their string is more powerful, what they are really saying is that they can somehow reduce the elastic energy that gets lost during the collision. If none of the energy is lost, then e = 1, and the ball will be served at speed $v_2 = 2MV_1 / (M + m)$. If all of the elastic energy is lost, then e = 0, and the ball will be served at speed $v_2 = MV_1 / (M + m)$. There is a factor of two difference here, but that is not really relevant because the rules of tennis require that the ball must lose about 45% of its stored elastic energy when it has a collision. So the manufacturers can increase the ball speed not by a factor of two but by something that is a lot less than a factor of two.

There are two ways to figure out how much elastic energy is stored during the collision. One way is to assume that e = 0, in which case all of the elastic energy is lost, and that tells us how much elastic energy was available. The other way is to change the frame of reference so that the momentum of the racquet is equal and opposite the momentum of the ball. Then the total momentum is zero before, during and after the collision. During the collision, both the racquet and the ball come to a complete stop, in which case all of the original kinetic energy gets converted to elastic energy. We will try the e = 0 method.

Suppose that e = 1, then, from Equations 12.4 and 12.5

$$v_2 = 2\,MV_1 / (M + m) \quad \text{and} \quad V_2 = (M - m)V_1 / (M + m)$$

The total energy before the collision is $\frac{1}{2}MV_1^2$, and the total kinetic energy after the collision is the sum of the racquet energy and ball energy:

Total energy after collision = $\frac{1}{2}MV_2^2 + \frac{1}{2}mv_2^2$

Substituting v_2 and V_2 from above, we get

$$\frac{1}{2}MV_2^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}M\left[\frac{(M - m)V_1}{(M + m)}\right]^2 + \frac{1}{2}m\left[\frac{2MV_1}{(M + m)}\right]^2$$

$$= \frac{1}{2}M\left[\frac{(M - m)^2V_1^2}{(M + m)^2}\right] + \frac{1}{2}m\left[\frac{4M^2V_1^2}{(M + m)^2}\right]$$

$$= \frac{1}{2}\left[\frac{M(M - m)^2V_1^2}{(M + m)^2} + \frac{4mM^2V_1^2}{(M + m)^2}\right]$$

$$= \frac{1}{2}MV_1^2\left[\frac{(M - m)^2}{(M + m)^2} + \frac{4mM}{(M + m)^2}\right]$$

$$= \frac{1}{2}MV_1^2\left[\frac{(M^2 - 2Mm + m^2 + 4mM)}{(M + m)^2}\right]$$

$$= \frac{1}{2}MV_1^2\left[\frac{(M^2 + 2mM + m^2)}{(M^2 + 2mM + m^2)}\right]$$

$\frac{1}{2}MV_2^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}MV_1^2$ (energy after collision equals energy before)

If e = 0, then from Equations 12.4 and 12.5 we get

$$v_2 = MV_1 / (M + m) \quad \text{and} \quad V_2 = MV_1 / (M + m)$$

so the total energy after the collision is

$$\tfrac{1}{2}MV_2^2 + \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}M\left[\frac{MV_1}{(M+m)}\right]^2 + \tfrac{1}{2}m\left[\frac{MV_1}{(M+m)}\right]^2$$

$$= \tfrac{1}{2}(M+m)\left[\frac{(MV_1)^2}{(M+m)^2}\right]$$

$$= \frac{(MV_1)^2}{2(M+m)} = \text{Total energy after collision}$$

The energy lost, E, is given by the energy before the collision minus the energy after the collision:

$$E = \text{Energy lost} = \tfrac{1}{2}MV_1^2 - \frac{\tfrac{1}{2}(MV_1)^2}{(M+m)}$$

$$E = \tfrac{1}{2}MV_1^2\left[1 - \frac{M}{(M+m)}\right]$$

$$E = \tfrac{1}{2}MV_1^2\left[\frac{m}{(M+m)}\right]$$

Since we have assumed that e = 0, all the elastic energy stored during the collision is lost. The elastic energy stored during a collision is therefore equal to E. In any other head-on collision between a moving mass M and a stationary mass m, the amount of elastic energy stored during the collision will be equal to E, but the amount of energy that is subsequently lost will be less than E if e > 0. The fraction, f, of the initial energy that is stored as elastic energy is therefore

$$f = \frac{E}{\tfrac{1}{2}MV_1^2} = \frac{m}{(M+m)} \qquad (12.6)$$

For example, if M = m then f = 1/2, meaning that half of the initial kinetic energy of the racquet gets stored as elastic energy in the ball and the strings and the racquet frame during the collision. If all that energy is recovered, then e will be 1.0, but if all that energy is lost then e will be zero. If M = 2m then f = 1/3, meaning that only one third of the initial kinetic energy of the racquet gets stored as elastic energy.

The ball is required to lose about 45% of its stored elastic energy and the strings lose about 5%. The string plane stiffness is typically about the same as that of the ball, so the elastic energy stored in the strings is about the same as that stored in the ball (see Chapters 13 and 32). About one quarter of the total elastic energy is therefore lost during a racquet and ball collision. If M = m, the

total energy loss amounts to about 1/8 of the initial energy of the racquet. If M = 2m, it amounts to about 1/12 of the initial energy of the racquet. Changing the energy loss in the strings or the racquet frame by 2 or 3% therefore has very little effect on the ball speed (Figure 12.3).

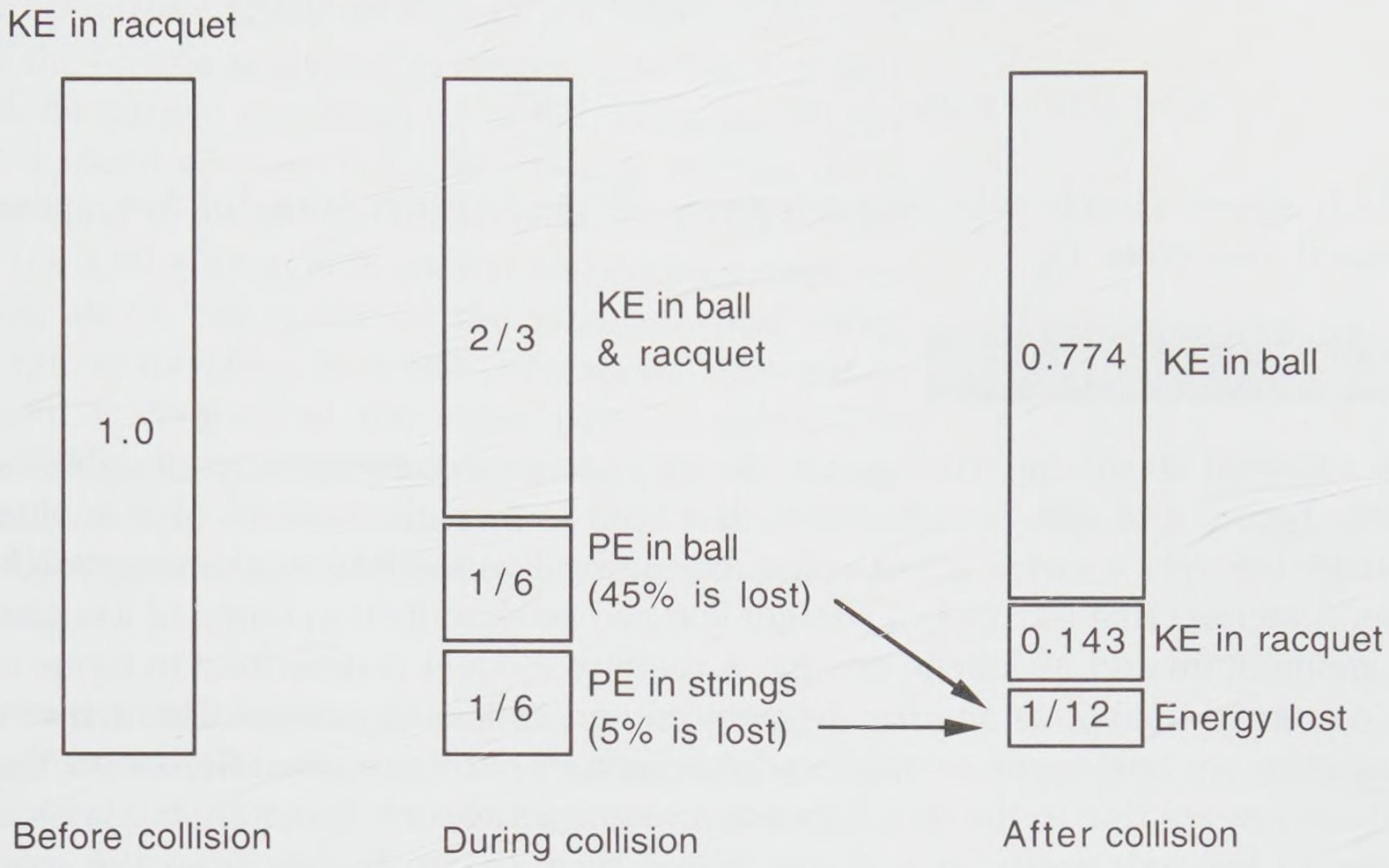


Figure 12.3

Energy fractions when M = 2m, f = 1/3, e = .866 and when the elastic potential energy (PE) is shared equally between the ball and the strings.

You might wonder why we are assuming here that M = m or 2m when the actual mass of a racquet (about 340 gm) is typically about 6 times larger than the mass of the ball (57 gm). That's because the effective mass of a racquet at the impact point is less than the actual mass, as we will shortly explain. It will take a fair amount of explaining, but consider this. If you hold a racquet by hand so that the strings are horizontal, and drop a ball near the tip, then the ball will bounce very weakly or not all. That's because the effective mass near the tip of the racquet is about 57 gm, about the same as the ball. We saw above that when two billiard balls collide head-on, one of them stops dead. The same effect occurs when you drop a 57 gm ball on a 57 gm racquet.

Calculation of the COR

For an impact in the middle of the strings, the effective racquet mass is typically about half the actual racquet mass. For a 340 gm racquet, this gives M = 3m. There is no energy loss due to vibrations of the racquet for an impact in the middle of the strings, so we only need to consider the losses in the ball and the strings. If M = 3m then the fraction of the initial kinetic energy stored as elastic energy during the collision is f = 1/4. If the ball and the strings are equally stiff, half of the elastic energy is stored in the ball and half in the strings. Given that the ball loses about half of its stored energy and the strings lose almost none, then 1/4 of the stored elastic energy is lost. The energy lost is therefore 1/16 or 6.25% of the initial racquet energy. From Equations 12.4 and 12.5, it can be shown that fraction of the energy lost when e is finite is

given by

$$f_L = 2E/(MV_1^2) = (1 - e^2)\ m\ /\ (M + m)$$

which indicates that $f_L = 0$ when $e = 1$ and it agrees with Equation 12.6 when $e = 0$. If $M = 3m$ and $e = 0$, then $f_L = 1/4$. However, if $M = 3m$ and if e is not zero, then $f_L = 1/16$ for realistic losses in the ball and the strings. Hence

$$1 - e^2 = 4/16 = 0.25 \qquad \text{so} \qquad e = 0.87$$

which agrees closely with measurements of the COR, at least for low speed impacts (see Note 1).

ROTATION

We assumed above and in Chapters 10 and 11 that racquets move in straight line paths before and after a collision with a ball. In fact, the motion of a racquet usually involves rotation about some axis as well as translation along a straight line. A racquet that moves in a straight line can be described in terms of its mass, its momentum and its kinetic energy. A rotating racquet is described in terms of its rotational inertia, its angular momentum and its kinetic energy. The first two quantities are analogous to mass and momentum but there are differences. The differences are due to the fact that when a racquet rotates about an axis, points close to the axis rotate at a lower speed than points further from the axis. Consequently we cannot describe a rotating racquet simply in terms of its speed in m/s. Instead we describe a rotating racquet or a rotating ball in terms of its angular speed, measured in revolutions per second or revolutions per minute or in radians/sec.

If a racquet is rotating at one revolution per second, then it rotates at 60 revolutions per minute and it rotates through 360 degrees per second. In physics we use another measurement of angle called the radian, and we do so for a good reason. One radian is the same as 57.3 degrees, and 2π radians is equal to 360 degrees. One revolution/sec is therefore the same as $2\pi = 6.28$ radians/sec.

Most students of physics are uncomfortable with radians when they first hear about them, but there is no logical reason to be afraid of radians. In fact, having 360 degrees in a circle instead of say 100 or 1000 is even less logical. We can blame the Babylonians who had troubles with fractions, and who found that 360 can be divided by lots of numbers without having anything left over.

The reason that we use radians is that it makes it easier to calculate the length of an arc of a circle. The length of the circumference of a circle of radius r is $2\pi r$ and it spans an angle of 360 degrees or 2π radians. The length of any section of this circle is found by simple proportion. For example, the length of a quarter of a circle is $\pi r/2$ and it spans an angle of 90 degrees or $\pi/2$ radians. This means also that the length of an arc spanning one radian is r, and the length, s, of an arc spanning θ radians is given by $s = r\ \theta$ (see Figure 12.4).

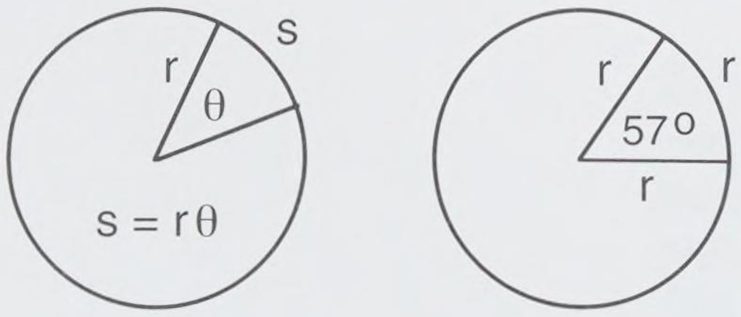


Figure 12.4

Relation between radius and arc length.

Relation Between Angular Speed and Actual Speed

If a racquet or any part of the racquet is rotating in a circle of radius r, and if it rotates by an angle θ in a time t, then it traces out a path of length $s = r\ \theta$ in

time t. The speed is v = s/t = rθ/t and the angular speed is defined as ω = θ/t (in radians/sec). Hence we have the useful results that

$$\mathbf{s = r\,\theta \quad and \quad v = r\omega} \qquad \mathbf{(12.7)}$$

Consider a racquet of length 70 cm that is rotating at 60 rad/sec about an axis 10 cm beyond the end of the handle as shown in Figure 12.5. The butt end of the handle is moving in an arc of radius 10 cm at a speed v = rω = 0.1 x 60 = 6 m/s, and the tip is traveling in an arc of radius 80 cm at speed v = rω = 0.8 x 60 = 48 m/s. It doesn't make much sense to talk about the speed of the racquet since every point is traveling at a different speed. But every point is rotating at the same angular speed (60 rad/s) and every point rotates through the same angle during the same interval of time (60 x 57.3 = 3438 degrees/sec or 3.438 degrees every 1 millisecond).

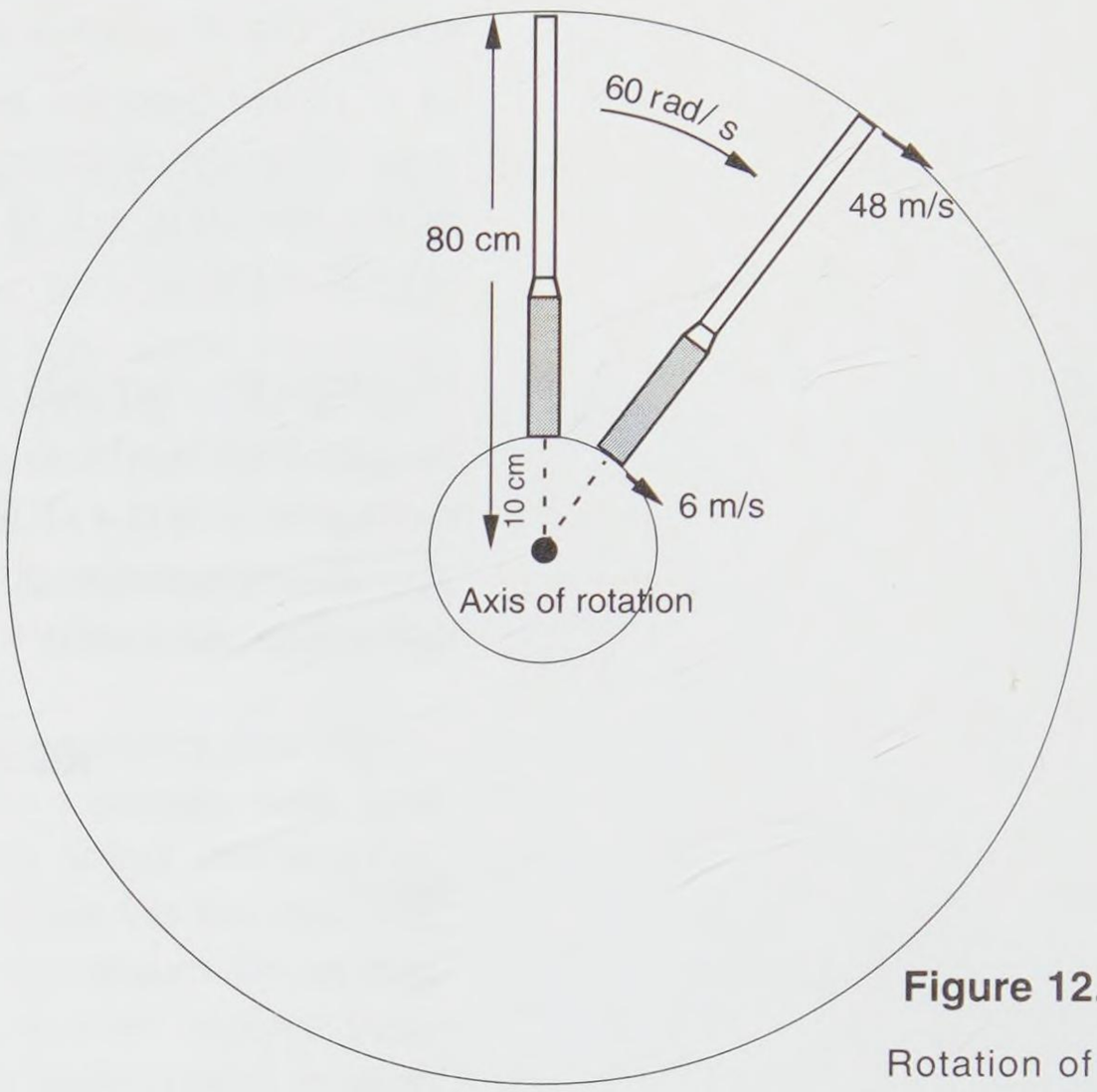


Figure 12.5

Rotation of a tennis racquet.

Armed with information on the angular speed of a racquet, its axis of rotation and values of the COR (e) or the ACOR (e_A) for that racquet, it is easy to locate the maximum power point of the racquet. An example is shown in Figure 12.6 where we have assumed that the racquet is swung as shown in Figure 12.5 with ω = 60 rad/sec. Note that values of e_A are shown in Figure 12.6 rather than values of e, since it easier to measure e_A than it is to measure e. The racquet speed is shown at three different points and is given by V = 60r since we are assuming that ω = 60 rad/sec.

As explained in Chapter 11, the serve speed at any impact point is given by $v = (1 + e_A)V$ where V is the speed of the racquet at the impact point. In the example shown in Figure 12.6, the serve speed is about 8% larger when serving from the middle of the strings than when serving from near the tip. Top players tend to serve from near the tip because the added height advantage reduces the chance of serving a fault. However, they usually customise their racquets by adding lead tape at the tip, which acts to increase e_A in that region. An increase in e_A from 0.1 to 0.2 would increase the serve speed near the tip to 1.2 x 42 = 50.4 m/s, in which case the serve speed near the tip would then be the same as the serve speed off the center of the strings. Further details on customizing racquets in this manner are described in Chapter 16.

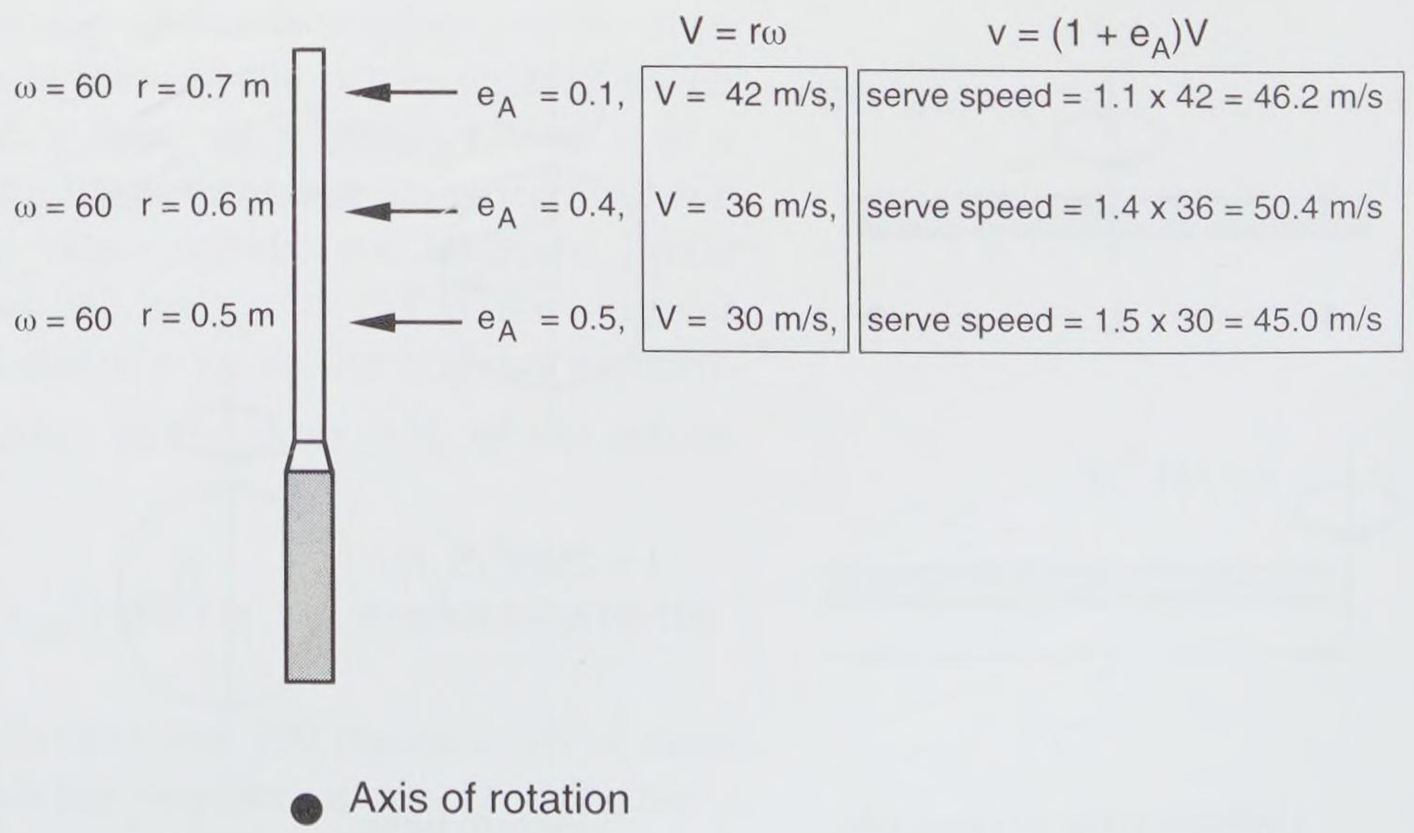


Figure 12.6

Serve speed at various impact points.

Moment of Inertia

A rotating racquet is moving and therefore has kinetic energy. The kinetic energy is $\frac{1}{2}Mv^2$ where M is the mass of the racquet, but what is v when every part of the racquet has a different speed? The answer is that v is an average speed, but it is not a simple average since we need to find the average value of v^2 rather than the average value of v. If the racquet is rotating about a fixed axis, then the average value of v is actually zero since the whole racquet is going nowhere – it is just stuck on its axis and is spinning round and round on the same spot.

Suppose we mentally divide the racquet into 100 small pieces all of mass m = M/100. The kinetic energy of each piece is $\frac{1}{2}mv^2$, where $v = r\omega$ is the speed of each piece, and r is the radius of each piece (i.e., the distance to the axis) . The kinetic energy of each piece is then $\frac{1}{2}mr^2\omega^2$, so the total kinetic energy of the whole racquet is

$$KE = \tfrac{1}{2}m(r_1^2 + r_2^2 + ... + r_{100}^2)\ \omega^2 = \tfrac{1}{2}I\,\omega^2 \qquad (12.8)$$

where

$$I = m(r_1^2 + r_2^2 + ... + r_{100}^2) \qquad (12.9)$$

I can be calculated for simple objects of simple shape and is called the moment of inertia of the object. For example, the moment of inertia of straight rod or beam of length L and mass M is $I = ML^2/12$ when the axis of rotation is at right angles to the rod and passes through the middle of the rod, as shown in Figure 12.7. The moment of inertia for several other objects is also shown in Figure 12.7.

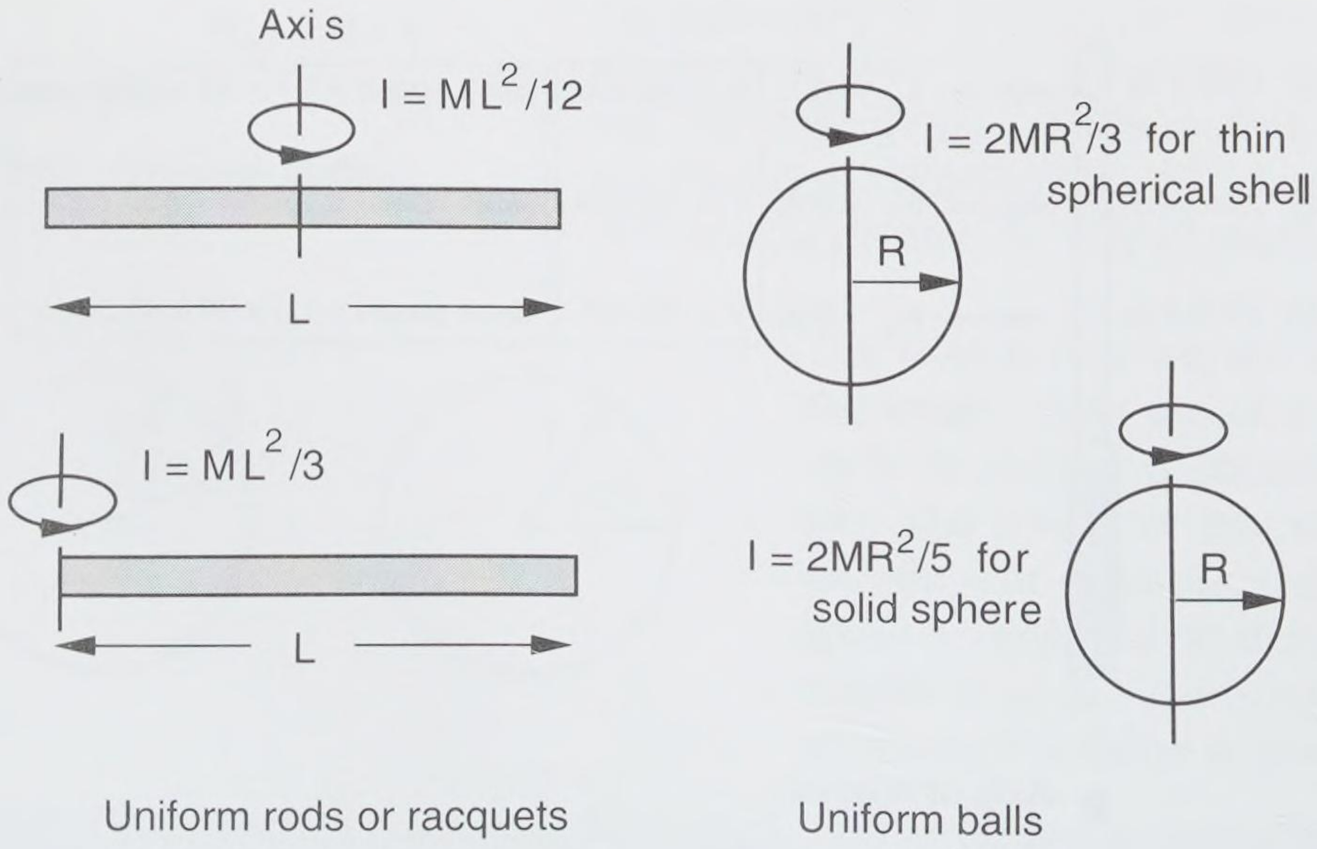


Figure 12.7

Moment of inertia for common objects.

A racquet is not a simple shape but its moment of inertia is almost the same as that of a straight rod of the same mass and length – at least when it is swung about its usual axis. In that case we often say that the moment of inertia is the swingweight of the racquet, despite the fact that the moment of inertia is not a weight and it is not even a mass. So, what is this thing called "moment of inertia"? There are two answers, and you can take your pick. The first is that it is the thing defined by Equation 12.9. The second is that is a measure of the resistance or inertia of an object to rotation. Consider two tennis balls, both of the same mass and diameter. One has all its mass near the outside of the ball and is hollow inside, just like a normal ball. The other is made from a small, 40 gm ball of lead in the middle and is surrounded by 17 gm of rubber foam designed so that both balls

bounce the same. The second ball has most of its mass in the middle and has a lower moment of inertia. The result will be that the second ball spins faster than the first since it has a lower resistance to rotation. Maybe I should patent that idea. If I made a golf ball like that I could hit it a lot further (due to the increased lift force).

Angular Momentum

A racquet that is rotating about a fixed axis is moving and therefore has momentum. If we divide a racquet of mass M into 100 small pieces each of mass m = M/100 then the momentum of each piece is mv. However if we add up the momentum of all 100 pieces the answer will be zero since for every piece moving left to right there is usually another piece moving right to left. Alternatively, if we look at the racquet in Figure 12.5 we see that the racquet spends half its time moving left to right and the other half (when it is below the axis) moving right to left. This is inconvenient since the momentum of any object rotating about a fixed axis is either zero, or its time average value is zero.

Consequently we need to define the momentum of a rotating object in a slightly different way to the momentum of an object moving in a straight line, and we give it a different name. The angular momentum of an object that is rotating about a given axis at angular speed ω is given by $I\omega$, where I is the moment of inertia about that axis. This is analogous to ordinary momentum Mv in that we replace the linear velocity v by the angular velocity ω and we replace inertia or mass M by rotational inertia I. The driving force in translational motion is an ordinary push or pull. The driving force in rotational motion is called a torque or a "moment", which acts to twist or rotate an object. When you use a screwdriver or a spanner or a steering wheel you apply a torque to rotate it. The torque or moment is given by the force exerted multiplied by the distance to the axis of rotation. The torque that spins a tennis ball when you hit it is the friction force between the ball and the strings (acting parallel to the string plane) multiplied by the radius of the ball. Everything concerning rotational motion is analogous to motion in a straight line except that we need to multiply or divide everything by the radius or the radius squared.

The formula for the angular momentum of any object is worked out by supposing that an object of mass M is made up of lots of little pieces each of much smaller mass m. If one particular piece has a mass m, a speed v, and is at a distance r from the axis of rotation, then its linear momentum is mv and we define its angular momentum by mvr. In other words, we take the linear momentum, multiply it by the distance to the axis and call it the angular momentum. From Equation 12.7 we know that $v = r\omega$ so the angular momentum of each piece is $mvr = mr^2\omega$. The angular momentum, AM, of the whole object is then

$$\mathbf{AM = m(r_1^2 + r_2^2 + ... + r_{100}^2)\,\omega = I\,\omega} \qquad \mathbf{(12.10)}$$

where we have assumed that the object is divided into 100 pieces each of mass m = M/100. Dividing the object into an infinite number of pieces will give a more accurate result. That is easy to do if you understand calculus, but we are assuming that most readers are not familiar with calculus. If you are, then you will probably know a lot about moments of inertia and angular momentum anyway.

Like linear momentum, angular momentum is conserved in a collision of two objects, provided there is no other force or torque acting on the colliding objects. If a diver does a somersault into a swimming pool, the diver will spin faster if he or she tucks in the arms and legs to make sure that all the different bits of the body are close to the axis of rotation. Adding up all the mr^2 values of each bit results in a small moment of inertia. While the diver is in the air there is no torque on the body (given that gravity acts through the center of mass) so angular momentum is conserved. The diver spins faster by reducing I and then stretches his or her arms and legs out to increase I and to reduce ω just prior to entering the water. During this time $I\omega$ remains constant so it is "conserved."

Collisions Involving Rotation of the Racquet

In Chapters 10 and 11 we confined our attention to racquets that impact the ball at the center of mass of the racquet, somewhere near the throat of the racquet. We did that for two reasons. One was to simplify the maths and the other was because we knew that we would eventually get the correct result at any other impact point if we replaced the actual mass of the racquet by its effective mass. It is now time to set the record straight and prove that we can indeed get such a result. However, we will do so only for a case where there are no energy losses, since the maths gets a bit too complicated if we allow for energy losses in the ball and the racquet. Energy losses are best treated in terms of the measured values of the ACOR for any given racquet, but for now we just want to look at the essential features of a racquet that rotates when it collides with a ball. We will allow the ball to strike the racquet at any point along the length of the racquet, but to simplify the calculations, we will assume that the racquet is initially at rest (see Note 2).

The situation is shown in Figure 12.8. A ball of mass m is incident at speed v_1 and strikes the racquet at a point at a distance b above the center of mass (CM) of the racquet. The ball bounces off at speed v_2, the CM of the racquet recoils at speed V_{cm}, and the racquet rotates clockwise about its CM at angular speed ω_2.

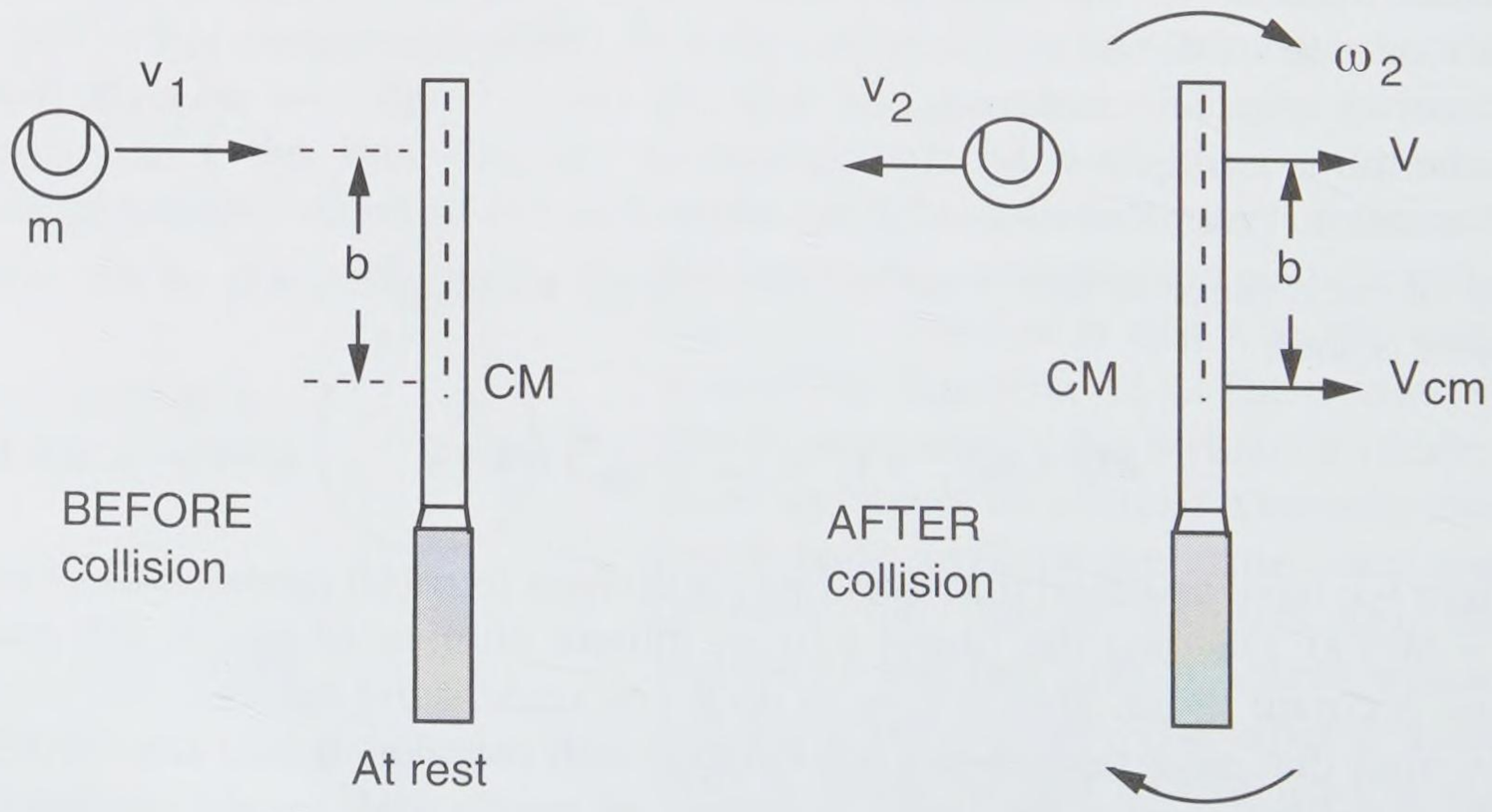


Figure 12.8

A ball incident on a racquet at rest.

Applying the conservation laws of linear and angular momentum as well as kinetic energy, we find for the situation in Figure 12.8 that

$$m v_1 = M V_{cm} - m v_2 \qquad (12.11)$$

$$m v_1 b = I \omega_2 - m b v_2 \qquad (12.12)$$

and

$$\tfrac{1}{2} m v_1^2 = \tfrac{1}{2} m v_2^2 + \tfrac{1}{2} M V_{cm}^2 + \tfrac{1}{2} I \omega_2^2 \qquad (12.13)$$

The ball is not rotating about its own axis, but it has angular momentum mvb about the CM of the racquet. In applying the conservation of angular momentum law, we need to consider the total angular momentum of the ball and the racquet about a common axis. In this case, the best axis to work with is the axis through the CM of the racquet. Like linear momentum, angular momentum is taken to be positive or negative depending on the direction of rotation.

From Equations 12.11 and 12.12 we find that

$$V_{cm} = m(v_1 + v_2) / M \qquad (12.14)$$

and

$$\omega_2 = mb(v_1 + v_2) / I \qquad (12.15)$$

Substituting Equations 12.14 and 12.15 into 12.13 gives

$$m v_1^2 = m v_2^2 + m^2 (v_1 + v_2)^2 / M + m^2 b^2 (v_1 + v_2)^2 / I$$

so

$$v_1^2 = v_2^2 + m(1/M + b^2 / I)(v_1 + v_2)^2 \qquad (12.16)$$

The quantity $1/M + b^2/I$ is not easily recognisable as anything of importance but it is equal to $1/M_e$ where M_e is the effective mass of the racquet at the impact point. Consequently we will define M_e by the relation

$$\frac{1}{M_e} = \frac{1}{M} + \frac{b^2}{I} \qquad (12.17)$$

in which case, Equation 12.16 can then be written as

$$v_1^2 = v_2^2 + m(v_1 + v_2)^2 / M_e$$

Expanding this out gives

$$v_2^2 (1 + m/M_e) + (2 m v_1 / M_e) v_2 - (1 - m/M_e) v_1^2 = 0$$

which is a quadratic equation with solution

$$v_2 = \frac{v_1(1 - m/M_e)}{(1 + m/M_e)} \qquad (12.18)$$

There is another solution, $v_2 = - v_1$, but this is of no interest unless the ball passes straight through the racquet and emerges unscathed on the other side. Equation 12.18 can be compared with Equation 12.5 and it will be seen that Equation 12.18 is the result that we would obtain if the ball collided head-on with a mass M_e. It is for this reason that we identify M_e as the effective mass of the racquet.

Substituting Equation 12.18 into 12.15 gives

$$\omega_2 = 2\, m\, b\, v_1 / [I\,(1 + m/M_e)] \qquad (12.19)$$

An interesting result is obtained if we consider the impact point on the racquet. This point recoils at a linear speed $V = V_{cm} + b\,\omega_2$. From Equations 12.14, 12.18 and 12.19 we find that

$$V = 2\, m\, v_1 / (m + M_e)$$

and hence the relative speed of the ball and the impact point after the collision is $V + v_2 = v_1$, which is the same as the relative speed before the collision. Consequently, the COR for this collision is 1.0, meaning that there is no loss of energy during the collision, which is what we assumed at the start. More importantly, it shows that in this situation the COR must be defined in terms of the relative speed of the ball and the racquet at the impact point, and not in terms of the speed of the CM of the racquet.

Effective Racquet Mass

The formula for the effective racquet mass is given by Equation 12.17. This can be calculated for any given racquet if we know or measure its moment of inertia, I, but most racquets of mass M and length L have a value of I that is close to that for a uniform rod of mass M, and length L, in which case $I = ML^2/12$. If we substitute this value of I into Equation 12.17 we find that

$$M_e = M/(1 + 12\, b^2/L^2) \qquad (12.20)$$

In the middle of the racquet, $b = 0$ and $M_e = M$. At the tip of the racquet, $b = L/2$ and $M_e = M/4$. Halfway between the middle and the tip, $b = L/4$ and $M_e = M/1.75$. The center of the strings is about 20 cm from the CM of a 70 cm racquet where $M_e = M/1.98$.

The effective mass is therefore smaller than the actual mass at all impact points other than the balance point in the middle of the racquet. The reason is as follows. If a force F is applied at the middle of the racquet then the whole racquet will accelerate in the direction of the force according to the relation $F = Ma$, where M is the actual mass of the racquet. The acceleration $a = F/M$ is inversely proportional to the mass M. A light racquet will accelerate rapidly to

high speed and a heavy racquet will accelerate slowly at a lower speed. Now suppose the force is applied at a point closer to the tip of the racquet. The whole racquet will still accelerate in the direction of F with a = F/M, but in addition, the racquet will rotate about its center of mass. As a result, the point on the racquet at which F is applied will move in the direction of F at a speed that is faster than when F is applied at the center of mass. In other words, that point behaves as if it had a mass that was less than the actual mass of the racquet since its acceleration is greater than F/M.

We can quantify this effect as shown in the Appendix of this chapter.

Example

Suppose that a 60 gm tennis ball traveling at 18 m/s collides with a 300 gm tennis racquet at rest, as shown in Figure 12.9. The racquet could be held in the hand, or it could be balanced with its butt end on the floor, or it could be suspended by a long length of string tied to the ceiling. If the ball hits the middle of the racquet at its center of mass, and if there is no loss of energy during the collision, the ball would bounce off the racquet at 12 m/s and the racquet would recoil at 6 m/s. The momentum before the collision is 18 x 60 = 1080 and the momentum after the collision is 300 x 6 - 60 x 12 = 1800 - 720 = 1080. The relative speed after the collision (12 + 6 = 18 m/s) is equal to the relative speed before the collision.

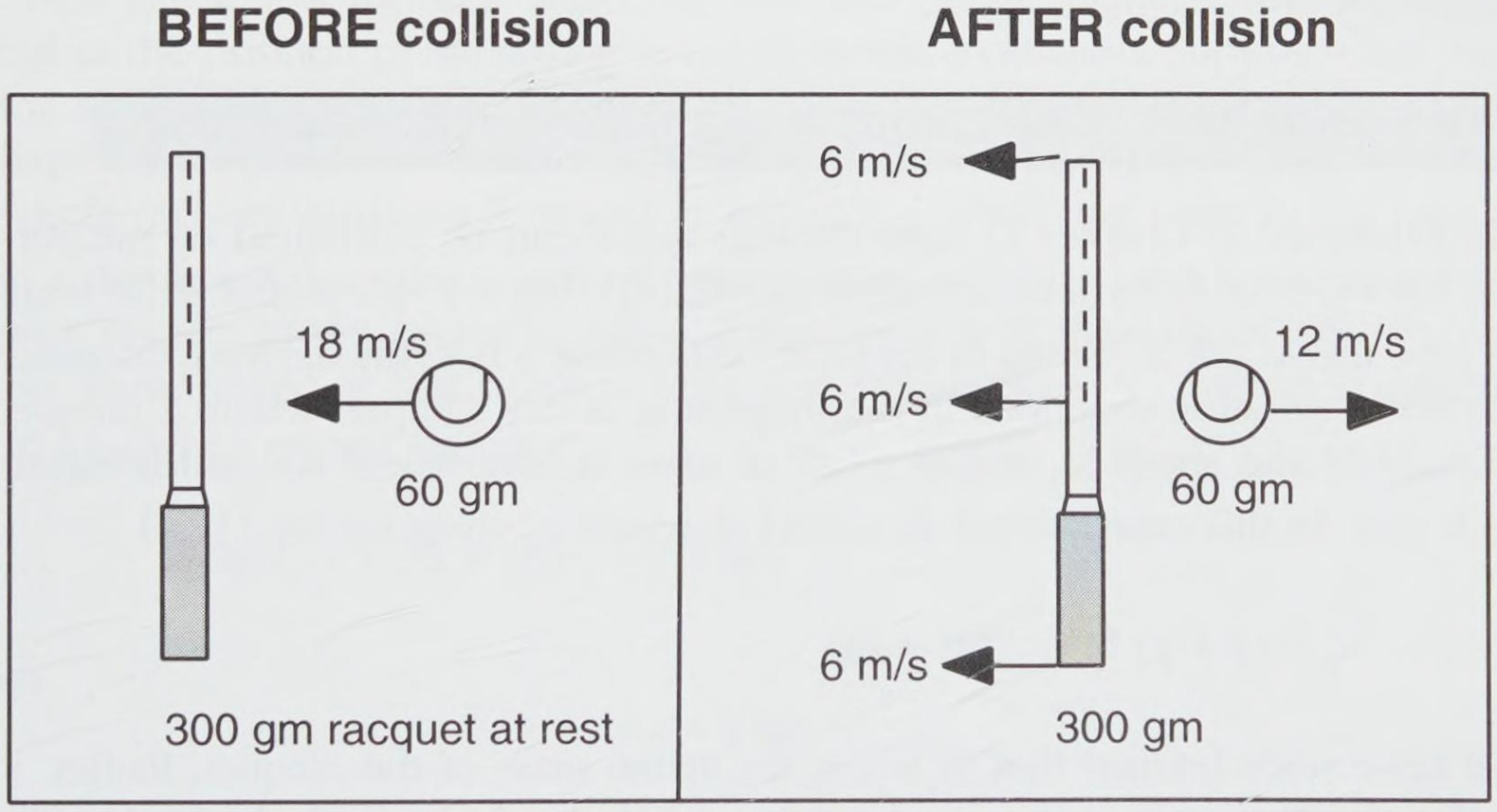
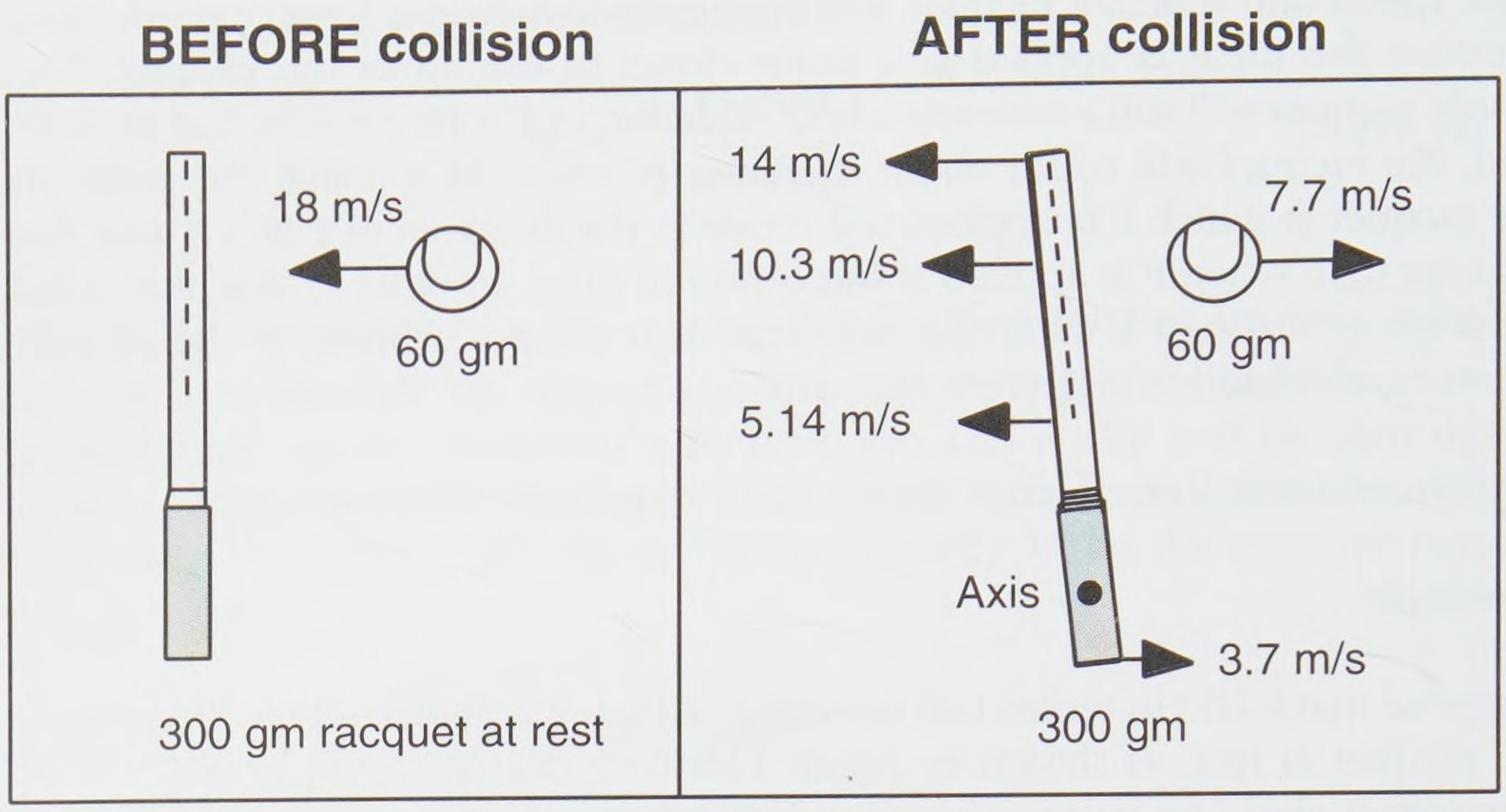


Figure 12.9

Now suppose the ball hits higher up, in the middle of the strings, as shown in Figure 12.10. The racquet recoils as before, but it also rotates around an axis near the end of the handle. As a result, the recoil speed of the racquet varies from one end to the other. This represents the combined effect of a recoil at 5.14 m/s (the speed of the center of mass) plus rotation around the center of mass.

These results were calculated assuming that linear momentum, angular momentum, and kinetic energy are all conserved. The ball ends up with 18% of its original energy and the racquet ends up with the rest. As far as the ball is concerned, it collided not with a 300 gm mass, as in the first example above, but with a 150 gm mass. The effective mass of the racquet is therefore 150 gm at this point, but it is the full 300 gm at the middle of the racquet. The relative

Figure 12.10



speed at the impact point is 18 m/s before the collision and 10.3 + 7.7 = 18 m/s after the collision. The linear momentum of the racquet after the collision is 150 x 10.3 = 1545 (which is also equal to 300 x 5.14). The total (linear) momentum is 1545 - 60 x 7.7 = 1080 as before.

The effective mass near the middle of the strings is about half the actual racquet mass for all racquets. About two inches from the tip, the effective mass is about 1/3 of the actual mass.

Estimating the ACOR from the Effective Racquet Mass

We explained in Chapter 11 how the ball speed can be calculated by measuring the COR (e) or by measuring the ACOR (e_A), but we side-stepped the issue of how the ACOR is related to the COR. This relation is easily derived. Consider the serve situation described at the beginning of this Chapter where a racquet of mass M and speed V_1 strikes a ball of mass m, and where the ball is initially at rest. In this case the ball is served at speed v_2 given by Eq. (12.4)

$$v_2 = (1 + e)\, MV_1 / (M + m)$$

We have since learned that M is not the actual mass of the racquet. Rather, it is the effective mass at the impact point, so M should be replaced by M_e. We also found, in Chapter 11, that v_2 can be calculated from a measurement of e_A, and is given by the serve speed formula

$$v_2 = (1 + e_A)V_1$$

If we equate these two expressions for v_2 and replace M by M_e we find that

$$\mathbf{e_A = \frac{(eM_e - m)}{(M_e + m)}} \qquad \mathbf{(12.21)}$$

This relation shows how e_A depends on the COR and the effective mass and it agrees nicely with experimental data on different racquets. For an impact in the center of the strings, e is typically about 0.88 and M_e is about half the actual

racquet mass. If m = 57 gm, M = 300 gm and M_e = 150 gm, then e_A = 0.36. For a heavier racquet with M = 400 gm and M_e = 200 gm, e_A = 0.46. Near the tip of a racquet e drops to about 0.7 due to energy lost in frame vibrations and M_e is about 1/3 of the actual racquet mass. If e = 0.7 and M_e = 300/3 = 100 gm, then e_A = 0.08. If e = 0.7 and M_e = 400/3 = 133 gm, then e_A = 0.19.

Axis of Rotation of a Racquet

As described in Chapter 6, the motion of a racquet can be regarded as consisting of two parts, one being a translation of the whole racquet in a straight line at the speed of its center of mass (CM), and the other being a rotation about an axis through the CM at angular speed ω. The combination of this translation and rotation means that the actual axis of rotation is shifted away from the CM. Just before a collision with a ball, the rotation axis is typically about 10 to 20 cm beyond the butt end of the handle. The location of the actual axis of rotation before the collision depends on the type of stroke and the amount of wrist action. During a collision, the rotation axis shifts to a point closer to the butt end or to some point under the hand, depending on the impact point on the strings. There is nothing a player can do to change the rotation axis during the collision, since the force of the ball on the strings is much larger than the force of the player's hand on the handle.

There is a simple formula, which we will now derive, that can be used to calculate the position of the axis of rotation during a collision. Suppose that, during the collision, the actual rotation axis is somehere near the butt end of the handle at a distance x from the center of mass. As shown in Figure 12.8, the whole racquet translates forwards at speed V_{cm}, while any point in the handle at a distance x from the center of mass rotates backwards at a linear speed xω with respect to the center of mass. If that point is at the axis of rotation, the net linear speed at that point is zero, and hence, $x\omega = V_{cm}$. From Equations 12.14 and 12.15, we find that

$$xmb(v_1 + v_2)/I = m(v_1 + v_2)/M$$

so

$$x = I/Mb$$

where I is the moment of inertia of the racquet about its CM. Note that if b = 0, then x = infinity, meaning that if the ball impacts the racquet at its CM, the racquet does not rotate at all. For most racquets, I is about $ML^2/12$ where L is the length of the racquet, so $x = L^2/(12b)$. In that case, the axis of rotation will be at the butt end of the handle, at x = L/2, if b = L/6. If L = 69 cm, then L/6 =11.5 cm, meaning that the center of percussion in this case is 11.5 cm from the middle of the racquet or 23 cm from the tip.

The actual rotation axis passes through a point known as the conjugate point with respect to the impact point, and the impact point is said to be the center of percussion for that particular conjugate point. Every point in the handle has a different center of percussion on the strings.

In deriving the formula for the effective mass of a racquet we have implicitly assumed that the racquet rotates about an axis through the conjugate point. It

does so during and immediately after the collision, but the axis of rotation before the collision is usually somewhere else. As a consequence, the kinetic energy of a racquet before the collision is not necessarily equal to $\frac{1}{2}M_eV^2$ and its momentum is not necessarily equal to M_eV, where V is the speed of the impact point. These expressions are exact if the racquet rotates about the conjugate axis defined above, or if V = 0, but they are only approximately correct if the racquet rotates about some other axis. Nevertheless, the collision between a racquet and a ball is correctly described, and the speeds of the ball and the impact point are correctly determined, if we assume that the impact point on the racquet is an object of mass M_e, traveling at speed V, and having kinetic energy $\frac{1}{2}M_eV^2$, momentum M_eV and angular momentum M_eVb. The actual kinetic energy of the whole racquet before the collision is typically about 10 or 20% greater than $\frac{1}{2}M_eV^2$ for a serve or a groundstroke. The correct formula for the kinetic energy of a racquet before the collision is given in Note 3 of Chapter 32.

Notes

1. The COR is normally measured by firing a ball at a racquet whose head is firmly clamped. In that case, all the initial ball energy is converted to elastic energy in the strings and the ball. If 1/4 of that energy is lost, the ball will bounce with $v_2^2 / v_1^2 = 0.75$ and hence COR = e = 0.87. The COR varies slightly with string tension and it also varies with ball speed. For example, at ball speeds around 10 m/s, the COR is typically about 0.9 on a racquet strung at low tension and about 0.85 on a racquet strung at high tension. Measurements at ball speeds around 40 m/s show that the COR decreases to about 0.80 on racquets strung at either low or high tension. The decrease in the COR is due mainly to the fact that the ball loses a greater fraction of its energy in a high speed collision.

2. A similar calculation can be done for a serve where the racquet is swung towards a ball at rest. In that case the resulting ball speed is the same as that for a head-on collision between the ball and a mass M_e incident at speed V, where M_e is given by Eq. (12.17) and V is the speed of the racquet at the impact point.

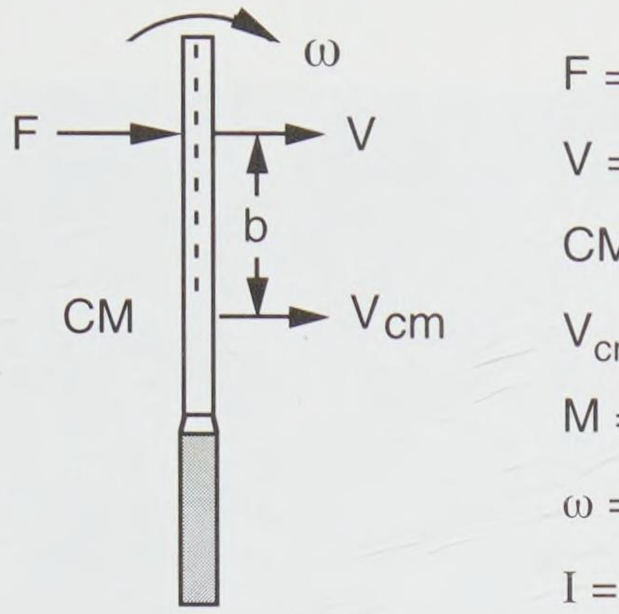
Appendix

Formula for Effective Racquet Mass for Impact Along Center Line

The effective mass M_e, of the racquet at an impact point lying along the longitudinal axis can be determined from the following relations (see Figure 12.11):

$$F = M_e a = M_e \frac{dV}{dt} \quad \text{and} \quad F = M \frac{dV_{cm}}{dt}$$

$$V = V_{cm} + b\omega \quad \text{so} \quad \frac{dV}{dt} = \frac{dV_{cm}}{dt} + b \frac{d\omega}{dt}$$



F = force of ball on racquet

V = speed of impact point

CM = center of mass

V_{cm} = speed of CM

M = actual racquet mass

ω = angular speed of racquet

I = moment of inertia about CM

Figure 12.11

$Fb = I \frac{d\omega}{dt}$ (where Fb is the torque about the center of mass.)

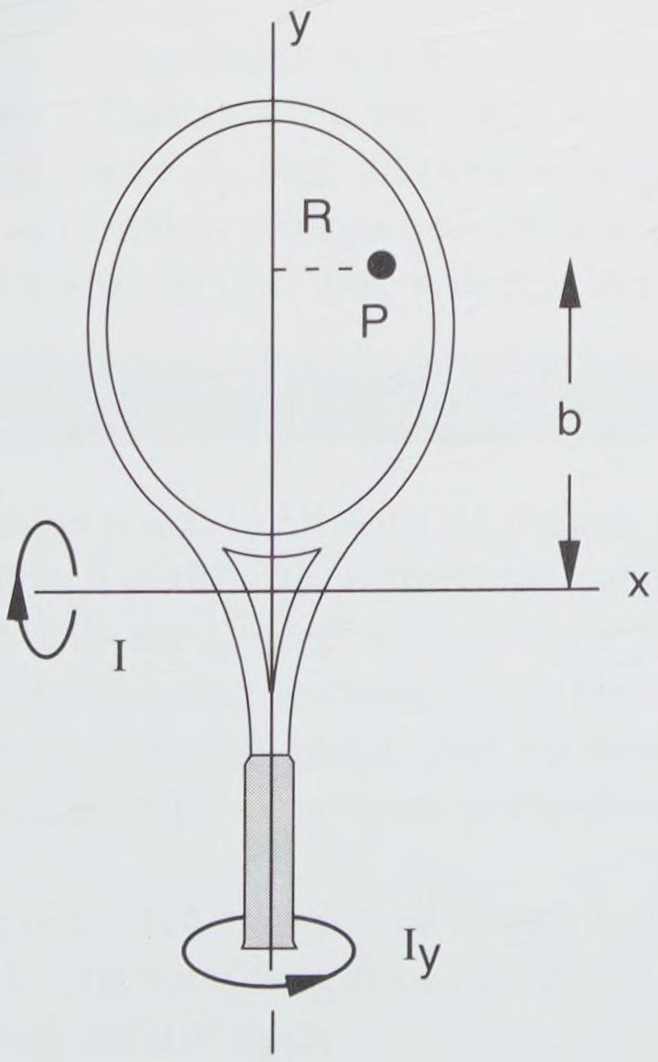
Hence, $\frac{F}{M_e} = \frac{F}{M} + \frac{Fb^2}{I}$ so, $\frac{1}{M_e} = \frac{1}{M} + \frac{b^2}{I}$

Or

$$\mathbf{M_e = \frac{M}{1 + Mb^2/I}} \qquad \textbf{(12.22)}$$

Formula for Effective Racquet Mass for Impact Anywhere

The effective racquet mass, M_e, of the racquet at any impact point can be determined from the following relations (see Figure 12.12):



P = impact point

y = long axis

x = axis through CM

R = distance from P to y axis

b = distance from P to x axis

M = actual racquet mass

I = moment of inertia about x axis

I_y = moment of inertia about y axis

Figure 12.12

$$F = M_e a = M_e \frac{dV}{dt} \quad \text{and} \quad F = M \frac{dV_{cm}}{dt}$$

$$V = V_{cm} + b\omega + R\omega_y \quad \text{so} \quad \frac{dV}{dt} = \frac{dV_{cm}}{dt} + b\frac{d\omega}{dt} + R\frac{d\omega_y}{dt}$$

$Fb = I \frac{d\omega}{dt}$ (where Fb is the torque about the center of mass.)

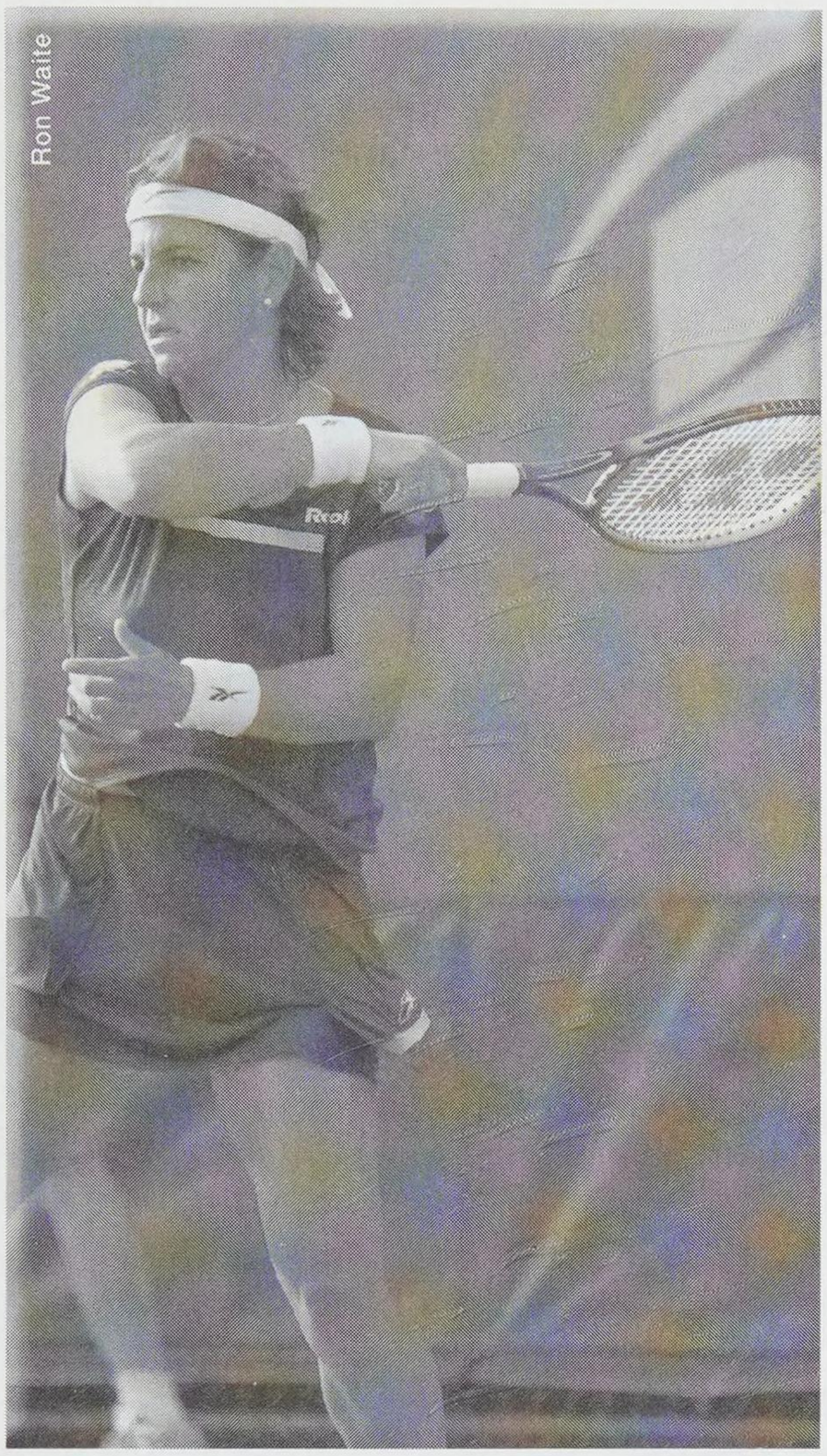
$FR = I_y \frac{d\omega_y}{dt}$ (where FR is the torque about the center of mass.)

Hence, $\frac{F}{M_e} = \frac{F}{M} + \frac{Fb^2}{I} + \frac{FR^2}{I_y}$

So,

$$\frac{1}{M_e} = \frac{1}{M} + \frac{b^2}{I} + \frac{R^2}{I_y} \qquad \textbf{(12.23)}$$

When R = 0, this formula is the same as Equation 12.22. If the impact point is closer to the edge of the frame, then R is not zero, and hence, R^2 increases, so $1/M_e$ increases, so M_e decreases. As a result, the ball will not bounce as well, and the speed of the ball will be reduced, regardless of whether the shot is a serve or a groundstroke.



Arantxa Sanchez-Vicario, Spain

13

Tennis Springs

Racquets, Strings, and Balls as Springs

By Rod Cross

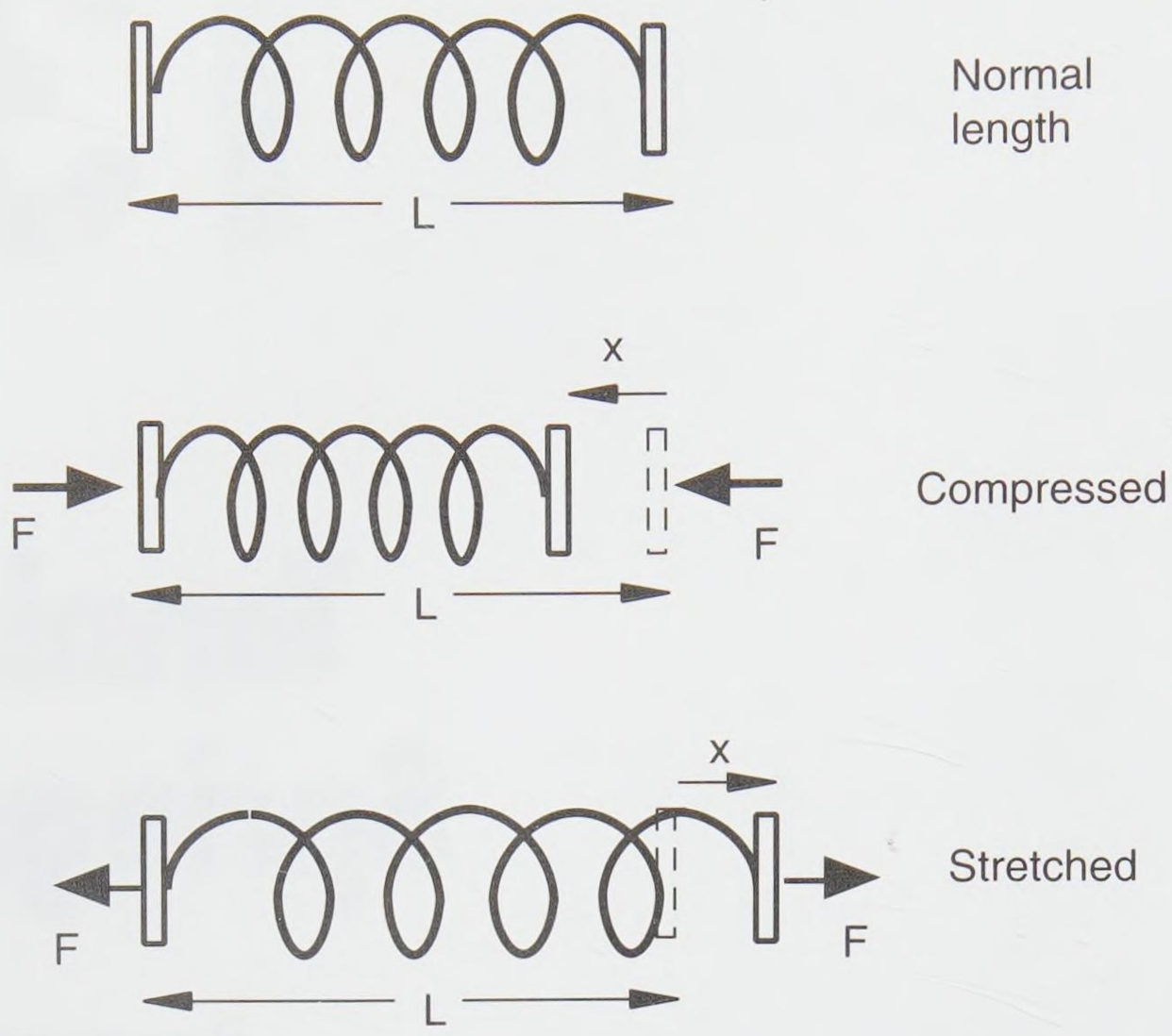
Tennis balls and tennis strings can be regarded as springs, for the simple reason that they ARE springs. They are not the usual type of metal spring wound into a coil, but they function in essentially the same way. In fact, all objects behave as springs, but not all objects are designed to compress or stretch as their main function. Strings are designed to stretch, balls are designed to compress and racquet frames are designed to be stiff so that they will not bend too far. In that respect, racquet frames are designed as stiff springs.

Stiff Springs and Soft Springs

Springs come in all sorts of shapes and sizes, but the most common variety is a metal coil. If the turns are wound tightly together, then they are called extension or tension springs, since they cannot be compressed further. If the coils are spaced so that there is a gap between each turn, then they are called compression springs. A compression spring is designed to be compressed. A compression spring can also be stretched in the same way as a tension spring, but it can't be stretched as far since the turns are already separated a certain distance, and since a spring will not recover properly if it is stretched too far.

Suppose that a spring of length L stretches by an amount x when subject to a pulling force F, as shown in Figure 13.1. The total length of the spring is then L + x. We define the stiffness (also called the spring constant) of the spring by the ratio:

Figure 13.1

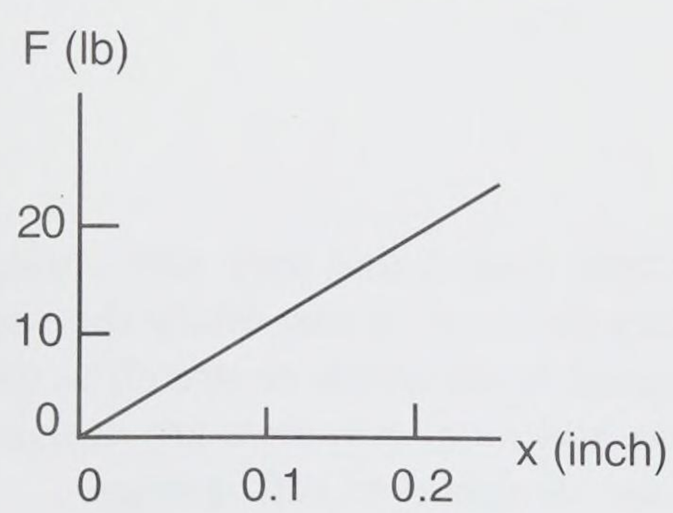


$$k = F/x \qquad (13.1)$$

If x is large, then k is small, and we say that the spring is soft. If x is small, then k is large, and we say that the spring is stiff. For example, suppose that a force of 10 lb causes the spring to stretch by 2 inches. Then k = 10/2 = 5 lb/in. If the spring stretches by only 0.2 inches, then k = 10/0.2 = 50 lb/in. The same formula applies if we compress a spring, but x then represents the decrease in length of the spring.

Coiled metal springs usually have the property that their stiffness is the same regardless of how far they stretch or compress. A typical force vs compression curve for a coil spring is shown in Figure 13.2. This spring compresses by 0.1 inch when the force is 10 lb, so k = 100 lb/in. It compresses by 0.2 inches when the force is 20 lb, so k = 20/0.2 = 100 lb/in as before. The graph of F vs x is a straight line passing through the origin, and k has the same value at any point on this line. This is typical of the behaviour of a metal spring, but tennis balls and tennis strings are not as obliging. Balls get stiffer as the force increases, provided the force increases slowly with time. If the force on a ball increases rapidly (as it does when the ball bounces), then the ball starts out being very stiff and becomes softer when the wall starts to bend.

Figure 13.2



Tennis strings also get stiffer as the force increases. There is no increase in stiffness for a rapid stretch, but there is decrease in stiffness for a slow stretch. A nylon tennis string will stretch by about one inch when a pulling force of 60 lb is first applied, but it keeps stretching further over time, even when the force is held constant. A string that stretches by say 20 mm initially will stretch about 2 mm further over the next few minutes, and then more slowly after that. As a result, the stiffness decreases by about 10% in the first few minutes and by another 10% over the next year or so.

Springs in Series or Parallel

If two identical springs are joined end to end (in series), then the stiffness of each spring is unaffected, but the combination is only half as stiff as each separate spring. The reason is that a pulling force F at each end will stretch each spring by an amount x, so the combination stretches by 2x. For example, if a 12 inch long tennis string stretches by one inch, then a 24 inch length of the same string will stretch by 2 inches when subjected to the same force, so it will be half as stiff as the 12 inch length of string. Even though it is the same string, stiffness depends on the length of the string.

The stiffness of a length of string also depends on its diameter. A thick piece of string will not stretch as far as a thin piece of string made from the same material. Suppose that you twist two identical lengths of the same string to make a single fat string. The strings are then in parallel. If a force F stretches each individual string by a distance x, then you will still need a force F on each individual string to stretch it by a distance x when the two strings are twisted together. However, the total force at each end of the twisted pair is then 2F, so the combination is twice as stiff as each individual string.

A complication with tennis strings is that the stiffness of a string increases as it stretches further. If a pulling force of 30 lb stretches a string by 1/2 inch, then a force of 60 lb will stretch it not by 1 inch, but by about 3/4 inch. When tensioned to 60 lb, a thin string will always stretch further than a fat string made from the same material. But the extra stretch when hitting a ball is about the same for both fat and thin strings since thin strings start to get stiffer at a smaller tension than fat strings. Thin and thick strings made from the same material start to get stiffer at the same stress = tension/area value. Chapter 35, String Gauges, explains this in more detail.

Potential Energy

An important property of a spring is that it can store energy. Energy stored in this manner is not kinetic energy due to motion but is classified as potential energy. A good example of this is the potential energy stored in the string and the arm of a bow. When an archer shoots an arrow, the string and arrow are drawn back by pulling on them. The work done by the archer is then stored as elastic potential energy in the bow. When the string is released, all (or most) of that energy is used to shoot the arrow. The stored elastic potential energy is converted to kinetic energy of the arrow.

The archer could simply throw the arrow by hand, but the arrow goes much further when it is released from the bow since more energy can be stored in the bow than can be given to an arrow just by throwing it. That's because the force applied by a person when throwing a light arrow is small compared to the force needed to stretch the string in a bow. As a result, more work is done by the archer in stretching the bow and more energy is thereby stored in the bow. For a similar reason, one can serve a tennis ball with a racquet faster than one can throw the ball, although in that case the serve speed is only about twice the thrown speed. The racquet is swung at the ball using an action similar to throwing the racquet, so the situation is not the same as the bow and arrow case. However, the force applied when swinging a racquet is larger than the force applied when throwing a light ball (since the racquet is heavier) with the result that more energy is given to the racquet.

Potential Energy Stored in a Mass

Potential energy is stored in any object when it is lifted vertically since work is required to lift it. When you let go, the potential energy is converted to kinetic energy as the object falls to the ground. The work W done by a force F in moving an object through a distance d (in the direction of F) is given by

Work = force x distance or W = Fd. (13.2)

When F is measured in lb and d in ft, then the units of work or energy are foot-pounds. When F is measured in Newton and d in metre, then the units of work or energy are Joules. One foot-pound = 1.356 Joule.

The minimum force needed to lift a mass m is mg, so if it is lifted off the ground through a height h, then the work done is mgh. If it is lifted by a force mg, it does not accelerate and there is no increase in kinetic energy. The work done is equal to the increase in potential energy. Hence, the increase in potential energy is mgh. A heavy object stores more potential energy than a light object, assuming both are lifted to the same height, since more work is required to lift the heavy object.

If the object is then dropped and falls to the ground, it gains an amount of kinetic energy $mv^2/2$ equal to mgh. The object therefore hits the ground at a speed given by $v^2 = 2gh$. This speed does not depend on the mass of the object. In theory, heavy and light objects fall to the ground at exactly the same speed. In practice, very light objects (e.g., a feather or a bit of paper) fall slower since air resistance has a relatively large effect on light objects. The force on an object due to air resistance depends on the area and the speed of the object, but not its mass. The force of gravity on an object depends on the mass of the object, but not its area or its speed (see Note 1).

An object thrown or hit vertically upwards at speed v has kinetic energy $mv^2/2$. It will rise to a height h before stopping, at which point all the kinetic energy is converted to potential energy. Since $mv^2/2 = mgh$, the object rises to a height $h = v^2/2g$. The height does not depend on the mass of the object. An object thrown twice as fast does not travel twice as high. It travels four times higher.

Potential Energy Stored in a Spring

The work required to stretch or compress a spring is given by (force) times (distance), but in this case the force required to stretch or compress the spring keeps increasing the more it is stretched or compressed. For a spring, the force F is given by F = kx. Since the distance compressed or stretched is x, one might expect that the energy stored in a spring is $Fx = kx^2$, but this is the wrong answer. The problem is that in order to compress a spring through a distance x, the force needs to start at zero when x = 0 and then it needs to keep increasing until F = kx when the compression is equal to x. The average force needed to compress a spring by an amount x is kx/2 and the correct answer for the energy stored in a spring is

Energy stored = Average force x distance = $kx^2/2$. **(13.3)**

This is the correct answer when the stiffness remains constant. If k varies as the spring is stretched or compressed, then the answer is roughly correct if k is taken as the average stiffness (see Note 2).

The Tennis Ball As a Spring

An approved tennis ball must squash by about 1/4 of an inch (6.5 mm) when subject to a load of 18 lb (8.2 kg or 80.4 N). It therefore has a stiffness k = F/x = 18/0.25 = 72 lb/in. or k = 80.4/0.0065 = 12,400 N/m = 12.4 kN/m. At least,

this is the stiffness for a slow compresion. As described in Chapter 37, k is actually larger than this if the ball is compressed rapidly.

The stiffness of a ball affects the way it interacts with the court and with the strings of a racquet. Consider what happens when a 57 gm tennis ball is dropped vertically from a height of 100 inches (2.54 m) onto a hard surface. It hits the surface at speed $v = \sqrt{(2gh)} = 7.06$ m/s and with kinetic energy 1.42 Joules. When the ball squashes, we can assume that all the kinetic energy is converted to potential energy so the ball squashes by an amount x given by

$$kx^2/2 = 1.42 \quad \text{so} \quad x = 15 \text{ mm} \quad \text{and} \quad F = kx = 188 \text{ N}$$

assuming that k = 12,400 N/m. The force of gravity on the ball is mg = 0.56 N, which is tiny compared with the force between the ball and the surface compressing the ball. The average force on the ball during its compression is 94 N, and the average acceleration upwards (or deceleration downwards) is given by $a = F/m = 94/0.057 = 1650$ m/s^2. The ball comes to a halt very rapidly. Since v = u - at, where v = final speed = 0 and u = initial speed = 7.06 m/s, the time taken to come to a stop is t = u/a = 7.06/1650 = 0.004 seconds. The stored potential energy is then converted back to kinetic energy as the ball expands, and it takes another 0.004 seconds to accelerate back to 7.06 m/s if none of the stored elastic energy is lost. This calculation indicates that the ball should spend about 0.008 seconds in contact with the surface. In fact, the time is typically about 0.005 seconds since (a) the ball does lose some of its stored elastic energy, and (b) the ball is stiffer during a rapid compression than during a slow compression. The point of the above calculation was not to get an accurate value of the collision time (see Note 3), but to explain how and why it is determined by the mass and stiffness of the ball.

A surprising feature of the ball bounce is the fact that the force on the ball is huge when it hits the ground. One could ask, "Where did that force come from?" It is the same sort of force that arises when you hit a hammer with a nail. It is very difficult, if not impossible, to push a nail into a block of wood with a hammer, but it is easy to hit the nail into the wood since the force on the nail is magnified when you hit it with a hammer. The force you exert on the handle to swing it is typically 500 times smaller, so where does this extra force come from? The simple answer is that it comes from the stiffness of the ground or the stiffness of the nail. When you swing a hammer or drop a ball, it acquires a certain amount of kinetic energy given by the force on the object times the distance moved. The hammer or the ball is subject to a relatively small force acting over a large distance before the impact. During the impact, the hammer or the ball is brought to rest by a large force acting over a small distance. The work required to stop the hammer or the ball is the same as the work needed to accelerate it from rest, so force times distance before the impact is the same as force x distance during the impact. The nail or the ground cannot compress very far since both are very stiff, but the hammer or the ball has enough energy to compress them a small amount which acts to generate very large forces during the impact.

Strings

When a string is installed in a racquet, it stretches a certain distance and is then tied to the frame so that it remains at that stretched length for the rest of its life. The force used to stretch the string is typically about 60 lb (27.2 kg or 267

N). Force and tension mean essentially the same thing here. We say that the string tension is 60 lb when the force applied at each end is 60 lb. The tension stays in the string when it is tied to the frame and after the stringing machine is disconnected and when you take the racquet onto the court. The string behaves as a permanently stretched spring. If you cut or break a string in your racquet, the string rapidly snaps back to its original length (almost).

While the string remains stretched, it stores a certain amount of potential energy. However, none of that energy is used by a player when hitting a ball. The work required to stretch all the strings when installing them in a racquet is about 100 Joules. This is the potential energy stored in the strings. The kinetic energy of a ball hit at 30 m/s is 25 Joules. In theory, there is enough energy in the strings to launch four balls each at 30 m/s. If the original energy stored in the strings could somehow be released, then the tension in the strings would drop every time a ball was hit, and the tension would drop to zero after about four hits. However, it is only when a string breaks that the tension drops to zero and the stored potential energy is then suddenly released.

The strings are tensioned in order to give them a certain amount of stiffness in a direction perpendicular to the string plane. A ball impacting on the strings pushes the strings and stretches them a bit further than when they were first installed. This adds to the stored elastic energy, and this extra energy is then immediately given back to the ball as it bounces off the strings. This can be repeated thousands of times, at least until the strings break. Players sometimes complain that strings lose their resiliance when they get old and they can't get any power out of the strings. This is not consistent with actual measurements. The string tension drops over time, but there is no loss in ball speed. If anything, the ball speed should increase very slightly when the string tension drops. What players may notice is that the ball stays on the strings a bit longer at low tension, which might give the impression that the ball comes off the strings more slowly.

The stiffness of the whole string plane, in a direction perpendicular to the string plane, is about the same as the stiffness of a tennis ball. As a result, a certain amount of elastic energy is stored in the ball when it squashes against the strings, and about the same amount of energy is stored in the strings. The strings always return about 95% of their stored energy, but the ball returns only about 55% of its stored energy.

How the Elastic Energy Is Shared

The fraction of the total elastic energy stored in the ball and the strings during a collision depends on their relative stiffness. The ball and the strings each behave like two springs pushing against each other, as shown in Figure 13.3. Despite what your intuition might tell you, the force of the ball on the strings is exactly equal to the force of the strings on the ball, regardless of how far the ball squashes and regardless of how far the strings stretch (see Note 4).

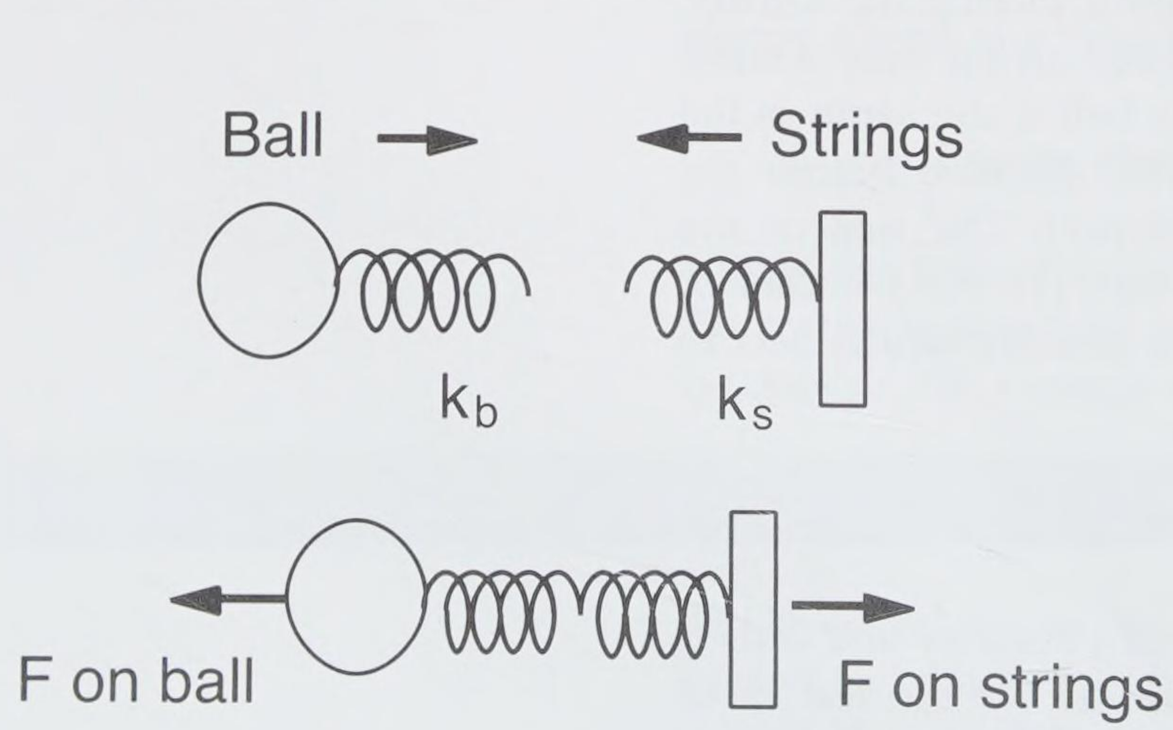


Figure 13.3

Suppose that the ball stiffness is k_b, the string plane stiffness is k_s, the ball compresses by an amount x_b and the string plane is deflected by a distance x_s. Then the force F on the ball is $F = k_b x_b$ and the force on the strings is $F = k_s x_s$. Since the forces are equal, $k_b x_b = k_s x_s$. This tells us that if the ball is stiffer than the strings, then it will compress by a relatively small amount and the strings will deflect by a larger amount. For example, if the ball is twice as stiff as the strings and if the string plane deflects by 20 mm, then the ball will compress by 10 mm. The energy stored in the ball is $k_b x_b^2/2$, and the energy stored in the strings is $k_s x_s^2/2$. Since the strings stretch twice as far, $x_s^2 = 4\ x_b^2$, but $k_s = k_b/2$, so the strings will store twice as much energy as the ball. This seems like it might make a big difference to the speed of the ball off the strings, but it doesn't. The reason is given below.

Effect of String Tension on Ball Speed

Suppose that a 57 gm tennis ball is incident vertically downwards on a hard, heavy surface (such as the court) at a speed of 10 m/s. The kinetic energy of the ball is then $mv^2/2 = 2.85$ Joules. It loses about 45% of its energy during the bounce (0.45 x 2.85 = 1.28 J) and the kinetic energy after it bounces is therefore 2.85 - 1.28 = 1.57 J. The bounce speed is given by $mv^2/2 = 1.57$ J so v = 7.42 m/s. The coefficient of restitution in this case (COR = bounce speed/incident speed) is 7.42/10 = 0.742, as required by the rules of tennis. If the ball bounced slower (and hence lower) or faster (and hence higher) than this, players would complain that there was something wrong with the ball.

Now suppose the ball is incident vertically downwards at 10 m/s on the strings of a racquet and that the head of the racquet is clamped so that it can't move. If the strings and the ball are equally stiff, then the kinetic energy of the incident ball (2.85 J) is shared equally between the strings and the ball, so they each store 1.425 J at the instant the ball comes to rest. At that instant, all of the initial kinetic energy of the ball is converted to potential energy in the ball and the strings. The strings lose 5% of their stored energy (0.05 x 1.425 = 0.071 J), the ball loses 45% of its stored energy (0.45 x 1.425 = 0.641 J) so the total energy loss is 0.071 + 0.641 = 0.712 J. The ball will bounce with kinetic energy (2.85 - 0.712) = 2.138 J and at speed 8.66 m/s. The coefficient of restitution in this case is 8.66/10 = 0.866. The ball bounces 16.7% faster and 36% higher off the strings than off a hard surface (Table 13.1).

Table 13.1

Conditions	Velocity In(m/s)	Energy In(J)	E to Strings (J)	E to Ball (J)	E Loss Strings (J)	E Loss Ball (J)	E Loss Total (J)	KE Out (J)	V Out (m/s)	COR
Concrete	10	2.85	0	2.85	0	1.28	1.28	1.57	7.42	0.742
$k_s = k_b$	10	2.85	1.425	1.425	0.0715	0.641	0.712	2.138	8.66	0.866
$2k_s = k_b$	10	2.85	1.9	0.95	0.095	0.428	0.523	2.328	9.04	0.904

If the strings are twice as soft as the ball, then the strings will store twice as much energy as the ball. The ball stores 2.85/3 = 0.95 J and the strings store 2 x 0.95 =1.90J. The total energy stored is then 0.95 + 1.90 = 2.85 J. The strings lose 5% of 1.90 J = 0.095 J, the ball loses 45% of 0.95 J = 0.428 J and the total

energy loss is 0.523 J. The ball will bounce with kinetic energy 2.328 J and at speed 9.04 m/s. In this case, the coefficient of restitution is 9.04/10 = 0.904. This is slightly higher than the previous case, but it is not a lot higher since the ball already bounces quite well off the strings, even when they are as stiff as the ball. This is an important point and it is often overlooked.

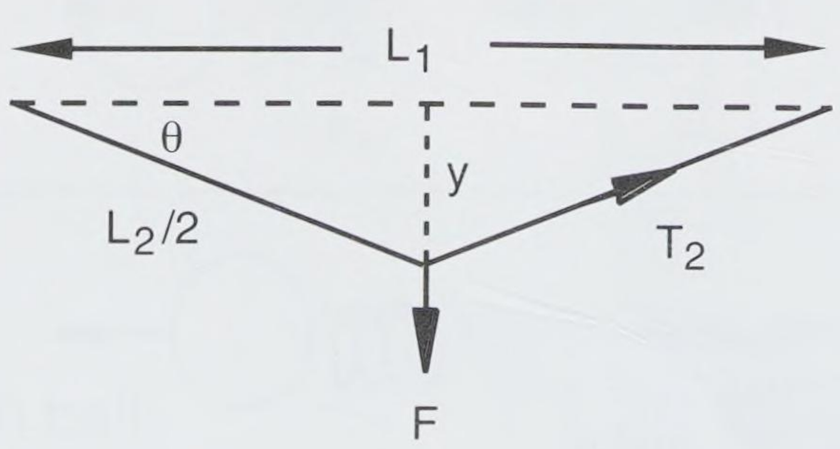
The point is that there is not much one can do to increase the bounce speed off the strings, just by decreasing the string tension or using a different type of string. To emphasise the point, suppose that the string plane is one million times softer than concrete. Decreasing the softness of a concrete slab by a factor of one million would increase the ball speed by only 16.7%, from 7.42 to 8.66 m/s. If the softness was decreased by an additional factor of one million so that there was no energy loss in the ball at all, then the ball speed would increase by another 12.6%, from 8.66 to 9.75 m/s (some energy being lost in the strings).

A factor of two decrease in string plane stiffness results in only a 4.4% increase in the ball speed off the strings (from 8.66 m/s to 9.04 m/s), and this is when the racquet head is clamped. The string plane stiffness is roughly proportional to string tension, but a factor of two change in string tension is unrealistic. Players might be prepared to change the string tension from 60 lb to 50 lb, but no one strings a tennis racquet at 30 lb. If the string tension is decreased from 60 to 50 lb, the increase in ball speed would be only about 1.5%. Furthermore, if the racquet head is not clamped, then the increase in ball speed off the strings is even smaller, as explained in detail in Chapter 12 on collisions. The basic reasons are that (a) only some of the energy of the incident ball (or the incident racquet) goes into stretching the strings or compressing the ball while the rest is used in changing the kinetic energy of the racquet, and (b) the speed of the ball off the strings depends primarily on the speed of the racquet rather than on the amount of energy lost in the ball and the strings.

Sideways Stiffness of a Single Stretched String

The stiffness of a string, in a direction perpendicular to the string, is usually a lot smaller than the stiffness for a lengthwise stretch. The transverse stiffness can be calculated with the aid of Figure 13.4. Suppose that the string has a length L_1 and is at tension T_1. A transverse force F applied in the middle of the string pushes the string sideways by a distance y. As a result, the string stretches slightly, by an amount that can be calculated from the geometry.

Figure 13.4



If the string increases in length from L_1 to L_2, then the right-angle triangle on each side has sides $L_1/2$, y and hypotenuse $L_2/2$. Pythagoras' theorem tells us that

$$(L_2 / 2)^2 = (L_1 / 2)^2 + y^2$$

If we multiply both sides by 4 then

$$L_2^2 = L_1^2 + 4y^2 = L_1^2 (1 + 4y^2 / L_1^2)$$

Now take the square root of both sides. Provided that y is much smaller than L_1, the answer is given to a good

approximation by

$$L_2 = L_1 (1 + 2y^2/ L_1^2) = L_1 + 2y^2/ L_1$$

where we have used the result that the square root of (1 + N) is approximately (1 + 0.5N) when N is much smaller than 1. For example, the square root of 1.04 is actually 1.0198 which is very close to 1.02.

The increase in string length is $x = L_2 - L_1 = 2y^2/ L_1$ and the increase in string tension is kx where k is the stiffness for a longitudinal stretch.

The tension increases from T_1 to T_2 where $T_2 = T_1 + kx$, so

$$T_2 = T_1 + 2ky^2/ L_1$$

The force F acting to push the string sideways is opposed by the effect of the string tension pulling back in the opposite direction. The tension does not act in the same direction as F, but it has a force component F_y in the y direction given by

$$\sin(\theta) = F_y / T_2 = y / (L_2 / 2) \qquad \text{so} \qquad F_y = 2yT_2 / L_2.$$

Each half of the string exerts a force F_y opposing F, so the string moves sideways until $F = 2F_y$, and then

$$F = 4yT_2 / L_2 = k_p\, y$$

where $k_p = 4T_2/L_2$ is the perpendicular stiffness of the string. Given that L_2 is essentially the same as L_1 when y is small, k_p is given to a good approximation by

$$k_p = (4 / L_1)(T_1 + 2ky^2/L_1) = 4T_2 / L_1$$

This equation shows that the string has a perpendicular stiffness even if y = 0, given by $k_p = 4T_1/L_1$. When y is very small, the stiffness depends only on the string tension T_1 and the length L_1 of the string. The transverse stiffness of a steel string would then be the same as that of a nylon string if both strings have the same length and are at the same tension. However, as y increases, k_p increases since the string tension increases. If y is larger than a few mm, the transverse stiffness of a steel string would become larger than that of a nylon string.

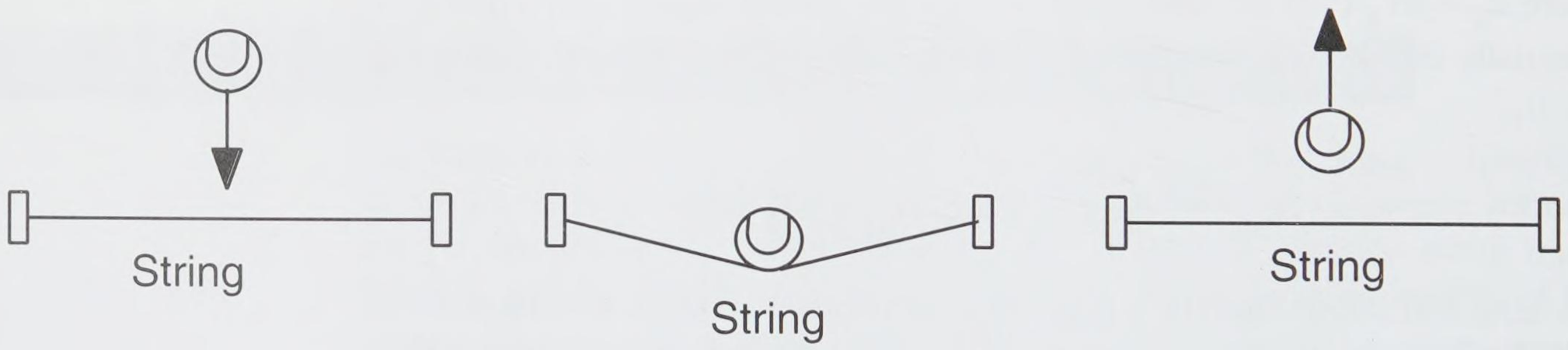
Consider a nylon string of length 30 cm = 0.3 m tensioned to 28 kg (274 N). Such a string is typical of a main string in a tennis racquet. A 30 cm nylon string has a longitudinal stiffness k of about 40,000 N/m. When y = 0, $k_p = 4 \times 274/0.3$ = 3,653 N/m. When y = 1 mm, the tension increases by only 0.27N so k_p is essentially the same. If y = 10 mm then the tension increases by 27 N and k_p increases slightly, to 4,013 N/m. If y = 30 mm then the tension increases by 243 N (24.8 kg) and k_p increases to 6,893 N/m, about double the y = 0 value.

Now consider a 30 cm kevlar string with k = 120,000 N/m, also tensioned to 28 kg. For small values of y, k_p is the same as that for a nylon string. If y = 10 mm, then the tension increases by 81 N and k_p = 4,733 N/m. If y = 30 mm, then the tension increases by 729 N and k_p increases to 13,373 N/m. A high speed ball impacting on a nylon string in a fully strung racquet will push the strings sideways by about 30 mm. However, it will not push a kevlar string that far since kevlar is a stiffer string. Just what happens can be calculated as described in the following section.

Impact of a Ball on a Stretched String

Suppose that a ball strikes a single string at right angles and in the middle of the string, as shown in Figure 13.5. If there is no energy loss in the ball or in the string, then the ball will bounce off the string at the same speed that it hit the string. This is not a good assumption for a tennis ball, but it is a good approximation for a metal ball. Impacting with a metal ball is a good way to test the properties of the string, since the change in ball speed off the strings provides a measure of the energy loss in the string. Experiments show that a metal ball or hammer bounces off a string at about 95% of the incident speed, regardless of the type of string and regardless of the string tension and regardless of whether the string is new or old. The only exception is the very first impact on a brand new string, in which case the ball bounces off the string at about 92% of the incident speed. In that case, a few weak bonds in the string get broken due to the large increase in tension during the impact. Once broken, they remain broken, and from then on the energy loss in the string remains small.

Figure 13.5



One might argue that the impact of a ball on a single string is not the same as impacting a ball on the whole string plane in a racquet. However, the effect is exactly the same. A 0.8 kg (1.8 lb) steel boule (as used in the French or Italian version of bowls) dropped from a height of 100 inches onto the strings of a head-clamped racquet bounces to a height of 90 inches and therefore rebounds off the strings at 95% of its incident speed. The bounce height is the same regardless of whether the strings are old or new and regardless of whether the strings are at high or low tension. Friction between the strings results in no significant or additional energy loss.

An interesting question is whether string tension or the type of string makes a significant difference to the behaviour of a string when a metal ball bounces off the string. At high tension, the string will be relatively stiff and the ball should bounce off the string sooner (rather than faster, as in higher speed). At low tension the string will stretch further. If it stretches further, the increase in tension during the collision will be larger. The question of interest is whether the large increase in tension in a string at low tension might be equivalent or almost equivalent in effect to an impact on a string that is initially at high tension.

A ball impacting in the middle of a string experiences a backwards force $F = -k_p y$ as described above. The equation of motion is therefore

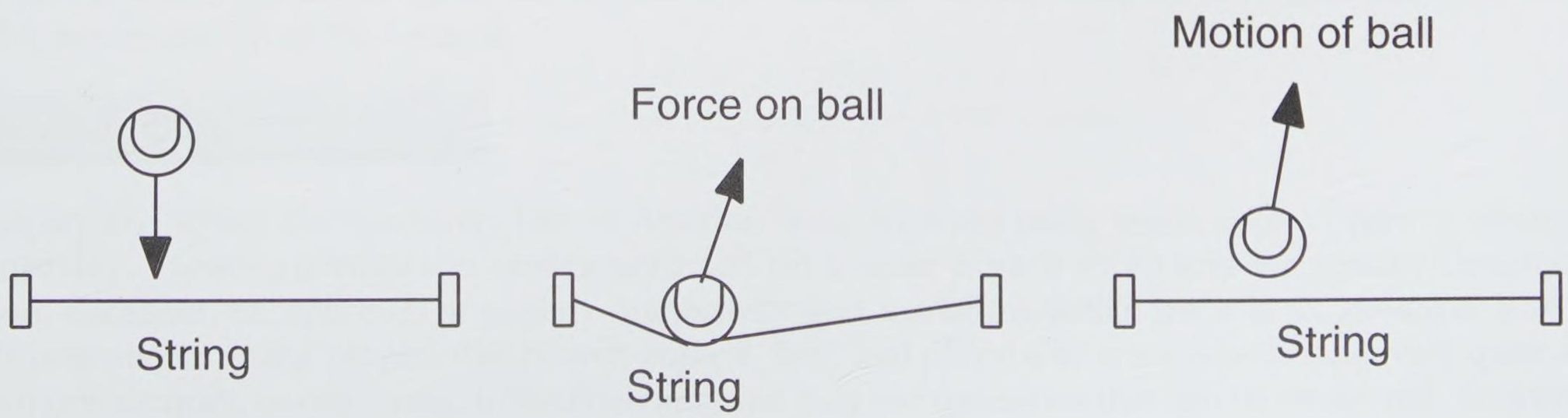
$$m\, d^2y / dt^2 = -(4y / L_1)(T_1 + 2ky^2 / L_1) \qquad (13.4)$$

where m is the mass of the ball. This equation can be solved analytically only when y is very small. This is not a very interesting case since all strings behave the same when y is very small. The more general case, when y is large, can be solved numerically. Typical results are described in Chapter 30.

Another interesting question is whether an impact towards one end of the string differs from an impact in the middle of the string. Right at the very end of a string, where it joins the frame, the string is infinitely stiff since the string will not deflect at all if the frame is rigidly supported. Consequently, the stiffness varies from a minimum at the middle of the string to a maximum at the end of the string.

There is another interesting effect that can occur when a ball impacts away from the center of the string. The tension in the string on each side of the ball exerts a net force on the ball towards the middle of the string. As a result, the ball will bounce back towards the center of the string, as shown in Figure 13.6. It is difficult to do this experiment with only a single string, but it is easy to see the effect by dropping a heavy steel ball on the strings of a head-clamped racquet, or by dropping a steel ball bearing on the membrane of a drum. If one drops a small steel ball or a golf ball on the strings of a racquet, the ball bounces at strange angles since the strings are lumpy and uneven. The effects of the lumps are evened out on a larger ball and when the strings deflect a large distance. This effect means that when a player mis-hits a ball near the edge of the frame, the ball will still bounce off the strings towards the intended direction.

Figure 13.6



Notes

Note 1. Since the force of gravity on an object increases with its mass, many people assume that a heavy object will fall faster since the force on the object is bigger. But force = mass x acceleration, so a heavy object needs a bigger force on it to accelerate at the same rate as a light object. The acceleration due to gravity, g, is the same for all objects, regardless of its mass.

Note 2. The amount of energy stored in the spring can be determined exactly from the area under the F vs x curve since the work is given by the integral of F.dx.

Note 3. When a ball bounces and compresses by an amount x, the force on the ball is $F = kx = -ma$ where a is the acceleration of its center of mass. If we assume that $a = d^2x/dt^2$, then the ball undergoes simple harmonic motion and is in contact with the ground for one half cycle. An accurate calculation of the impact time is not possible since (a) the center of mass moves a smaller distance than x by a fraction that changes with time, and (b) k does not remain constant during the bounce.

Note 4. For every action there is an equal and opposite reaction, but this idea is sometimes hard to accept. For example, if you are standing at the net and your partner serves a ball into the back of your head, the ball pushes on your head with the same force as your head pushes on the ball. Your head didn't deliberately push on the ball, but it is obvious from the fact that the ball bounces backwards that something pushed the ball backwards. That something was your head. Similarly, if you drop a ball, it falls to the ground because the earth exerts a force downwards on the ball. The ball exerts an equal and opposite force on the earth, pulling it upwards, but the earth is a lot heavier than the ball and its motion is undetectable.

Mark Philippoussis, Australia

14

Racquet Power

The Factors Influencing Power

By Rod Cross

Summary

Racquet power is a quantity that is hard to measure since it varies with the impact point on the strings and with the type of shot. Heavy racquets are more powerful than light racquets if they are swung at the same speed. If light racquets are swung faster than heavier racquets, then the power is about the same. Most of the power in a shot comes from the player's arm, with the result that all racquets have about the same maximum power, regardless of racquet mass, string tension, or string type. Racquet stiffness has no effect on power when the ball is hit in the middle of the strings, but stiff racquets are more powerful when the ball is hit nearer the tip of the racquet.

Introduction

Racquet and string manufacturers like to describe their products using terms such as power, control, feel, and playability. These quantities are commonly rated on a scale from 0 to 10 and are usually described as being good, excellent, exceptional, or superb. Subjectivity and marketing aside, there is an assumption by the manufacturers and by many players that power, control, feel, and playability are quantities that vary quite significantly from one racquet, or one string, to another, and that they are quantities that can be measured. However, all these

terms and adjectives are meaningless from a purely scientific point of view since none of them have ever been measured. Racquet power could, in principle, be measured, but power is a quantity that is hard to pin down in terms of a clear and simple definition. In this respect, power is a bit like happiness. Everyone knows what it is, but no one knows how to measure it.

Over the last few years, I have measured the properties of about 300 different models of strings. Every string differs from the next, sometimes by a large factor in terms of the amount they stretch, but there is one factor that is essentially the same for every string. Every string stores elastic energy very efficiently when it is stretched rapidly, and it returns between 95% and 97% of that energy when it returns to its original length. As a result, the difference in power between any two strings is almost undetectable. Any difference in power between different strings must therefore be related to the effect of the strings on energy loss in the ball. At low string tension the impact is softer and the ball is less deformed, so there will be a smaller energy loss in the ball. Different strings stretch different amounts on impact with a ball, with the result that the increase in tension during the impact varies from one string to the next. For example, the increase in tension is small with a natural gut string, larger with a nylon string, and largest with a kevlar string. This affects the softness of the impact and it affects the energy loss in the ball (as well as energy lost in frame vibrations), but the energy loss in the string itself is negligible, regardless of the type of string.

As to racquets, research is equally clear that, (a) most of the power comes from the player's arm rather than the racquet, and (b) the difference in power between any two racquets is likely to be less than 5%. My own tests, and tests by others (see Tennis Magazine, August 1997), have shown that players can serve a ball almost as fast with an old wood racquet as with a modern graphite racquet. The advantage of modern racquets is not that they are a lot more powerful, but they allow players to use or control the power more effectively. When combined with the increased strength and fitness of modern players, the result is that the modern game of tennis is played at a much faster pace than in the old wood racquet era.

Factors Affecting Power

1. Racquet power refers to the speed of the ball off the strings. But the speed of the ball depends on many factors, not just the power of the racquet. For example, the speed of the ball depends on how hard a player chooses to hit the ball and on how hard the player is able to hit the ball. A sensible definition of racquet power therefore requires some stardard way of hitting the ball, keeping either the racquet speed or the racquet energy or some other parameter constant. Just what this parameter should be has not yet been determined.

2. If one were to measure racquet power by swinging all racquets at the same speed, then heavy racquets would all show up as being more powerful than light racquets. And the same could be said if balls were fired at a stationary racquet. In each of these specific cases, more mass means more power, but there are other circumstances (such as an unwieldy 5 lb racquet) where more mass could mean less power.

3. For a serve, the speed of the ball is directly proportional to the speed of the racquet. Power therefore depends on both racquet mass and racquet speed.

For a groundstroke, the speed of the ball off the strings also depends on the speed of the incoming ball. This is another parameter that would have to be kept constant in order to compare the power of different racquets.

4. Under normal playing conditions, the speed of a swung racquet depends primarily on how fast the player chooses to swing it. If the player chooses to swing it as fast as possible (at the risk of miss-hitting the ball) then light racquets can be swung faster than heavy racquets. Just how much faster is something that has never been measured precisely, and is something that will depend on the type of stroke and the individual player. For example, players who use a long backswing on groundstrokes can accelerate the racquet to a higher speed than players who use a short backswing. Players with a long backswing are likely to get more power with a heavy racquet and players with a short backswing are likely to get more power with a light racquet. However, this is also something that has never been measured. Some players seem to be able to generate very high racquet speeds using a short backswing and a rapid punching action when they strike the ball.

5. Despite the fact that light racquets can be swung faster than heavy racquets, players don't necessarily do so. Swinging a racquet as fast as possible is the best way to miss-time the shot or miss-hit the ball. A player swinging a racquet at a controlled speed will get more power from a heavy racquet if the racquet speed is kept constant. In general, players tend to swing a racquet at a controlled speed for groundstrokes, volleys, and second serves, but they swing faster for a first serve or an overhead smash.

6. If a ball is dropped on a stationary, hand-held racquet, the ball bounces best at the throat and worst at the tip. If one swings a racquet, the racquet travels fastest at the tip and slowest at the throat. These two factors (racquet speed and bounce efficiency) must be factored together to determine the ball speed off the strings, and the answer will vary depending on the relative speed of the ball and the racquet at the impact point. For example, the point of maximum power is near the throat of a racquet when the ball is traveling a lot faster than the racquet prior to the impact, since that is where the ball bounces best. At the opposite extreme where the racquet is travelling a lot faster than the ball, the maximum power point is closer to the tip of the racquet, since that is where the racquet is traveling fastest.

7. The point of maximum power depends on how the racquet head speed varies from the tip to the throat, and that depends on the type of shot and the amount of wrist action. In a wristy shot, the tip might travel twice as fast as the throat. In a volley, the tip and throat might travel at the same speed.

Racquet mass, speed, and bounce are the key elements in power. Despite the complications outlined above, some definite statements can be made about racquet power by combining existing experimental data with the laws of physics. The essential physics is that, (a) when a player swings a racquet, a certain amount of energy is given to the racquet, and (b) the amount of energy transferred from the racquet to the ball depends mainly on the energy loss in the ball and the energy lost to vibrations of the racquet frame. A surprising result is that there is very little one can do to change a racquet so that it works more efficiently in transferring energy out of the arm and into the ball. If the ball is hit in the middle of the strings, then there is no energy loss due to vibrations. One could get more power by eliminating most of the energy loss in the ball, but that is not allowed by the rules of tennis.

Physics of Power

The essential physics of racquet power is described in Chapters 10 to 13. We can summarise the main results as follows:

Suppose that a ball of mass m is served from a point on the strings where the effective mass of the racquet is M_e. The effective mass is about half the actual racquet mass in the middle of the strings, and it drops to about 1/3 of the actual racquet mass at a point about 5 cm from the tip. The ball can't be served any closer to the tip since it would impact on the frame of the racquet. If the impact point is headed towards the ball at speed V and if the ball is served without any significant spin, then the ball will be served at a speed v given by

$$v = (1 + e_A)V \qquad (14.1)$$

where e_A is a number less than 1.0 called the apparent coefficient of restitution (ACOR). It is a measure of how well the ball bounces off the strings, and it depends on the effective mass of the racquet at the point where it bounces. The bigger the effective mass, the better it bounces. The formula for serve speed shows that serve speed is equal to the racquet speed V plus a small amount that depends on how well the ball bounces off the strings.

The ACOR decreases from the throat area to to the tip area and is typically about 0.5 in the throat, 0.4 in the middle of the strings and about 0.1 near the tip. The ACOR is measured by firing a ball at right angles to the strings of a stationary racquet, and the values obtained using this method agree closely with the expected result that

$$e_A = \frac{(e - m / M_e)}{(1 + m / M_e)} \qquad (14.2)$$

where e is the coefficient of restitution (COR). The COR is typically about 0.88 in the middle of the strings, but it decreases to about 0.70 near the tip and throat of the racquet due to the energy loss caused by vibrations of the frame.

When a racquet is swung to serve a ball, the speed at the tip is greater than the speed at the middle of the strings. The decrease in e_A near the tip is largely compensated by the increase in V, with the result that the serve speed does not depend strongly on the impact point. Nevertheless. the serve speed does vary with the impact point. The maximum ball speed results for an impact somewhere between the middle of the strings and the tip, depending on whether the racquet is head heavy or head light and whether any lead tape has been added near the tip of the frame.

Suppose that the actual racquet mass M = 0.32 kg, the effective mass M_e = 0.16 kg, and e = 0.88 for an impact in the middle of the strings. Then e_A = 0.39. For such a racquet, the serve speed is given by $v = (1 + e_A)V = 1.39V$. In order to serve a ball at 200 kph, the racquet speed must be V = 200/1.39 = 144 kph. If you use a heavier racquet with, say, M = 0.36 kg and M_e = 0.18 kg, then e_A = 0.43 and v = 1.43V. In this case the racquet speed needs to be 140 kph to serve a ball at 200 kph. It is clear from this example that the serve speed depends mainly on the racquet speed and it depends only slightly on the mass of the racquet.

Similarly, the ball speed does not depend very much on the strings or the string tension. Changing the string tension has only a small effect on e or e_A , and a small change in e_A has an even smaller change on the ball speed, as described in Chapters 13 and 32.

Maximum Possible Racquet Power

In theory, maximum possible racquet power would be obtained by eliminating all energy losses in the ball and the racquet. In that case, e = 1 and then

$$e_A = \frac{(1 - m / M_e)}{(1 + m / M_e)} \qquad (14.3)$$

Given that the serve speed is $v = (1 + e_A)V$, one would like to make e_A as large as possible. The largest possible value of e_A is 1.0, which means that M_e must be a lot bigger than m. For example, if $m/M_e = 0.1$ then $e_A = (1 - 0.1)/(1 + 0.1) = 0.82$, and if $m/M_e = 0.01$, then $e_A = (1 - 0.01)/(1 + 0.01) = 0.98$. In the limit where $e_A = 1.0$, the serve speed would be 2V. However, the racquet would be too heavy to swing very fast. If we consider a lighter racquet with M = 0.32 kg, $M_e = 0.16$ kg, and if e = 1.0 for an impact in the middle of the strings, then $e_A = 0.475$. For such a racquet, the serve speed is given by $v = (1 + e_A)V = 1.475V$. This is the maximum possible serve speed for such a racquet, using an illegal ball with no energy losses. The serve speed is then 6% faster than that for a legal ball where v = 1.39V. The serve would be almost impossible to return since the ball would bounce off the court about twice as high as a legal ball.

At the other extreme, one could reduce the serve speed by using a ball with e = 0. Such a ball would act like plasticene and would not bounce at all off the court. Nevertheless, it would bounce off the strings since the strings will stretch and eject the ball. If the ball and the strings have about the same stiffness, then e will be about 0.5 for an impact on the strings, in which case $e_A = (0.5 - m/M_e)/(1 + m/M_e)$. For example, if m = 57 gm and $M_e = 160$ gm, then $e_A = 0.106$ so v = 1.106V. The serve speed would then be 20% slower than for a legal ball where v = 1.39V, but the ball would be impossible to return since it would roll along the court instead of bouncing.

Estimating Racquet Power

We have shown how racquet power depends on racquet mass and the coefficient of restitution, but it also depends on racquet speed. It should be possible to measure the ball speed for any given racquet speed and impact point, but no one knows how fast one should swing a racquet or what string to use and at what tension in order to measure power. Power depends on who swings the racquet. If the same racquet is given to 10 people and they are asked to swing the racquet as fast as possible, then 10 different answers will be obtained. Similarly, person A might swing a lighter racquet 10% faster, but person B might swing the lighter racquet 5% faster.

The only racquet speed that could be used to test racquet power without ambiguity is zero speed. One can fix the speed of the incoming ball and measure the speed of the outgoing ball. But the result will still not be a measure of rac-

quet power since power depends on the speed at which any given racquet can be swung.

One can estimate the effect of racquet speed on racquet power by making some assumptions about racquet speed vs racquet mass. For example, one could assume that a racquet that is 10% lighter can be swung either 5% or 10% faster and then calculate the increase in ball speed in both cases, for both a serve and a groundstroke. This is really only guessing what might happen, but we can improve the guess using some simple physics. For example, if the initial energy of the racquet is the same for all racquets, then a 10% decrease in M will require a 5% increase in racquet speed given that the racquet energy is proportional to M x speed squared. However, this suggests that if M is reduced to zero, then the racquet speed will increase to infinity. Reducing the racquet mass to zero will not increase the racquet speed enormously since the mass of the arm places a limit on how fast one can swing a racquet. Consequently, if the racquet mass is decreased by 10%, then the increase in racquet speed will be less than 5% and probably around 2% or 3%. An extreme example would be to decrease the racquet mass from 20 gm to 10 gm. In this case, the mass is decreased by a factor of 2, but there would be almost no change in the racquet speed.

When a player serves a ball, the player swings the racquet in a complicated, loopy path over the shoulder, behind the back and then upwards and forwards back over the shoulder. The initial action is to position the racquet for a sudden explosive acceleration just before impact. The essential features of the acceleration phase can be described by assuming that the racquet is swung through a distance d by applying a force F. The work done is Fd, and hence, the kinetic energy of the racquet just before impact is $\frac{1}{2}MV^2 = Fd$. The force is therefore given by $F = MV^2/(2d)$. If the player serves with a heavier racquet then the force that is applied to the racquet will not necessarily be the same. Try pushing a tennis ball through the air and then push a brick wall. It is obvious that the force you can exert on the wall is much larger than the force you can exert on a tennis ball. If the force applied to swing a heavy racquet was the same as the force on a light racquet then MV^2 would remain the same and hence the kinetic energy of the racquet would be the same. In that case, a player could swing a racquet of zero mass at infinite speed. A more likely result is that the total kinetic energy of the arm plus the racquet will remain roughly constant when the racquet mass is varied.

Suppose that when M is increased, F increases and V decreases in such a way that FV stays constant. In physics, force times velocity is equal to power and is measured in watts. The power supplied to a moving racquet (or any other object) is defined to be the rate at which its kinetic energy increases. In other words, power = force x distance/time = force x velocity. If FV remains constant when the racquet mass is increased, then $MV^3/(2d)$ = constant, so M x V cubed will remain constant. In that case, a 10% decrease in M would result in a 3.3% increase in V.

As defined in physics, "power" is not the same as the "power" of a racquet since racquets don't deliver any power on their own. We can talk about the power of an engine or the power of a player since engines and players act to do useful work. A racquet on its own can't jump up and serve a ball. It is just an instrument that helps the player transfer energy out of the arm and into the ball. Consequently, when we talk about the power of a racquet we are really

talking about the relative efficiency by which the racquet acts to transfer energy from the player's arm into the ball. In that respect we can say that a racquet is more efficient than say a pencil or a table tennis bat, but common useage is to say that it is more powerful.

The exact relation between racquet speed vs racquet mass is not known, and probably varies from one player to another anyway. However, we can model the effect on serve speed by assuming that the racquet speed is given by $V = k/M^n$ where M is the actual racquet mass, k is a constant and n is some number between 0 and about 0.5 that describes how V varies with M. For example, if n = 0.5 then the racquet energy will remain constant regardless of the racquet mass. If n = 0 then the racquet speed will remain constant regardless of M. If n = 0.33 then the power input from the arm to the racquet will remain constant. Calculations for various values of n are shown in Figure 14.1. It was assumed arbitrarily that V = 80 mph when M = 300 gm, regardless of the value of n, simply so we could see how different values of n affect the serve speed. It was also assumed that $M_e = M/2$ and e = 0.85 in order to calculate some typical values of e_A.

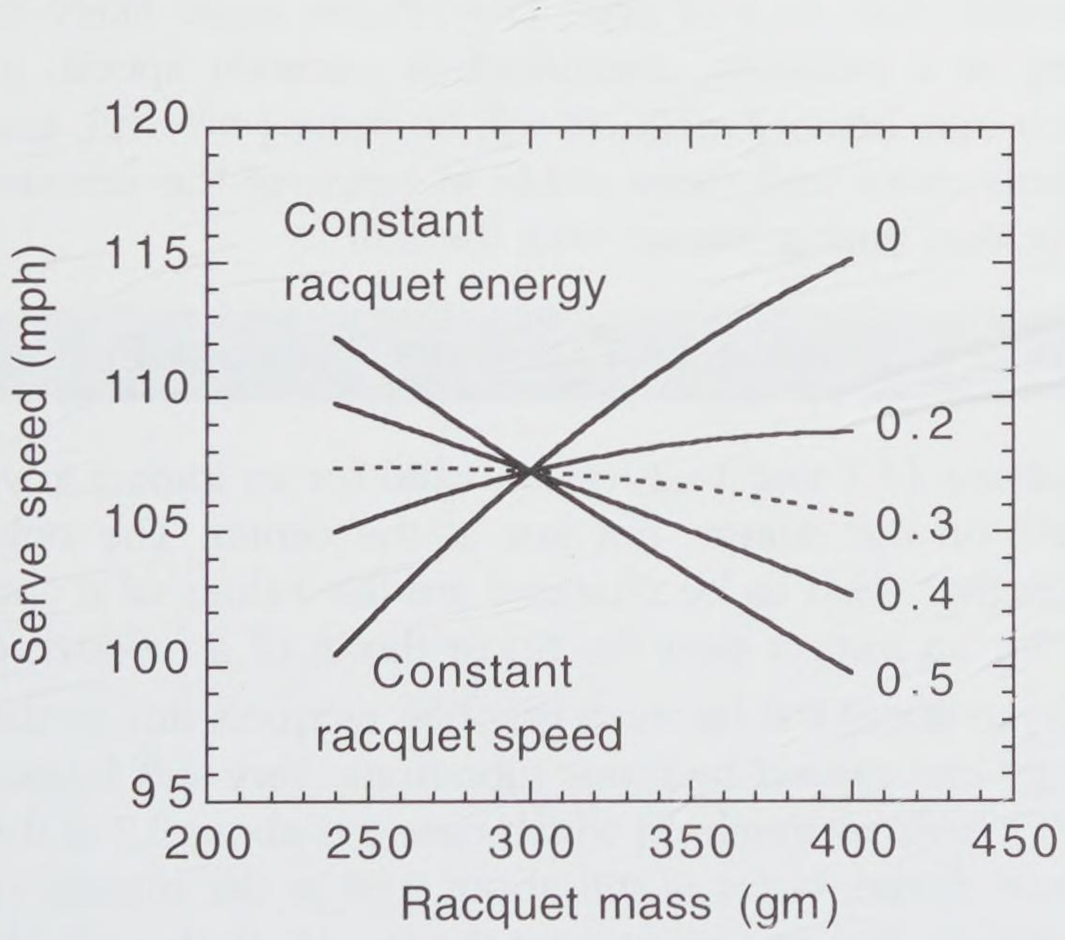


Figure 14.1

The biggest variation in serve speed results if we assume that the racquet speed remains constant when the racquet mass is increased. In that case, increasing the racquet mass from 250 gm to 400 gm results in a 15% increase in serve speed (from 100 mph to 115 mph). If we assume that the racquet energy stays constant, then there is an 11% decrease in serve speed. However, it is more likely that n will be about 0.3, in which case there is a 1.9% drop in serve speed when the racquet mass is

The Moving Racquet versus the Free Racquet at Rest

By Howard Brody

Abstract

Is there anything that can be learned about the interaction of a ball and a tennis racquet that can be determined using a moving racquet (mechanically swung) that cannot be determined from firing balls at a free racquet at rest? It has recently been proposed that a machine that swings racquets be constructed as a method for measuring the "power" of a tennis racquet. This sidebar will claim that tests involving a moving racquet can provide NO information that tests using a free racquet at rest cannot provide.

Introduction

Tennis racquets are tested by measuring the speed of rebounding tennis balls that have interacted with the strings. This can be done in a number of ways – with a racquet at rest being hit by a ball of known speed, with a racquet of known speed hitting a ball at rest, or having a moving ball strike a moving racquet. In each case the velocity of the rebounding ball and the location of the impact on the strings are recorded. If all the desired information can be obtained using a system with the racquet at rest, it is clearly the easiest method for testing the racquet, and it is the method that has been used for years by racquet manufacturers and sports scientists. Swinging the racquet gives no new information and it adds to the experimental difficulties as well as adding possible measurement errors.

Frames of Reference Argument

When a tennis ball rebounds off of a racquet at rest, the ratio of the rebound ball speed to the incident ball speed is called the apparent coefficient of restitution (ACOR). The word "apparent" is used to distinguish this quantity from the actual coefficient of restitution (COR), which takes into account the recoil speed of the racquet. It has been shown[1] that the value of ACOR obtained with a freely suspended, static racquet (the racquet is at rest in the laboratory frame of reference) can be used to find the ball rebound speed for a moving racquet hitting a moving ball (both in the tennis court frame of reference). This can be done using Newtonian transformations from one frame of reference to another and the result is the ball speed formula:

$$V_{out} = e_A \cdot V_{in} + (1 + e_A) \cdot V_r$$

where V_{out} is the ball speed coming off of the racquet, V_{in} is the ball speed approaching the racquet, and V_r is the racquet head speed at the impact point, all in the

Continued on page 166

tennis court frame of reference. This is an exact formula for the normal component of ball and racquet head velocity, and no approximations have been made in deriving it.

The value of ACOR used in the ball speed formula depends upon the impact location and the relative ball-racquet velocity, so data must be taken in the lab over a range of incident ball velocities and impact locations. The value of ACOR will depend, among other parameters, on the racquet weight and weight distribution, which will also affect the racquet head speed, V_r, when swung by a player.

Racquet Head Speed

How the racquet head speed varies with racquet weight and racquet weight distribution is at present poorly known for individual players or for players in general. It is felt that players swing heavier racquets at a slower speed, but there is no data to confirm this. Research of this type is presently under investigation at Loughborough University, and a report which measured a few players serving is available[2]. If a machine were constructed to swing a racquet, it would probably be designed to input constant energy, constant impulse, constant force, constant torque, or some function of one of these physical characteristic that tests reveal for typical players. In addition, the effective pivot point of the swing would have to be determined to determine how V_r varied along the face of the racquet. If such information were available, then it would be possible to predict the racquet head velocity from the known racquet's physical parameters and the laws of motion without resorting to the machine. Therefore, the racquet swinging machine would not be needed since a simple calculation using the parameters used in designing the machine would give the value of V_r.

The results of this calculation for V_r and the measured values of ACOR could be substituted into the ball speed formula, and the value of V_{out} would be obtained for any value of V_{in}, the incident ball velocity. A similar argument will hold for any type of bat, racquet, or paddle used in a sporting event.

1.) Brody, H., The Physics of Tennis III, American Journal of Physics, 65, 981-987, Oct 1997.
Brody, H, How Would a Physicist Design a Tennis Racquet? Physics Today, 48, 26-31, Mar 1995.

2.) Mitchell, S.R., et al, Head Speed versus Racquet Inertia in the Tennis Serve, Journal of Sports Engineering, 3, 99-110, 2000.

increased from 250 gm to 400 gm. If n = 0.2 there is a 2.3% increase in serve speed.

We conclude from the above analysis that there is unlikely to be any significant variation in serve speed with racquet mass. This conclusion is based on the following observations:

- It is reasonable to assume that racquet speed will decrease as racquet mass increases, hence $n > 0$.

- It is unlikely that the racquet energy will remain constant, since this assumes that the arm mass is negligible, hence $n < 0.5$.

- Players who like light racquets are unlikely to switch to heavy racquets (or vice versa) just to get a few percent increase in serve speed, so a change in racquet mass from 250 gm to 400 gm is not a realistic option for most players.

If a player wants to serve x % faster, the best solution will be to swing the racquet x % faster, and that might mean more gym work or a better diet rather than a heavier racquet. The same conclusion does NOT apply to groundstrokes. In that case, players are more likely to swing at a relatively controlled or constant speed, in which case heavier racquets will be more powerful, less maneuverable and more stable in terms of the amount of rotation during impact with the ball.

Effects of Frame Stiffness on Racquet Power

Equations 14.1 and 14.2 remain valid for an impact anywhere on the strings, not just at the center. The only things that need to be changed are the values of e and M_e. For an impact near the tip or throat of a racquet, e drops to about 0.6 for most graphite racquets due to the energy loss caused by frame vibrations. Very stiff frames don't vibrate as much, in which case e is about 0.7 at the tip and throat, but it is still about 0.88 in the middle of the strings. For an impact near the tip, M_e is about 0.3M and e_A is close to zero. For an impact near the throat, M_e is about 0.7M and e_A is about 0.5. In each of these cases, V is the speed of the racquet at the impact point, not just the speed at the middle of the strings. The best procedure is to actually measure e_A at various points on the strings rather than calculate it, and then calculate v from Equation 14.1. Typical results are described in Chapter 16.

Stiff racquets are generally more powerful than flexible racquets since less energy is wasted in frame vibrations.

The exception to this general rule is that frame stiffness makes no difference to power for an impact at the vibration node near the middle of the strings since frame vibrations are absent for such an impact. For example, if the frame stiffness is increased so that the frame vibrates at 200 Hz instead of 125 Hz, then e_A near the tip will increase from about 0.10 to about 0.18 for a racquet of mass M = 340 gm. This translates to an increase in serve speed of 7% since $(1 + e_A)$ is increased by 7% (Cross, Sports Engineering, 2000). The serve speed from the tip is slightly less than the serve speed from the middle of the strings, but elite players tend to serve from a point near the tip due to the added height advantage.

References

R. Cross, Flexible beam analysis of the effects of string tension and frame stiffness on racquet performance, Sports Engineering 3, 111-122 (2000).

A. Nathan, Characterising the performance of baseball bats, American Journal of Physics, in press (2003)

Anastasia Myskina, Russia

15

Determining On Court Ball Speed

Do-It-Yourself Measurements of Ball Speed

By Howard Brody

Measuring Serve Speeds

Radar Gun. As soon as you hear the words "How fast is the ball moving?" the first thing you think of is the radar gun and the serve speeds you see displayed in lights at big tournaments. Radar guns are traditionally used for measuring serve speeds, but you never seem to see them measuring the speed of a groundstroke. If they are calibrated correctly and aimed properly, radar guns give you the speed of the ball as it leaves the racquet. Clearly, if the gun is not pointing at the ball impact location, it cannot give you the correct speed off the racquet. A tennis ball loses speed due to air resistance, and if the gun does not see or pick up the ball's trajectory until it crosses the net, for example, the answer will be 10 to 20 miles per hour slower than the ball's speed off the racquet. There is a second effect due to the ball's direction of motion. If the ball is hit at an angle relative to a line between the radar gun and the racquet (for example, the gun is aiming down the center and the shot is hit cross court), the gun reading will be lower than the true ball speed, and this correction is given in Table 15.1.

Table 15.1

Change in radar gun reading due to pointing of gun at an angle to the ball flight direction.

Angle Degrees	Cosine of Angle	Reduction in Reading of 100 mph serve
0	1.000	0
5	.996	-0.4
10	.985	-1.5
15	.966	-3.4
20	.940	-6.0
25	.906	-9.4
30	.866	-13.4

As tennis coaches, many of you would like to be able to tell your students exactly how many miles per hour they are hitting their shots. But you don't own a radar gun, and you have no intention of buying one, considering its limited use and high cost.

VCR/Camcorder. On the other hand, many of you already own a camcorder with a high-speed shutter and a VCR capable of single frame or single field advance. You can use this TV equipment to determine the speed of a serve or a groundstroke with acceptable accuracy.

The major disadvantages of this proposal are that you do not get an instant answer as you do with a radar gun (you have to play the tape back in the VCR) and you have to do some work, where the radar gun just has to be pointed and the speed is displayed.

There are two ways to measure ball speed with the tape you obtain from your camcorder. The direct method of measuring exactly how far the ball moves for each frame or field advance of the tape is difficult. It requires special camera placement in addition to having the area around the hitter calibrated as to size versus picture image. It is not a recommended method unless you are knowledgeable about optics and physics.

Stop Watch. There's a second method which will determine the ball speed off the racquet. It's done by measuring the average ball speed and applying a simple formula. For many years, I have been using a computer program that determines a tennis ball's trajectory and I have noticed an interesting result.

The ball speed at the receiver's baseline is approximately half the speed of the ball coming off the server's racquet. (There are slight modifications depending on court speed and type of shot, but to the accuracy that we need, these can be neglected.)

The distance from baseline to baseline is 78 feet. So if you could measure the time it took the ball to travel that distance, you would have measured the average ball speed. My computer output has shown that the ball speed off the racquet head is 4/3 times this average ball speed.

If the time it took for the ball to go from baseline to baseline is 0.85 seconds, the average speed would be 78 feet/0.85 seconds = 91.8 feet/second. Using the 4/3 factor predicted by the computer, this would correspond to a speed of 122 feet/second (4/3 x 91.8) off the racquet head.

To convert feet/second to miles/hour, you divide by 1.467, and this would give you 83.4 miles/hour. Doing all the above arithmetic and combining it into one formula (V = (4 x 78)/(3 x 1.467 x t) = 70.9/T), the ball speed off the racquet head (in mph) would be 70.9 divided by the time (in seconds) for the ball to go from baseline to baseline. In this case, the formula would be: ball speed = 70.9/time.

Now if you can handle a stopwatch sufficiently well enough to be able to time the ball between when it's hit and when it reaches the baseline, then you can get the ball speed off the racquet head immediately. Just divide 70.9 by the stopwatch time in seconds.

The problem is that most of us cannot do that with the needed accuracy, so you should use the camcorder and VCR to determine the time. During single advance of the tape during playback, you just count the number of frames or fields between when the ball is hit and when it reaches the baseline. Each frame is equivalent to 1/30 of a second and each field is equivalent to 1/60 of a second (0.01666 sec).

If you advanced your VCR 51 times in tracking the ball from baseline to baseline, and it was a single field advance, then 51 x 1/60 = 0.85 seconds. If you have a single frame rather than single field advance, 25 advances would correspond to 0.833 seconds and 26 advances would correspond to 0.867 seconds. Just plug the time you measured into the (70.9/time) formula and you have the ball speed as it leaves the racquet.

Corrections. There are some small numerical corrections that you may want to make since not all balls travel exactly 78 feet each time they are hit, as some go crosscourt, some are hit from behind the baseline, etc.

The numerical constant used (70.9) must be modified for those cases. If the ball were hit from three feet behind the baseline, the distance would increase from 78 to 81 feet, and the numerical constant (to be divided by the time) would be 70.9 x 81/78 = 73.6.

As an example, a wide serve hit from the T, but returned from the baseline at the doubles line, would have traveled exactly 80 feet, not 78 feet. For this case, the ball speed (in mph) off the racquet head would be 70.9 x 80/78 divided by the time in seconds. If the time was again 0.85 seconds, the initial ball speed leaving the strings would have been 85.6, not 83.4 mph.

If you compare the results you get from this method to the results obtained with a radar gun, you may discover it may give somewhat higher serving speeds. This could be caused by a number of things, among them: 1) a radar gun out of calibration, 2) a radar gun being used at an angle, or 3) the fact that the radar gun does not pick up the ball until it has moved a certain distance off the racquet. (A typical serve will lose 0.6 mph for each foot it moves through the air.)

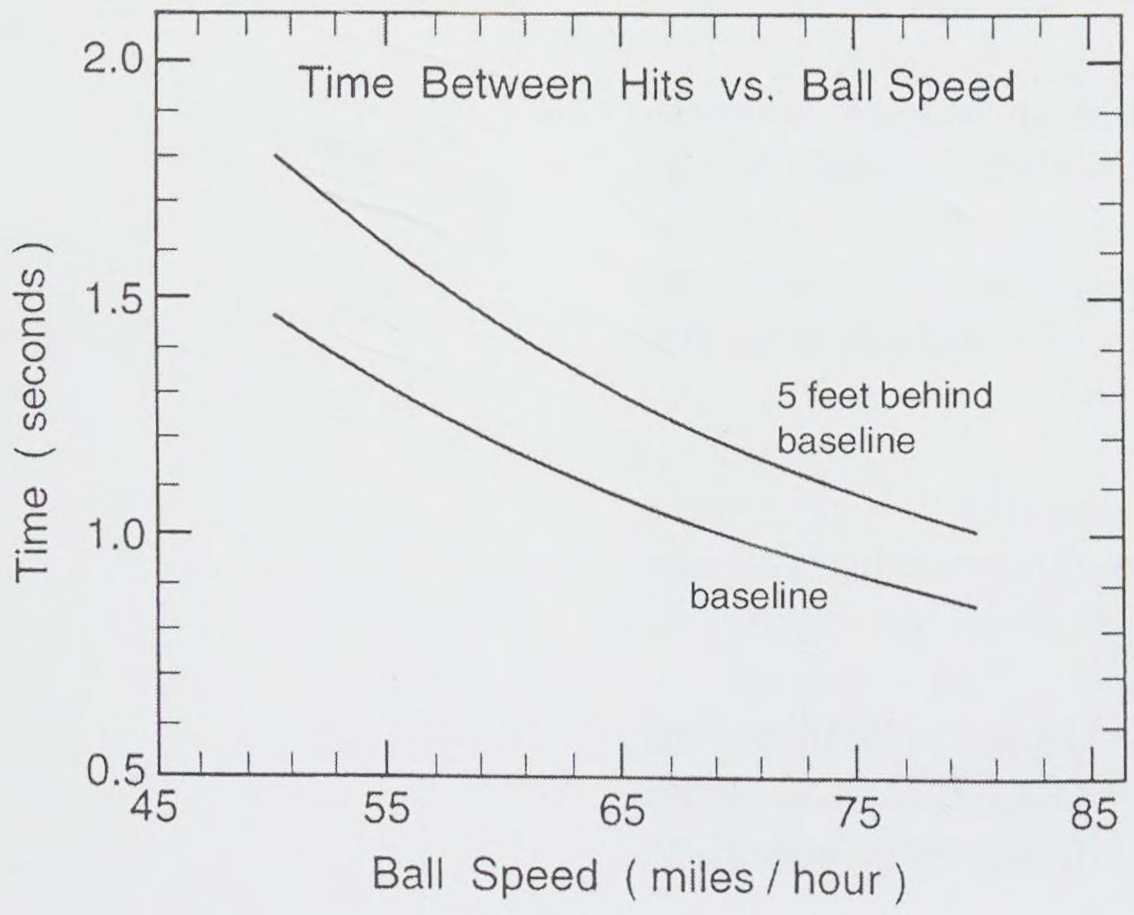
If you think about it, this time interval you have measured is actually more important to you as a tennis player than the ball speed. A discussion of the importance of this time interval will be provided in Chapter 24 on Strokes and Time.

Measuring Groundstroke Speeds

You can also use the VCR/Camcorder method to time the speed of a groundstroke, but as with the serve, it is a bit complicated. However, there is also a very simple method for measuring the speed of the ball as it leaves the racquet during a groundstroke rally that only requires a stopwatch. It also has the advantage of giving an immediate answer (you don't have to get out the VCR and count frames).

If two players are having a groundstroke rally (in a practice session or in a match), and you time how long it takes, on average, for the ball to go from racquet to racquet, Figure 15.1 tells you how fast the ball was moving when it left the racquet. That's all there is to it – except for a few details, which follow.

Figure 15.1



1) It is very difficult to time a single ball passage with enough accuracy using a stopwatch to make this method practical. To get around this problem, you should measure the time it takes for a 3-, 4- or 5-shot (or even more) rally, and then divide by 3, 4, or 5, as the case may be.

This will reduce the timing error by a factor of 3, 4, or 5. If a 5-shot rally (ball crosses the net five times) takes 6.9 seconds with a timing uncertainty of +/-0.2 seconds in measurement error, a single shot takes 6.9/5 = 1.38 seconds with an uncertainty of only 0.2/5 = 0.04 seconds.

You have the same total uncertainty or error in timing whether you time one shot or 10 shots. However, when you measure a multiple shot rally, the uncertainty for a single shot is divided by the number of shots hit, greatly reducing the error.

2) The time of travel depends on where the players are standing when they hit the ball (on the baseline, behind the baseline, etc.). So the calculation has been done (and the chart shows the results) for both a baseline to baseline rally and for a rally with both players five feet behind the baseline. If they are at some other position, such as 2-1/2 feet behind the baseline, then the curve will be halfway between the two curves shown.

As an example, if you had timed a four-hit rally (four complete ball transversals) and it took 5.2 seconds, the time between hits would be 1.3 seconds (5.2/4 = 1.3). Looking up 1.3 seconds on the chart, you would see that the original ball speed off the racquet would have been 55 miles per hour if the players were both on the baseline, 65 miles per hour if both players were five feet

behind the baseline, and 60 miles per hour if they were both 2-1/2 feet behind the baseline (or one was on the line and the other five feet behind it).

Note that you cannot take the 78 feet (baseline to baseline), 88 feet, or any other number of feet, and simply divide it by the time to get the ball speed off the racquet. That calculation will give an average speed of the ball over its entire flight and not the speed as it leaves the strings. This is because the ball slows up considerably due to air resistance and its bounce. These factors, which produce Figure 15.1, have been included in the computer program.

3) If instead of hitting shots parallel to the sidelines, the rally is going corner to corner. That distance is 82.5 feet, which is halfway between the two curves. (A cross-court rally from 2-l/1 feet behind the baseline would be similar to the five feet behind the baseline data.)

Court surface and spin will make only a slight difference. All calculations were done assuming the ball bounces about six to eight feet inside the baseline (which is typical for most groundstroke rallies). It was also assumed that both players are hitting the ball equally hard (which is the usual case for a rally) and they are both hitting with the same pace from shot to shot. If these last two conditions are not met, the result you get will be approximately the average of the ball speed off the racquet.

This measurement of groundstroke ball speed does not have to be done "live," as it will give the correct results when you use a video of a rally. (You could also then measure the speed of a single shot, as opposed to the average speed, by the single frame/field advance method previously described.) From the video replay, you can also determine the average groundstroke speed for some of the old greats, such as Rod Laver. If you could trust the speed of your movie projector (your replay was at the same number of frames per second as it was when taken), you could also determine groundstroke speed for Bill Tilden and Don Budge from old movie clips.

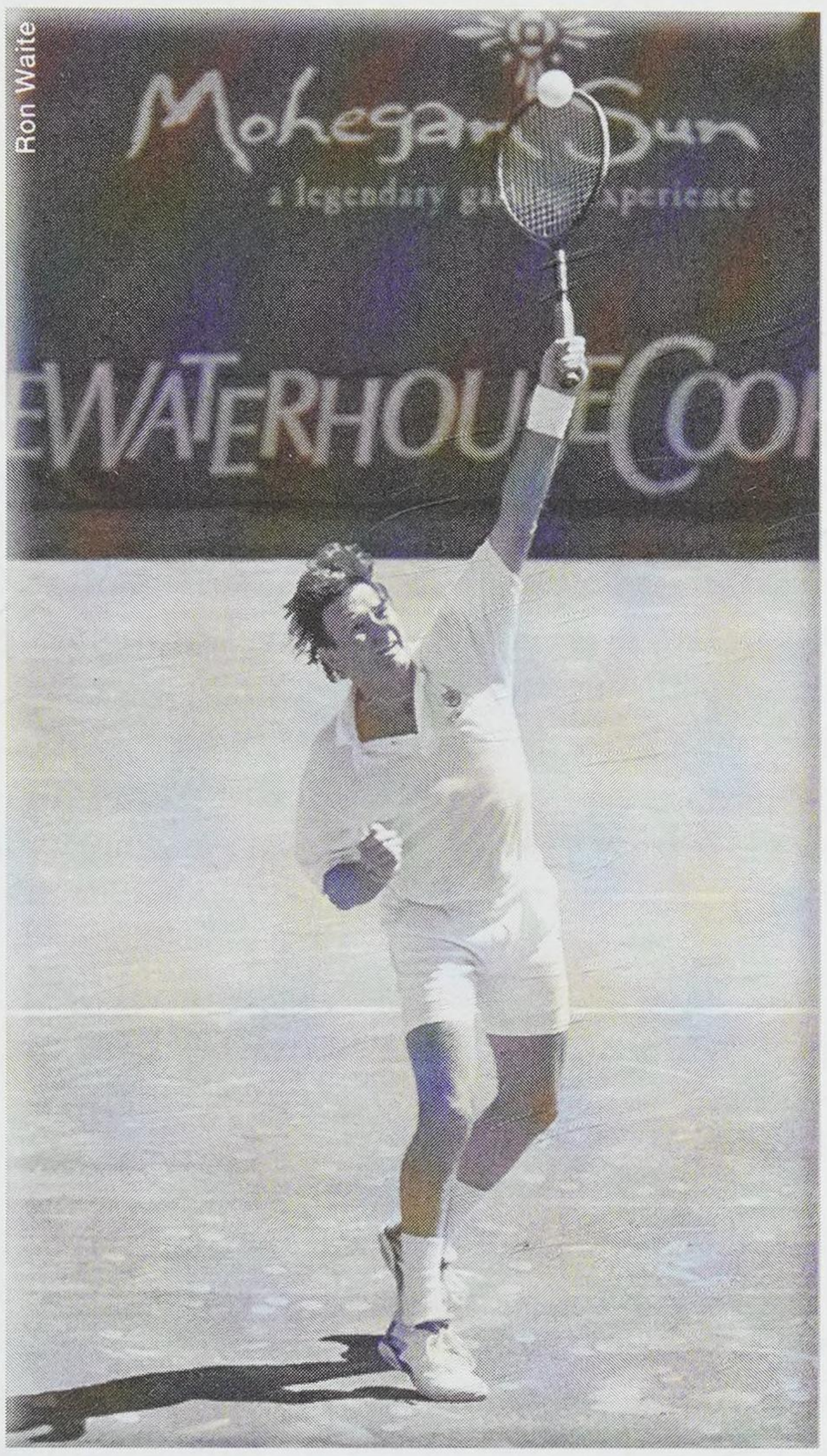
As a test of all this, I timed two touring pros as they were warming up. When they started, the time for five shots ran from about 7.6 to 8.3 seconds. As they warmed up, the five-shot time was reduced to about 7.2 seconds. After a few more minutes, they reached times like 6.95 seconds for the five hits from about three feet behind the baseline.

Michael and Carl Chang were hitting from about four feet behind the baseline and the times were about 6.9 seconds for the five shots. Then MaliVai Washington replaced Carl and the time for five shots dropped to about 6.5 seconds, but they were a bit closer to the baseline.

16

Customizing Racquets

Adding Lead Weight to Affect Performance



Jimmy Conners, United States

By Rod Cross

Introduction

Customizing a racquet means modifying it in some way to suit the user. The most common way is to add a string dampener. This deadens the sound of the strings, but that is all it does. It does not have enough mass and it is not located in the right spot to reduce vibrations of the racquet frame. The human hand is a much better vibration dampener. The next most common way to modify a racquet is to change the grip. Players can replace a grip with a better one at low cost, or they can add tape or plastic heat-shrink under the grip, or add an overgrip, to increase the grip size. Professionals may even have their grip custom made to suit their exact hand shape and to suit the way they grip the handle.

The term customizing most commonly refers to the fact that self-adhesive lead tape can be added at one or more spots on the frame to change the way the racquet feels and performs. The average player rarely bothers to add lead tape, but the professionals insist on it, partly to ensure that every one of their racquets feels exactly the same. Professionals are provided with racquets for free, but each racquet is usually a bit different from the next. For example, it is usual for racquets to differ in weight by up to 5 gm, even if they are identical models. Rather than shaving 5 gm off the heaviest one, it is simpler and better to add 5 gm to the lightest one. Alternatively, a player might like to add 10 gm to the heaviest racquet and 15 gm to the lightest one so that all the racquets are a bit heavier than the racquets provided. Extra weight should be added in the right spot, not just anywhere in

the handle or around the tip. Adding weight changes the balance of the racquet, which changes the way it feels when you swing it. It changes a whole bunch of other things as well, and we will examine each of these things in turn.

Balance Point

The balance point of a racquet refers to the location of its center of mass (or its center of gravity to use an older term). It is easy to locate the balance point by balancing a racquet on one finger, or on the edge of a ruler, or some other narrow edge. It is usually close to the middle of the racquet, about 27/2 = 13.5 inches (343 mm) from the butt end of the handle. If it less than 13.5 in. from the end of the handle then the racquet is said to be head-light. More than 13.5 in, then the racquet is head-heavy. The balance point will shift towards the tip of the racquet if extra weight is added at the tip, by an amount that is easy to calculate. Suppose the mass of the racquet plus the strings is M, its length is L, and its balance point is located at a distance x from the butt end of the handle. Now, suppose we add a small mass m at the tip. The balance point will then be at a distance B from the butt end, where B is given by

$$B = \frac{(mL + Mx)}{(M + m)} \qquad (16.1)$$

The balance point shifts a distance y = B - x = m(L - x)/(M + m).

For example, suppose that M = 320 gm, L = 690 mm, x = 340 mm and m = 10 gm. Then B = (10 x 690 + 320 x 340)/330 = 350.6 mm and y = 10.6 mm. The balance point, therefore, shifts towards the tip by 10.6 mm, changing the racquet from head-light to head-heavy in this case, the point halfway along the racquet being 345 mm from the butt end. Since these values are typical of most racquets, it is a general rule that an extra 10 gm at the tip will shift the balance point by about 10 mm. If the racquet is heavier than 320 gm then 10 gm at the tip will shift the balance point by a slightly smaller amount.

What if we add a mass m at the butt end instead of the tip? Then the balance point will be at a distance

$$B = \frac{Mx}{(M + m)} \qquad (16.2)$$

Using the above numbers, B = 320 x 340/330 = 329.7 mm so the balance point is shifted towards the butt end by 10.3 mm. It doesn't matter whether m is added at the tip or the butt, the shift caused by adding m is about the same, except that the shifts are in opposite directions. Adding only 5 gm, the shifts will be half as big.

Suppose you want to match two racquets so they have exactly the same mass and the same balance point. Suppose that one racquet is heavier than the other by G gms, and the balance points are also different. There are many ways to match the two racquets, but the simplest way is to add G gms to the lightest racquet. If you add it either at the tip or the butt end, then you might shift the balance point too far, or not enough. If the shift is too big, then you will need to add the extra mass at a point closer to the middle of the racquet. A good result is obtained by adding equal weights at the 3 and 9 o'clock positions since that makes the racquet more stable in terms of rotation around the long axis. But what if the shift is not enough? Then you will need to add more than G at

one end of the lightest racquet and then add a bit extra to the heavier racquet. If you add mass at the balance point of the heavier racquet, then the balance point of the heavier racquet is unaltered. However, the aim would probably be to shift both balance points closer together, since that way you would need to add only a relatively small mass to the heavier racquet.

Swingweight

The swingweight of a racquet refers to its resistance to rotation when you swing it. The actual weight itself represents the resistance to motion in a straight line. If you didn't support the racquet, it would fall in a straight line to the ground. To stop it falling you need to lift or push upwards with a force equal to the weight of the racquet. Swinging a racquet is a different action. If you swing a racquet with a certain force, then a racquet with a small swingweight will rotate faster than one with a large swingweight. The best way to appreciate the difference between swingweight and actual weight is to tape a small metal object, like a bolt, to the racquet, then swing it. It doesn't matter where you tape the object, the total weight of the racquet plus the object will be the same. But if you tape the object to the tip of the racquet, then the racquet will feel a lot heavier when you swing it than if you tape the object onto the handle. The reason is that an object at the tip travels faster than an object on the handle and a bigger force is needed to accelerate it to a higher speed.

The swingweight of a racquet is known technically as the moment of inertia or the rotational inertia. It is approximately equal to $ML^2/3$, but it will be a fraction higher if the racquet is head-heavy, and it will be a fraction lower if the racquet is head-light. For example, if M = 0.3 kg and L = 69 cm, then the swingweight is approximately 0.3 x 69 x 69/3 = 476 kg cm^2. If you add a mass m at the tip of the racquet, then the swingweight will increase by an amount mL^2. Consequently, the swingweight will increase by a fraction 3m/M or by 300m/M%. For example, if M = 300 gm and m = 10 gm, then the swingweight will increase by 300 x 10/300 = 10%. Most people would notice the difference easily. However, if extra mass is added in the handle, the increase in the swingweight is much smaller and would not be noticeable. If you add a mass larger than 10 gm in the handle, then the racquet will definitely feel heavier since the actual weight of the racquet is increased even if the swingweight is not affected.

The above formula for swingweight refers to rotation of the racquet about an axis through the butt end, and it gives an accurate measure of the effect of swinging a racquet about an axis through the wrist. However, swingweight is normally measured or specified for a rotation axis 10 cm from the butt end of the handle, at a point near the first finger. The reason is not because players swing the racquet about that axis, but because the racquet needs to be clamped by the handle in a machine to swing it when measuring the swingweight. The swingweight measured this way is less than the value $ML^2/3$ by an amount equal to 100M. For example, if $ML^2/3$ = 476, and M = 0.3 kg, then the measured swingweight will be 476 - 30 = 446 kg·cm^2.

When a racquet is swung by a player (instead of by a machine), the racquet rotates about several different axes all at once. There is rotation about axes through the wrist, the elbow, and the shoulder, as well as rotation about the hips. For each of these axes, the racquet has a different swingweight. Adding

a weight at the butt end of the handle has no effect on the swingweight about an axis through the wrist, but it increases the swingweight about an axis through the elbow or shoulder. The effect is fairly small since the total swingweight about the elbow includes the swingweight of the hand and forearm as well as the racquet.

Racquet Power

Adding a small weight at the tip of a racquet increases the power of a racquet by a small amount. Adding weight anywhere in the head will increase racquet power, but adding weight at the tip is the most effective. The trade-off is that the racquet is harder to swing and it is less maneuverable. But if the racquet is very light to start with, or if you have strength to spare, then the extra power could be useful. The extra power is relatively small since you won't be able to swing the racquet as fast if you increase its swingweight. For a controlled shot where racquet head speed is kept the same, the extra weight will have a more noticeable effect on racquet power.

A weight added at the tip of a racquet shifts the point of maximum power closer to the tip. For a serve, this gives the player an added height advantage. For a groundstroke, the point of maximum power is closer to the center of the strings. The point of maximum power depends on the speed of the incoming ball as well as the speed of the racquet. If the racquet is initially at rest or moving slowly compared with the speed of the ball, then the maximum power point is near the throat of the racquet, since that is where the ball bounces best.

For a serve, the maximum power point is near the tip, since that is where the racquet is moving the fastest. Calculations of the ball speed with 30 gm added at various spots are given in Figures 16.1 and 16.2. These calculations are based on measurements of the ball speed off an initially stationary racquet of mass 332 gm (Cross, 2001).

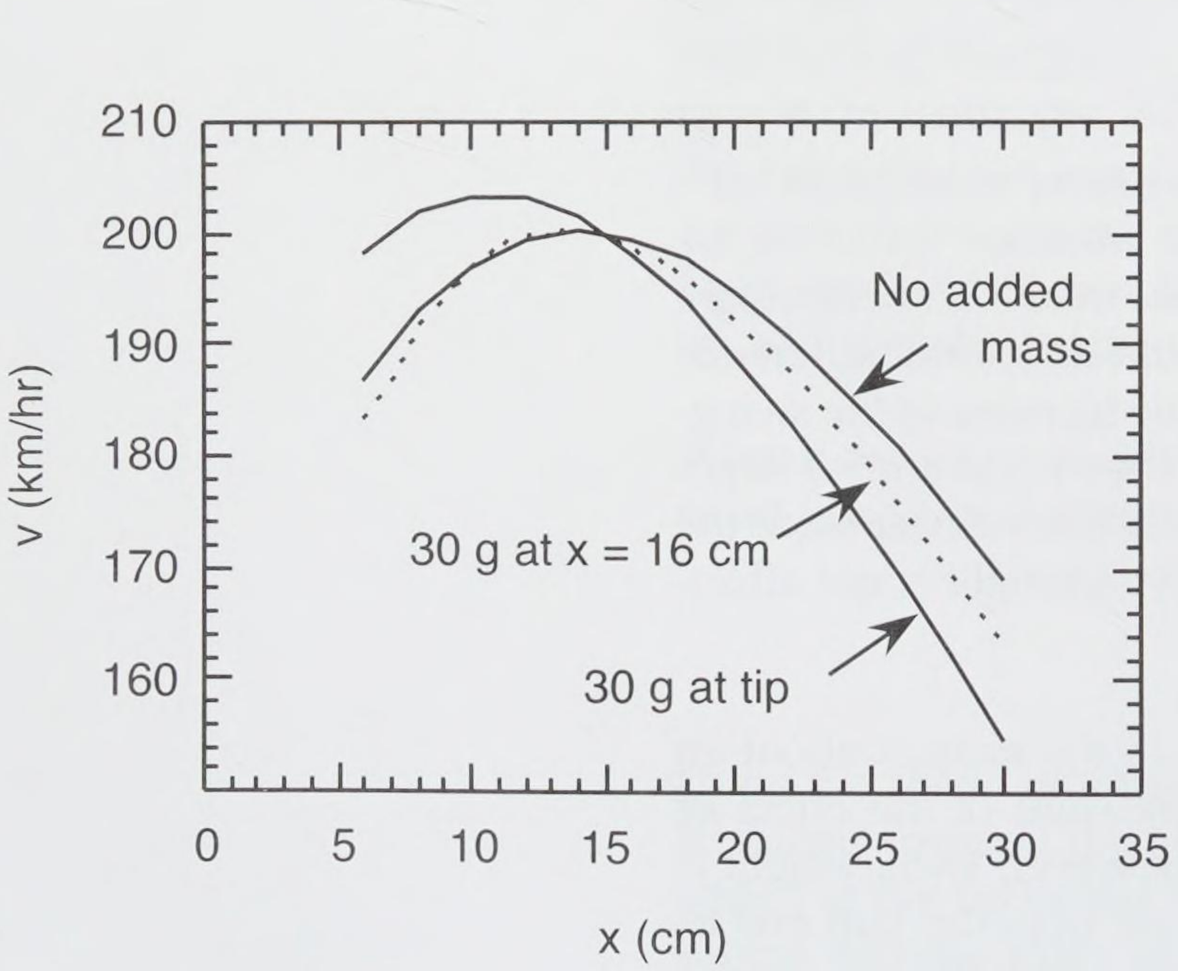


Figure 16.1

Serve speed with 30g added at the tip or 16 cm from the tip. The serve speed is shown vs the impact distance, x, from the tip.

The racquet speed was decreased slightly as the swingweight increased, keeping the ball speed fixed (200 km/hr) at x = 15 cm.

Rotation About the Long Axis

If a player hits a groundstroke near the top or bottom edge of the racquet, then the racquet will tend to rotate about its long axis. The hand will stop it spinning around this axis, but the racquet will rotate at least a few degrees, enough to alter the intended trajectory of the ball. The best solution is to hit the ball in the middle of the strings, but no one is perfect. The next best solution is to use a racquet that doesn't rotate easily around the long axis. The way to do that is to make sure that the material in the frame is as far from the long axis as possible and it is as heavy as possible. The extreme case would be a very heavy racquet with a huge head. However, it is not necessary to go to that extreme. A more practical solution is to use a moderately large head, or to add small weights at the 3 and 9 o'clock positions. Adding

a weight at the tip will have no effect at all in this respect, since the axis passes straight through the 12 o'clock position. For the same reason, additional mass in the handle will have no effect.

An ice skater can spin around rapidly on one spot by keeping the arms close to the body. In order to slow down, the skater pushes the arms out, away from the body. Similarly, a diver doing a triple somersault tucks the arms and legs close into the body in order to spin as fast as possible, and then stretches out to slow the rotation speed. It's the same with a racquet. Mass close to the spin axis has very little effect on reducing racquet rotation, but mass well away from the axis reduces the rotation speed. The moment of inertia about the long axis is the important factor. Old wood racquets with small heads had a small moment of inertia, which probably explains why players using wood racquets did not hit the ball as hard as the modern players. The larger head in modern racquets gives much better rotational stability and allows the player to hit the ball hard with reduced rotation about the long axis.

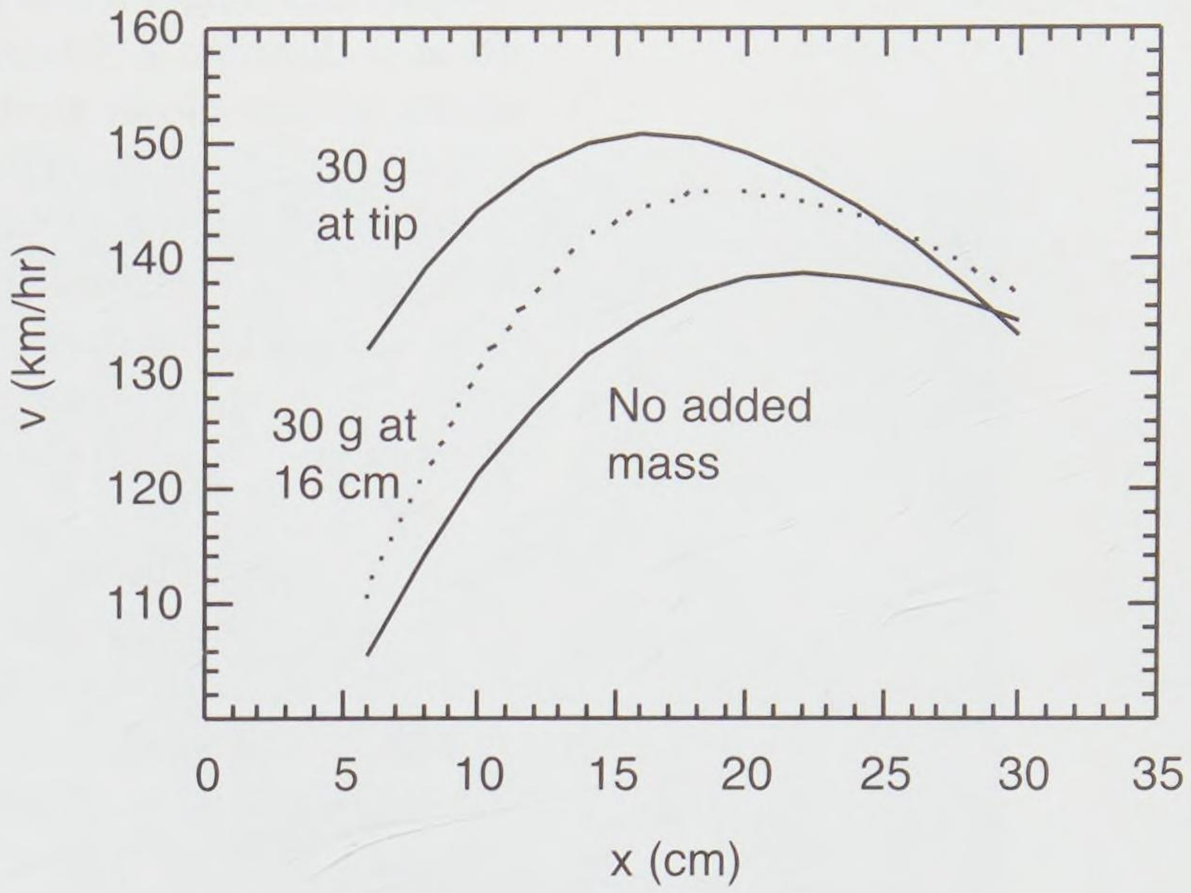


Figure 16.2

Ball speed vs impact distance from the tip, x, for a groundstroke when the racquet speed is kept constant. The ball is incident at 20 m/s (72 km/hr) and the racquet is swung so that the tip is rotating at 27 m/s (97 km/hr) and the throat (at x = 30 cm) is rotating at 18 m/s (65 km/hr).

Another method of reducing the rotation angle slightly is to increase the string tension. The rotation angle increases with the rotation speed and it increases with time. At high tension the ball spends a shorter time on the strings and hence the rotation angle is reduced. This is possibly the main reason that players notice an increase in ball control at high string tension.

Shifting the Sweetspots

The sweetspots of a tennis racquet are near the middle of the strings. Adding a weight at the tip of the racquet shifts the sweetspots closer to the tip. One of the spots is technically known as the fundamental vibration node and the other is called the center of percussion. If a racquet hits a ball at the vibration node, then the racquet will not vibrate, regardless of how stiff or flexible the racquet is. That point will shift by about 16 mm closer to the tip if an extra 10 gm is added to the tip, or 32 mm if 20 gm is added to the tip. If weights are added to the 2 and 10 o'clock positions on the frame, then the vibration node is not shifted at all since those points on the frame never vibrate, regardless of where the ball strikes the strings. Adding weights at 3 and 9 o'clock has only a tiny effect on shifting the vibration node in the middle of the strings. Adding weight in the handle has essentially no effect on shifting the vibration node in the middle of the strings.

An impact on the strings at a point on the long axis causes the racquet to rotate about an axis located at a point in the handle. That point is called the conjugate point. Every point in the handle has a different COP in the strings. For

example, if the conjugate point is right at the butt end of the handle, and if the racquet has length L and has a uniform mass distribution, then the COP is located at a distance L/3 from the tip. The center of percussion (COP) is located about 40 mm closer to the throat than the vibration node. If a racquet strikes a ball at the COP, then the racquet will vibrate a small amount, but there is no sudden movement of the butt end of the handle one way or the other (apart from the back and forth vibrational motion). As a result, the shot feels nice. The shot feels nice anywhere around the center of the strings, not just at one or two tiny spots, since the level of shock and vibration drops significantly in this region.

Adding 10 gm at the tip will shift the COP about 5 mm closer to the tip. Adding weights in line with the COP (near the 3 and 9 o'clock positions on the frame) does not shift the COP at all. Adding weight in the throat will shift the COP slightly closer to the throat. Removing weight from the throat (when the racquet is manufactured) shifts the COP slightly towards the tip. Adding weight at the conjugate point in the handle does not shift the COP at all.

Summary

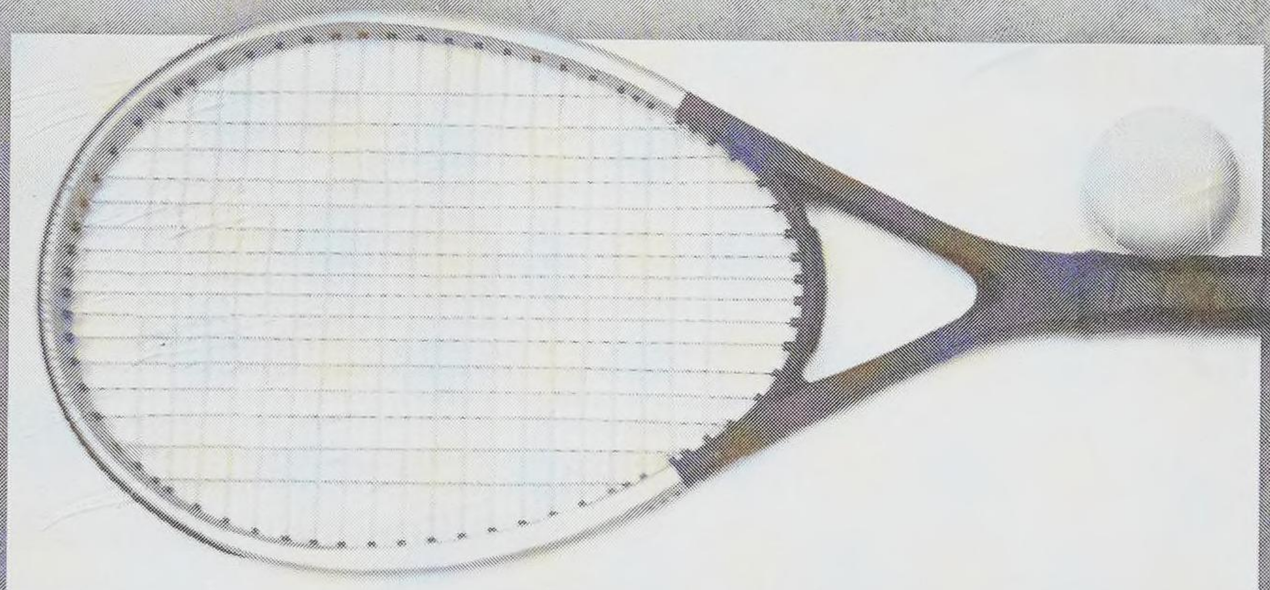
The addition of weights to a racquet changes almost every physical parameter of the racquet. However, there are particular locations where an added weight may have no effect at all on some of these parameters. For example, (a) a weight added at the balance point does not shift the balance point, (b) a weight added in the handle does not change the swingweight of the racquet about an axis through the handle, (c) a weight added at the center of percussion or the corresponding conjugate point does not shift the center of percussion, and (d) a weight added at a vibration node on the frame does not shift the location of the vibration node and it does not change the vibration frequency of the racquet.

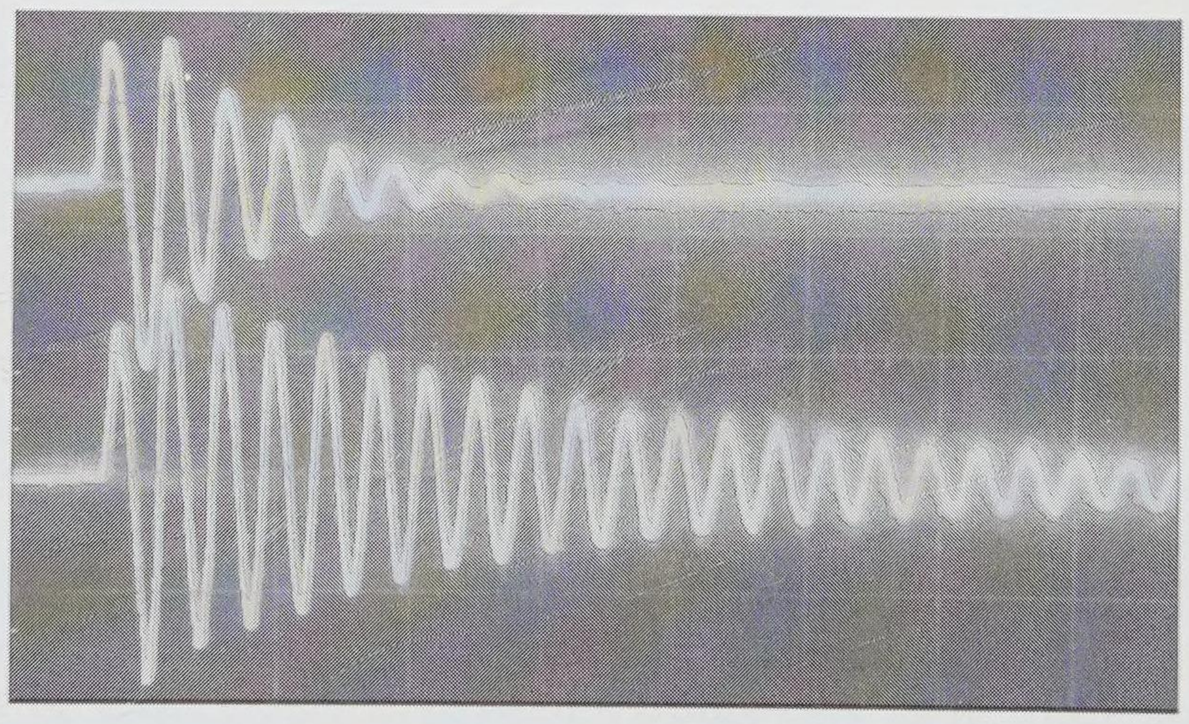
Reference

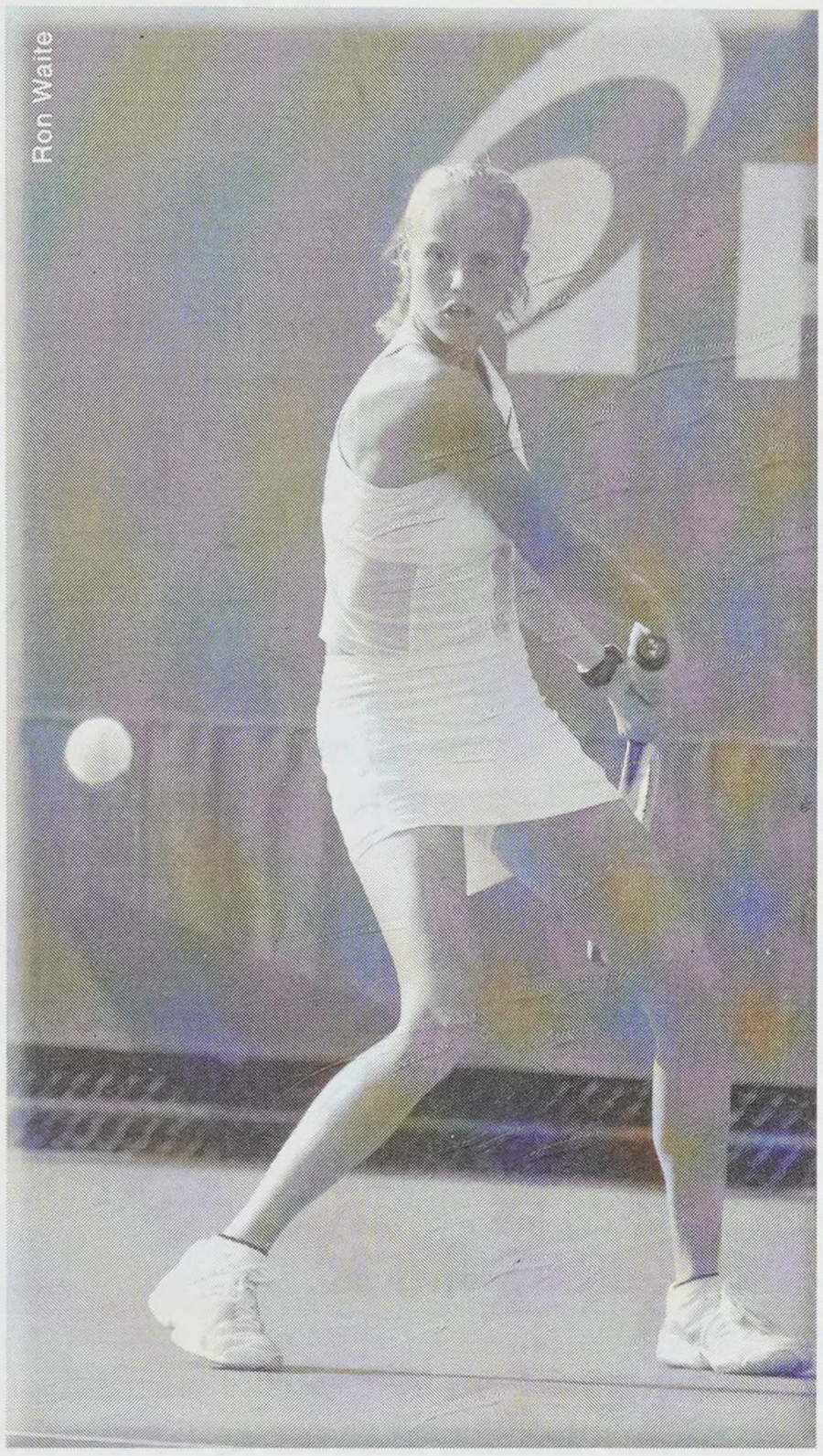
R. Cross, Customizing a tennis racket by adding weights, Sports Engineering 4, 1-14 (2001).

PART TWO

STROKES







Daja Bedanova, Czechoslovakia

17

Racquet Technology and Strokes

How Equipment Has Changed the Game

By Howard Brody

When you watch tennis being played today, if you are as old as the authors of this book, you immediately notice that the tennis racquets are quite different from the wooden racquets that were in use 30 or more years ago. The modern racquet is larger in both head length and head width, considerably lighter, less flexible, made of a reinforced plastic material, and it is possibly an inch or so longer. When you watch the more proficient young players hitting, you also see that the style of tennis strokes has also changed over that period of time. The classic, smooth, flowing groundstrokes that were displayed by the tennis champions of the first 2/3 of the 20th century have been replaced by a game that emphasizes power from the baseline and the ability to end the point with a single swing anytime the opponent is slightly out of position, or hits a shot that bounces short. Is it possible that the changes in stroke mechanics are a direct result of the changes in racquet technology, or did the strokes evolve independently of the racquet changes? What this chapter will explore, is how the new technology in tennis racquets has allowed players to modify the way they hit the ball, and get away with it.

Traditional Racquets

The old standard wood racquets that were used by most players up until the 1970 era, weighed at least 14 to 15 ounces, had a neutral balance, and had a small head in both length and width. Because of the structural limitations of wood, in order to fabricate a racquet light enough for a player to comfortably swing, the frame could

not be made very thick, which resulted in some degree of flexibility, particularly near the tip. In addition, the strength-to-weight ratio of wood dictated that the head could not be too large if you wanted to string it at a reasonable tension.

Modern Racquets

The modern racquets are molded from a graphite-reinforced material (plus, on occasion, other exotic reinforcing materials), weigh 9 to 11 ounces, and are often handle light. It is even possible to construct racquets with a weight as low as 7 ounces, yet have a head that is quite large and produce a frame that is exceedingly stiff and durable.

Classic Strokes

The classic groundstroke used by most players up until the middle of this century was a long flowing swing with good, early preparation, and a long follow through. The players stood sideways and smoothly accelerated the racquet through to the hitting region as the body weight was transferred forward (you stepped into the ball). The shot was often hit flat, or with some topspin, but occasionally players preferred to use a small amount of slice (underspin) on the backhand.

New Strokes

The modern forehand groundstroke that many players have adopted has an open stance with a semi-western or western grip. The players coil the body, and then uncoil it as the shot is hit, bringing the big trunk muscles into action. This uncoiling, rotational motion often causes the player to leave the ground during, or at the end of, the shot. The result is that the ball is hit with excessive topspin and usually very hard.

In today's tennis, even the youngest juniors seem to be whipping their racquets around with abandon. Years ago, probably only the exceptional player would have had the physical ability to have done that with the old, heavy wood frames. The average player and the recreational player needed the long, flowing swing to accelerate the old, heavy racquet up to hitting speed. In addition, this gradual acceleration gave the player much more control of the racquet head and allowed the player to hit the ball at approximately the same location on the racquet face each time it was swung. This was essential, since various impact locations on the old racquets often responded quite differently (the "Sweetspot" was small). A ball hitting an inch or so away from the desired location on the head might easily end up in the net or go long. With these old racquets, to get the ball to land consistently in the court required a consistent impact location on the strings, which could only be accomplished with a controlled swing.

Sweetspots

With the quick, whipping swing that many players use today, it is more difficult to hit the ball at exactly the same location on the head each time the racquet is swung. However, due to the characteristics of the modern racquet and the heavy topspin strokes used, the resulting ball trajectory is much less sensitive to the exact location of the ball impact on the strings. (The racquet is more

"forgiving" or it has a large "Sweetspot".) If there is a preferred location to hit the ball (such as the center of the strung area), the new racquets give you more latitude for error of impact location, both in a direction across the racquet face, and along the main axis of the frame.

Stability

Because the new frames are much wider than the old wood racquets, they are much more stable against twisting when the ball impact point is not along the principal axis. The physical property of the racquet that produces this stability is called the polar or roll moment of inertia. The larger this moment, the less the racquet will twist on off-center hits and the less the power degrades as the ball impact point moves off the axis. This moment of inertia is proportional to the weight of the racquet and to the square of the width of the racquet head. A 10-inch wide head (the size of a typical oversize racquet) is 25% wider than the old 8-inch wide wooden frame, so it has a moment of inertia that is over 50% greater. This more than makes up for the 25% reduction in weight that comes with the newer racquets. This increase in polar moment reduces the racquet twist on off-axis impacts (hence reduces the ball's errant angle due to the twist), and keeps the rebound ball speed from changing too much on such off-center hits. Both of these effects give the player a larger margin for impact location error in the striking of the ball. In addition, the use of topspin gives the player a much larger "window" of acceptable angles to hit into, if the shot is to land in the court.

Power

When the power of a racquet is measured in the laboratory, balls are fired at the frame and the ball rebound speed is measured for various impact locations. For a typical racquet, the ball rebound speed is a maximum for impacts near the throat and the ball rebound speed falls off as the impact location moves toward the tip. As a general rule, the further the impact location is from the balance point, the lower is the ball rebound speed in the lab. In addition, stiffer racquets tend to have more power than flexible racquets, particularly closer to the tip. Many of the new racquets are head heavy, which means their balance point is further up in the head. Because of this, the maximum power point moves up, away from the throat. As the racquet head is made stiffer, the power degradation near the tip is reduced. These results are for a situation where the racquet is not being swung, but is at rest in the laboratory.

When the racquet is swung, the tip is moving somewhat faster than the throat and the ratio of these two speeds depends upon the exact nature of the swing. A very wristy, whipping motion will have a greater tip to throat speed ratio than the old classic swing. Because the tip is moving faster than the throat, the maximum power point moves up higher in the head. When the physics of all of these factors (the racquet response and player swing) is combined to predict the actual playing characteristics of the racquet, the new frames with the modern style of swing show a uniformity of power response over a large area of the head. This is contrasted by the old frames, coupled with the classic swing, where the ball had to be hit at exactly the same place every time to get a uniformity of response.

For the classic racquet and classic swing, if the ball impacted beyond the center of the head, there was a loss of power and the ball might not clear the net.

For those old wooden racquets, if the ball impacted closer to the throat than the center of the head, the result was more power, and it was likely that the shot would go over the baseline. With the modern racquet and modern swing, these sorts of variations in ball impact location will result in very little variation in resultant ball speed, compared to the ball speed when the impact is at the center of the head. This means that if the ball is struck an inch or two from the center of the head, the result will still be a good shot.

If a player had the physical prowess to swing a heavy, classic racquet in the modern, wristy manner, any small resulting miss-hits might end up spraying the ball all over the court. The modern racquets, due to their forgiving nature and their lighter weight, have allowed players to adopt a new style of swing and still get the ball to land in the court where they want it.

Court Surfaces and Strokes

There is a second reason why strokes using the western and semi-western grips are seen more frequently these days than 30 years ago. From the beginning of tennis, the game was played on grass, which is a low bouncing, fast surface. Three of the four "Grand Slam" tournaments were on grass, as were many of the other tournaments. (For example, the USTA was known as the USLTA, where the L stood for lawn.) The western and semi-western grips are not optimized for fast, low bounces, but are ideal for medium or slow paced, high bouncing surfaces. Today the professional grass court circuit lasts at most a month, so most players learn, practice, and play on either hard courts or clay courts. Consequently, the new grips and the new types of strokes have evolved along with the new racquet technology to match the predominant court surfaces.

"Bring Back the Wood Racquet"

Over the years there has been a cry (particularly by a certain TV commentator) to bring back the old wood racquets and return tennis to the sport it was 30 or 50 years ago. There are complaints about the speed of the game, the shortness and rareness of a rally, and the new style of stroking that has developed. This may sound fine on television, but when it comes to drafting the rules of tennis, there are some problems with this suggestion. Exactly what is a wood racquet? The so-called wood racquet in use 40 years ago was actually a frame made of wood, glues, fibers, and other materials. In other words, it was a composite, just as today's racquets are composites. Would the International Tennis Federation (the group responsible for preserving the basic character of the game of tennis and enforcing the rules of tennis) have a rule saying the frame must be 75% or 90% wood? Would graphite fibers and epoxy be outlawed, even if the remaining 90% of the frame were wood? How would the ITF enforce any such rule? Would they have to take apart each and every racquet model from each and every racquet manufacturer and do an analysis of the composition? This does not look like a promising solution. That is why the ITF has opted for a different solution to the perceived problem.

There is no doubt that the new, graphite composite racquets, having only between half and two-thirds the mass of the old wood racquets, allow you to generate more power (rebounding ball speed off the head). It is also clear that the new lightweight, oversize, extremely stiff frames have a bigger "sweetspot" in that they allow you to miss-hit the ball a bit more without severe penalty (in other words, they are more forgiving of player error).

There is general agreement that the new racquet technology has allowed the game to change. Beginners find it easier to learn to play and they reach a level of competence where they enjoy the game sooner – at least before they give up in disgust because they perceive that tennis is too hard to learn. The average recreational player seems to get more enjoyment when playing with the new equipment that is available. The women's professional game has gone from a slow, long rally contest to an exciting game. The women (as represented by the WTA) absolutely do not want the game to change back to the way it was 50 years ago.

Then what is the problem that is bothering so many? It seems to be centered around the men's professional circuit, particularly on fast courts, such as the grass at Wimbledon, some indoor carpets, and certain acrylic surfaces, such as used at the US Open. People claim the game is getting too fast, and this often means that it is becoming serve dominated and rallies are rare. Is this true? Has the game become faster or more serve dominated over the years? This was a question the ITF asked, and the ITF Technical Commission attempted to answer, with a research project reviewing decades of Grand Slam data.

It was hypothesized that if the serve were becoming more dominant in the men's professional game, then the rate of tie-breaks would be increasing and this rate should depend on the surface being played on. It was also hypothesized that none of these tie-break correlations would appear in the women's game, where the serve is not as important. The four Grand Slam tournaments were chosen for analysis since they have four different court surfaces, have both men and women competing, and probably have the least turnover of players from tournament to tournament during a given year.

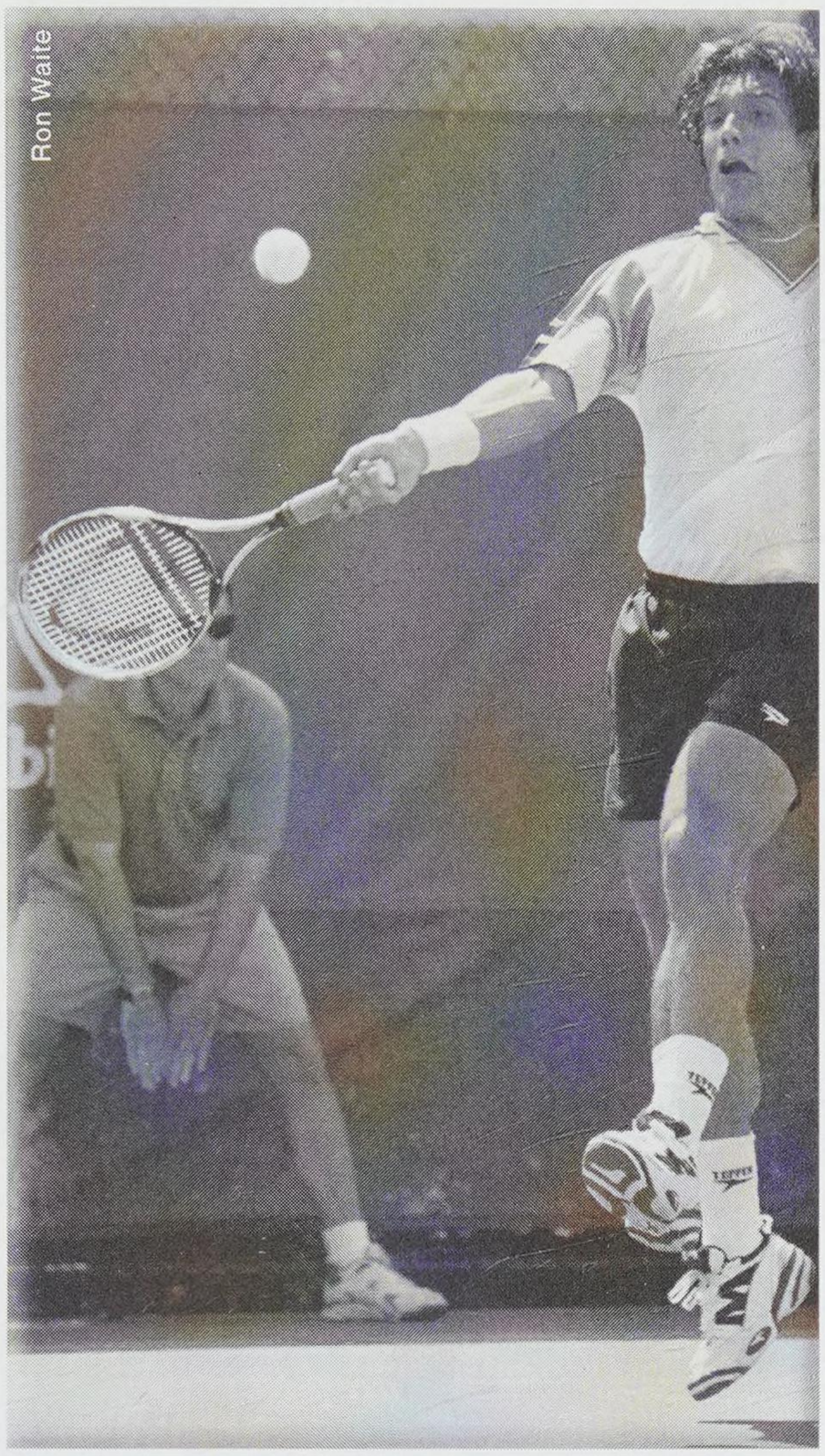
The results of the analysis of the past 20 years of Grand Slam events (over 90,000 sets) showed a strong correlation of rate of tie-breaks to perceived court surface speed for the men and no correlation for the women. Among individual men playing on the grass at Wimbledon (a very fast court surface), there was a strong correlation between tie-breaks played and both average first serve speed and maximum first serve speed. There was also a slow but steady increase in the rate of tie-breaks over time at all four Grand Slam tournaments for the men, but no such trend for the women.

This clearly shows that the men's game is getting faster and becoming more serve dominated. Is this a problem? That is a matter of opinion. If this trend continues for 10 or 20 more years, will it harm the game of tennis and will spectators begin to lose interest? There is some chance of this happening. Will people want to see the serve and volley game at Wimbledon become a serve ONLY game? Tie-breaks are exciting, but if more than half of all the sets played end up in a tie-break because no one ever has their serve broken, is that good, exciting tennis? Having done the research, the ITF Technical Commission had to see what it could do to control the speed of the game, if it became necessary in the future.

A number of possible means to slow the game down were carefully analyzed. Many of these ideas, when subjected to scientific scrutiny, were discovered to not be effective. Other means (such as changing the dimensions of the tennis court) were considered too expensive, since there are over 100,000 tennis courts in the world. Many possible solutions to the perceived problem would have adversely impacted the recreational game of tennis (the ITF wants one set of rules for the game – both professional and recreational). The solution adopt-

ed after several years of testing and analysis was to allow (not mandate) the use of a larger (6% in diameter) tennis ball. The use of this larger ball will slow down the game and reduce the serve dominance, and at the same time probably improve the quality of the tennis played by the recreational player. In addition, due to its increased cross-section (12%) the larger ball should play better at higher altitude than a standard size ball.

The ITF at its 2001 annual general meeting, approved the use of the larger ball (called a Type 3 ball) for use on any surface, in any tournament or match, recreational or professional. The "old" standard ball (called a Type 2 ball) is still allowed to be used in any tennis match as well.



Hernan Gumy, Argentina

18

Impact Location

Where To Hit on the Racquet

By Howard Brody

Where on the Racquet Head Should You Hit the Ball?

Most players have no idea as to exactly where on their racquet head they hit the ball. And, if they were interested, they would probably have no way of determining the ball impact location, let alone modifying it. They might have an idea of where they hit the ball ON AVERAGE from the wear on their strings or where the yellow ball fuzz is caught on the strings, but not the location of a specific single shot. Is the question of where the ball strikes the strings only of academic interest, or can it be of some practical use to a tennis player?

Ask a player where they aim to strike the ball, and they will often say "the sweetspot – wherever that is." Some players aim to hit the ball where it feels good, while other players aim for the center of the head because that gives them the largest margin of error for hitting a clean shot. Wouldn't it make sense for racquet designers to have the sweetspot of their racquet in the center of the head? Then many players would be satisfied and part of the problem would be solved. But not really!

There are three distinct sweetspots on a racquet, where it "feels good" when you hit the ball there. They are the ball impact location that produces the least shock or jar to the hand (called the center of percussion or COP),

the ball impact location which results in the least racquet frame vibration (called the node), and the ball impact location where the player gets the maximum "power" on the shot. The location of minimum shock, or jar, and the location that minimizes vibrations are two obvious places where it feels good. The maximum power location is the place where the ball zings off the racquet with the least effort on the player's part, so it also feels good.

This maximum power point may also tend to produce shots that also maximize control and minimize errors, so it may be the sweetest of the sweetspots. This "power" spot is the location that a player should aim for. Unfortunately, you can't paint a spot on the strings at that location because it is not a fixed point since it depends on the racquet swing. On the other hand, the location of the node and COP are fixed by the racquet parameters (length, weight distribution, etc.), and with modern, oversize racquets, they are usually near the center of the head. With the old, small headed wood racquets, these two sweetspots were near the racquet throat. When Howard Head (who invented the Prince racquet) enlarged the size of the racquet head, the center of the head came out close to the location of these two sweetspots, not by moving their position, but by moving the location of the center of the strung area. This was one of the reasons for the immediate popularity of the oversize racquet. The maximum power point is determined by those same racquet parameters (length, weight distribution, etc.), plus the type of shot that is being hit, the incoming ball speed, and the peculiarities of an individual's stroke. It can be anywhere from the throat area to near the tip, depending on the details of the stroke. That's why you cannot nail down its location and paint a spot on the strings there.

If you add weight to your racquet by placing lead tape near the tip, then you will move all three sweetspots up, away from the throat, and closer to the tip. Placing extra weight at the 3 and 9 o'clock locations of the head will probably move the power point up a bit, but will not markedly change the location of the node or COP. (It will give better stability against twisting for off-axis impacts.)

In one well known study[1], where the ball impact location was measured for the groundstroke of a number of good players, it was reported that the node location was struck rather than the COP location by most of the players tested. However, the study was faulty in that it did not determine the location of the power point, so it might have been a preference for that point over the COP point that was being observed.

As a general rule, the maximum power point moves away from the hand as the incoming ball speed decreases relative to the racquet head speed or as the stroke becomes more "wristy" and the head is moving much faster than the throat. If your opponent hits a hard shot, you want to hit the ball on your racquet a bit closer to your hand (see Note 1). The same holds true with respect to court speed. You will probably do better to hit the ball closer to your hand on grass courts and further away on clay courts. The limiting case is the serve, where the ball speed is zero and the racquet snap gives you effectively a very wristy stroke. For a flat serve, you want to strike the ball well above the center of the head of the racquet to get the maximum ball speed. As a bonus, this extra height at which the ball is struck allows you to get in more serves.

With a modern racquet, groundstrokes should be hit not too far from the center of the head. It is even possible to have a slightly different optimum ball

impact location on the strings for forehand and backhand, depending on the mechanics of your stroke. With the classic groundstroke that was seen 40 or so years ago, when used with the wood racquets that were popular then, the ideal impact location for maximum power was below the center of the (small) head, near the throat. With today's modern lightweight racquets and strokes using a more flexible wrist, the power point has moved up somewhat and its location is now nearer the location of the other sweetspots.

That still leaves the problem of knowing exactly where on the strings the player actually hits the ball. For the serve, that is an easy experiment to do. Take the ball, chalk it (with school blackboard chalk), and then serve that ball. There will be a clear pattern of chalk residue left on the strings showing exactly where the ball was struck. On a flat serve, if that mark is not between the center of the head and the tip, you are probably not serving to your full potential. Groundstrokes are more difficult to assess, since chalking the ball may not work too well. You might inspect your strings for wear, but this will only produce meaningful information if you are very consistent in your stroking and you get a lot of hits out of a set of strings. Another method is to coat or paint the strings with something and see where the wear in the coating occurs. A final method is to use a video camera (with a fast shutter) and watch a number of shots taken from behind or in front of the player.

Transverse Direction

Up to now in this chapter, only locations along the long axis have been discussed. Is there a problem in the transverse direction – away from the long axis? For a flat shot, the closer the ball impacts to the axis, the better the shot. Then there will be no twisting or turning of the racquet leading to a ball exiting at an unwelcome angle. There will be no loss of power, since off axis impacts degrade the rebounding ball speed. But this is all for flat shots.

What about spin (topspin, backspin, and slice) shots? When you hit a shot with spin on it, the ball does not enter and leave in a direction normal to the string plane. The ball impacts at a particular point, then slides or rolls a bit, and comes off of the strings at a different location. If the ball initially impacts close to the racquet axis, then rolls or slides for a distance, the ball may strike or come close to striking the frame before fully exiting from the strings. This would greatly reduce your margin of error for miss-hitting a spin shot.

To try to increase the margin of error, the ball should initially impact off-center, roll or slide across the racquet axis, then exit off-center the other way. This will allow you to hit harder shots with more spin while reducing the chances of the ball hitting the frame.

Conclusion

How can all of this help you? If you are like most players, you just hit the ball, and over the years you have developed a hand-eye pattern that probably has the ball impacting close to the location that this chapter is recommending. But you can do somewhat better. The next time someone hits a particularly hard shot at you, make a conscious effort to strike your return just a little closer to your hand, toward the throat of your racquet. If the shot coming toward you is somewhat slower than average and you want to hit your return hard, then try to meet the ball a little bit further out on the strings, in the direction of the

tip. Both of these suggestions will tend to optimize your racquet's performance over the performance you will get if you are hitting the ball at the same location on the strings every time you swing. On your serve, make sure that you are hitting the ball above the center of the head.

Notes

1. It is well known in baseball, that when the pitch is a fastball across the outside of the plate, you should "go with the pitch" and hit to the opposite field rather than try to pull the ball down the line. This is because batters do not have the luxury (or time) to reposition themselves and turn an outside pitch into one closer to their body. Tennis players can move and have the option of hitting the ball on the racquet closer or farther away from the throat.

References

1. Impact probability distribution, sweet spot, and the concept of an effective power region in tennis racquets. Herbert Hatze Journal of Applied Biomechanics, 1994, 10, pgs. 43-50

Steffi Graf, Germany

19

Improving Your Serve

Physics Principles for Getting the Ball In

By Howard Brody

Introduction

Using a knowledge of the laws of physics, a player can improve the serve percentage (get a larger fraction of the serves to go in) by changing parameters so as to increase the vertical angular acceptance of the shot. One obvious way to do this is to reduce the speed of the serve. However, a slower serve may be counter-productive, in that it could lead to a net loss of more points than it gains, depending on the opponent. If it is desired to maintain a high speed serve, detailed examination of ball trajectories and an analysis of racquet dynamics show how to get more serves to go in. There will be an improvement in serve percentage if the ball is hit when it is higher, which is why taller players, being able to reach higher, have a distinct advantage when serving. There also will be an improvement in the vertical angular acceptance (or "window") when the ball is hit with the racquet moving upward in order to produce topspin. A third method of increasing the serve percentage is to toss the ball up higher and hit it as it is falling downward. It is shown how each of these suggestions can increase the vertical angular acceptance for the serve being good, which, in turn, should lead to an increase in serving percentage.

Many players love to hit their first serve as hard as they can, but unfortunately, it doesn't go in very often when they really smash the ball. The reason this happens may be caused by several separate effects; the biomechanics of their swing, the physics of the ball trajectory, and/or the racquet they may be using.

When players try to hit the ball harder, they tend to lose some control of their racquet and end up spraying their shots around. The result is that the ball hits the net or goes long more often than it goes in. To remedy this problem requires athletic skill, correct stroke mechanics, and practice. Lots of practice! The other causes of a poor serving percentage, the physics of the ball trajectory and not having optimum equipment, are the subject of this chapter. Using the information developed here, it might be possible to have a better serve without additional hours of practice.

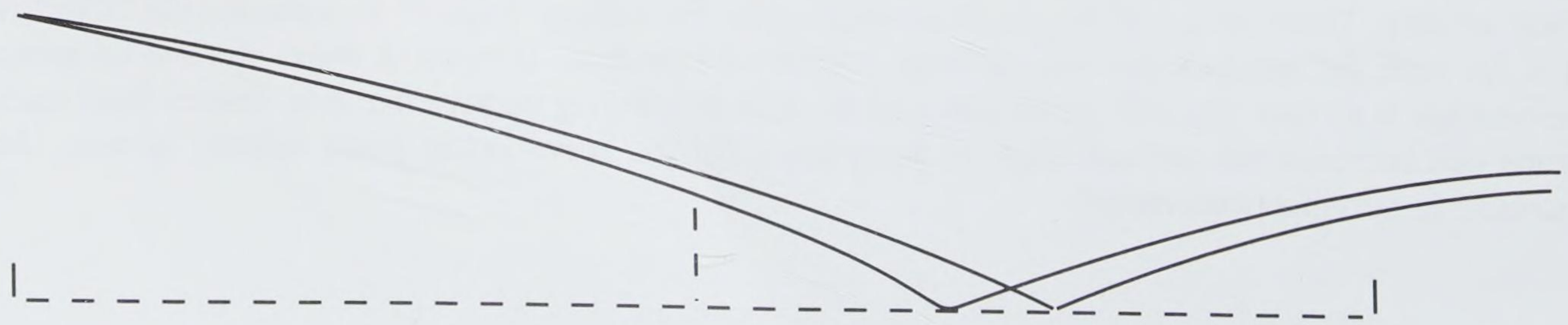
Vertical Angular Acceptance (Window)

On the serve, once the ball impact height and location, as well as the ball speed and spin have been fixed, there is a minimum angle the ball must have as it leaves the racquet just to clear the net. Keeping all the conditions fixed, there is a maximum angle that the ball can have and not bounce beyond the service line (Figure 19.1). The difference between the minimum and maximum angle is the vertical angle of acceptance for that serve to be good. At the maximum angle (corresponding to bouncing on the service line), the ball clears the net by a certain margin. This margin of clearance over the net can be considered a window of acceptance that the ball must be hit through if the serve is to be good. If a ball reaches the net below the window, the net is hit. If the ball crosses the net above the window, it will bounce beyond the service line, and be a fault.

If this window (or clearance over the net) is large, the serve is more likely to go in, compared to a serve that must pass through a small window. This is particularly true when the variation in the initial angle from one serve to another is large, due to biomechanics of the stroke. The variation in vertical angle of the ball off of the racquet on the serve can be reduced by improving the stroke mechanics, taking lessons, practicing, and not being too ambitious. However, there is a limit for each player beyond which they cannot go, regardless of how much they practice. At this point, the only way to increase the serve percentage is to increase the window.

Figure 19.1

Trajectories for a ball that just clears the net and a ball that lands on the service line.



The size of the window, or clearance at the net through which the ball must pass, depends on the ball speed off of the racquet, the height at which the ball was hit (impact height), the spin of the ball, and the server's location on the court. It is clearly advantageous to arrange these parameters so as to maximize the window which will allow more serves to go in, if at the same time, a serve can be hit that will win points (or prevent your opponent from winning points). We will examine each of these parameters and see how to modify the serve mechanics, equipment, or strategy in order to improve effectiveness.

Ball Speed

A computer program has been written that calculates the vertical clearance window of a serve as a function of each parameter. First, we will examine what happens as the ball speed is varied. The results of this calculation are shown in Figure 19.2. From these data, it is apparent that the harder the ball is hit (increased ball speed off of the racquet), the smaller the window through which the ball must pass in order to be a good serve. The physical reason for this result is that the force of gravity is pulling the ball down into the service box, and on slower shots, gravity can more readily change the ball's direction and get it to bounce in the box.

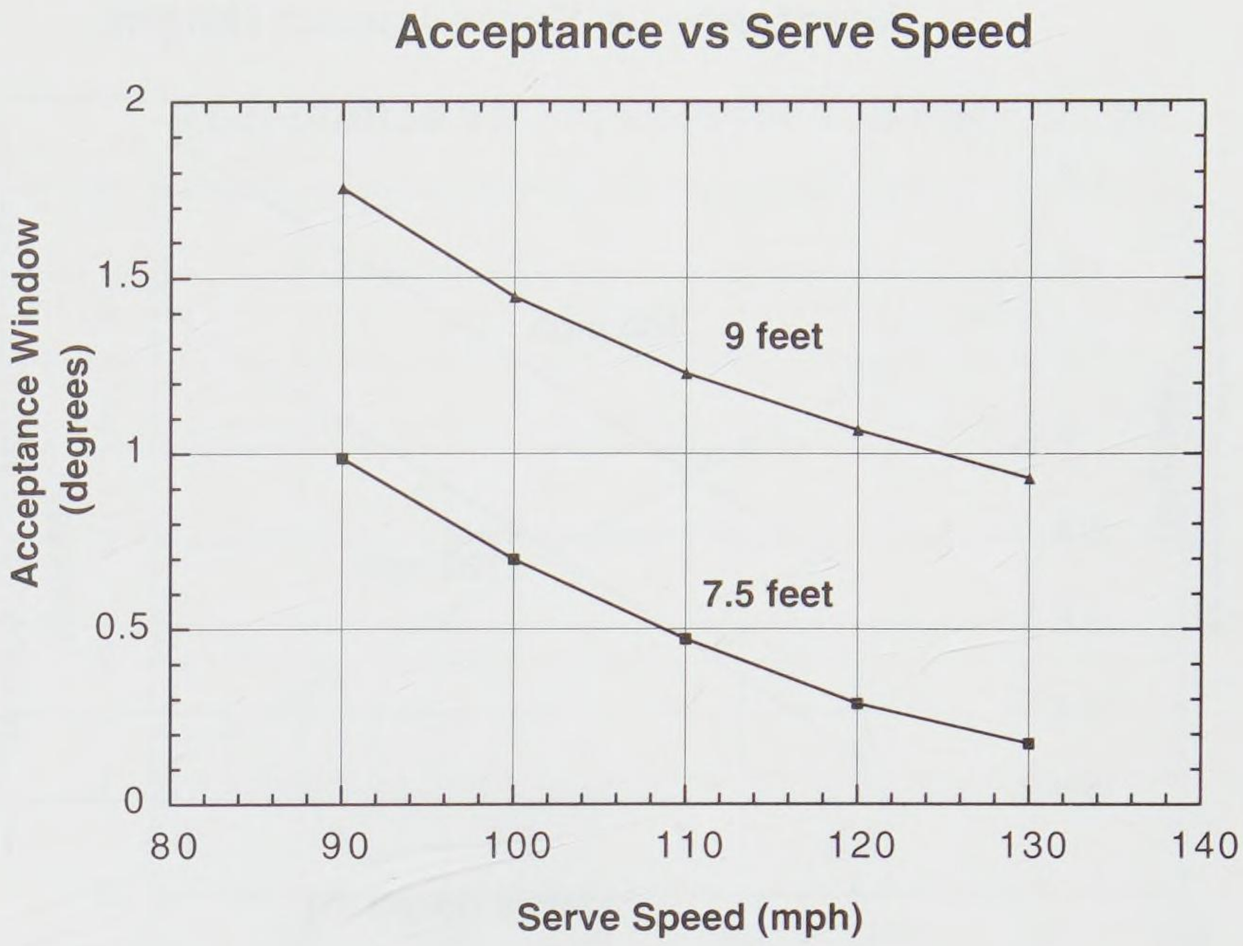


Figure 19.2

Angular acceptance versus ball speed for serves hit at 7.5 and 9 feet above the ground (impact height).

Many players, finding that their hard serve is not going in a reasonable fraction of the time, switch to a slower serve in hope of raising their serving percentage. Just reducing the ball speed from 100 mph (with a clearance over the net of 7.5 inches) to 90 mph (with a window of 9.8 inches) will increase the chances of the serve going in by 30%. It is obvious that more serves will go in through a 9.8 inch window than a 7.5 inch window. If only 50% of the first serves go in at 100 mph, 65% of the first serves will go in at 90 mph. This corresponds to about 5 EXTRA first serves going in for each set of play.

However, there is a penalty for this action. All the first serves are now at 90 mph, and the result might be a loss of more points per set than when the 100 mph serve is used. Even if the reduction in ball speed leads to a net gain in points won, many players would be opposed to the idea, since they have pride in how hard they can hit that first serve, even if it rarely goes in. Therefore, we will examine another parameter.

Impact Height

The ball's height, when struck by the racquet (impact height) also influences the size of the clearance over the net. Figure 19.3 shows the result of a computer calculation of the window size versus the impact height. (Note, this is the ball's height when struck, not how high you throw the ball on the service toss.) The graph clearly shows that the window opens up as the impact height

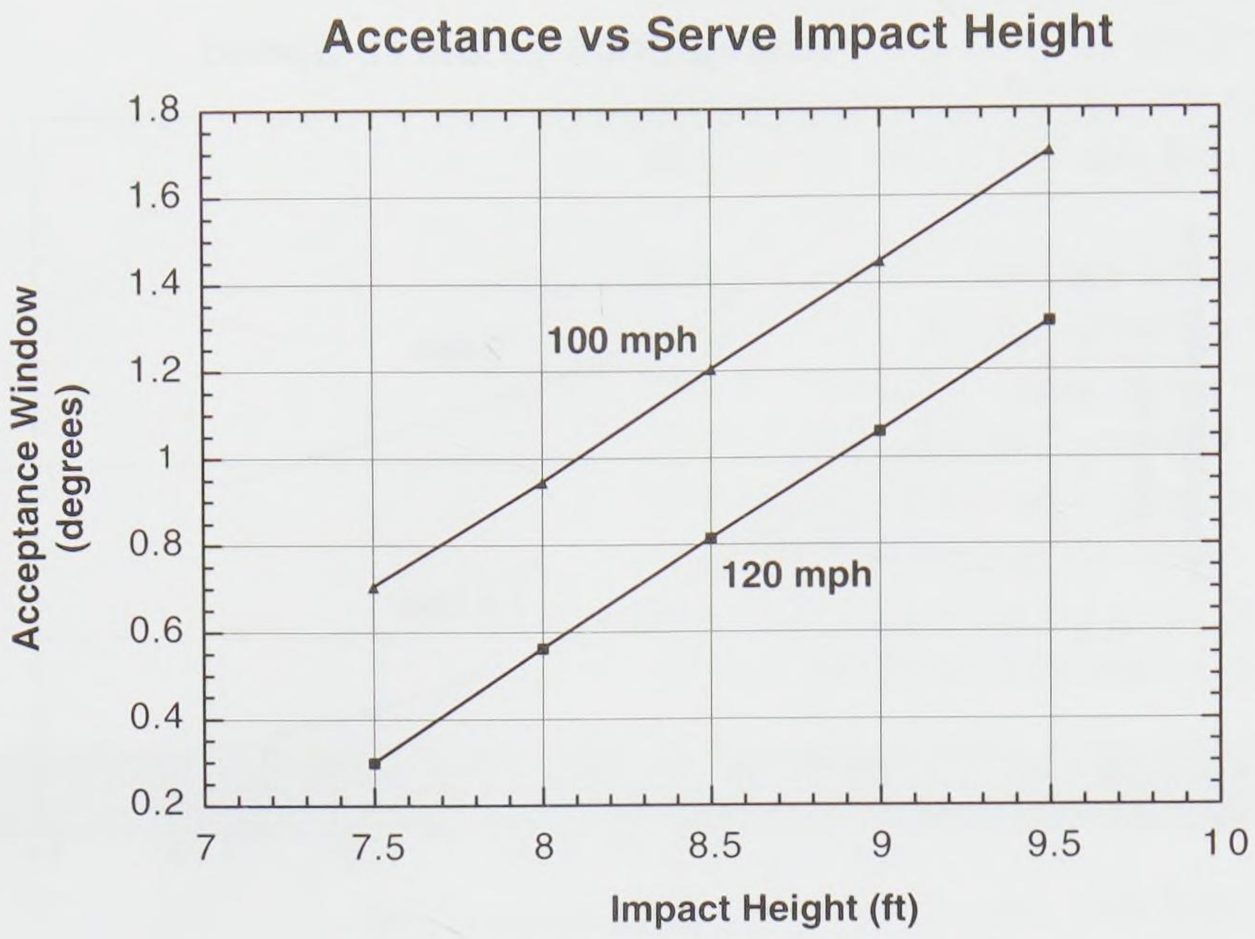


Figure 19.3

Angular window versus impact height for 100 and 120 mph serves.

increases. All players know this fact, and make use of it when they serve overhand rather than underhand. These data also show the severe disadvantage a short player has when attempting to get hard serves to land in the court. If we look at the graph for the 100 mph serve, originally hit from a height of 8 feet, and if it is desired to continue serving at 100 mph, to get the 30% extra serves going in will require that the ball impact point be 6 inches higher. Is that possible?

A measurement of serve impact heights was made using members of the University of Pennsylvania tennis teams. Each player was given a racquet with a piece of chalk taped to the head. They were asked to stand in front of a wall and then to mark the wall with their arm extended upward as if they were serving. Several players were then asked to repeat the measurement, but reaching as high as they could (on their toes, fully extended, etc.). The new marks on the wall were 5 to 6 inches above the older, flat-footed marks. Since many players actually leave the ground when they serve, this 5 to 6 inch increase in impact height is the minimum that one might expect from a good server over a flat-footed server. As a general rule, a good server impacts the ball at a height above the ground of approximately 1.5 times that players height.

If a player is hitting the ball flat-footed and also not stretching and extending fully upward, they may be taking as much as a 30% penalty in first service percentage. The first serve percentage can also be increased by hitting the ball slightly higher on the racquet head, holding the racquet a bit lower down on the grip, or using a longer racquet (if at the same time the impact height is increased by that amount). Every extra inch of impact height corresponds to about a 5% increase in the first serve percentage for the average player hitting a 100 mph serve, so even an increase of a single inch will be significant in the course of an entire match.

Topspin

Some players may already be hitting the ball at as high an impact location as is physically possible. What can they do now to get more serves to go in? Hit with topspin. Figure 19.4 shows how topspin opens the acceptance window and allows more serves in play. That is great advice for some, but for many players, trying to consistently hit serves with topspin is more difficult than getting that hard first serve to go in. In addition, because the biomechanics of the topspin serve usually has the racquet moving upward (as well as forward) at impact, the ball must be contacted well below the peak of the swing. Unless a fair amount of topspin is put on the shot, some of the extra window opening added by topspin may be lost to the lowering of impact point, and in addition, for some players, the loss of control may negate any other advantages.

Higher Ball Toss

But there is a way of getting some topspin on serves and opening up the acceptance window — almost for free. Throw the ball up higher on the toss! If the serve is hit perfectly flat, but the ball is falling when struck, the resultant shot will acquire some topspin. The amount of topspin is not large when compared to the big kick the top pros give to their serve, but every bit helps. (And the more marginal the serve is now, the more it will help.) The faster the ball is falling when struck, the more spin it gets. Therefore, the higher the toss, the more spin and the bigger the window will be.

Figure 19.5 shows how the acceptance window is increased as topspin is added to the serve because the toss is higher, but keeping everything else exactly the same. (The timing probably will have to change a bit to compensate for the higher toss.) Note that a simple toss of only 6 inches above the impact point will increase the number of serves going in by 12% over a toss to exactly the impact point. The next time you see videos of Steffi Graf and her high toss serve, think about this.

For years the advice has been, toss the ball to exactly the correct hitting height, then crush it while it was essentially at rest. Now the advice is to throw the ball above the eventual contact point and hit it on its way down. Which is better? What are the disadvantages and advantages of each recommendation?

It is clear that it is easier to hit a stationary ball than a moving ball, but on a higher ball toss, is the ball moving sufficiently fast so that this is a problem? Probably not, particularly if the toss is only 6 inches to a foot or so above the eventual impact point. It all depends on how good is a player's timing. You can attempt the following experiment and find out.

Try hitting a forehand with the frame of the racquet, not the strings, by turning the racquet 90 degrees (so the string plane is parallel to the ground). Throw the ball up

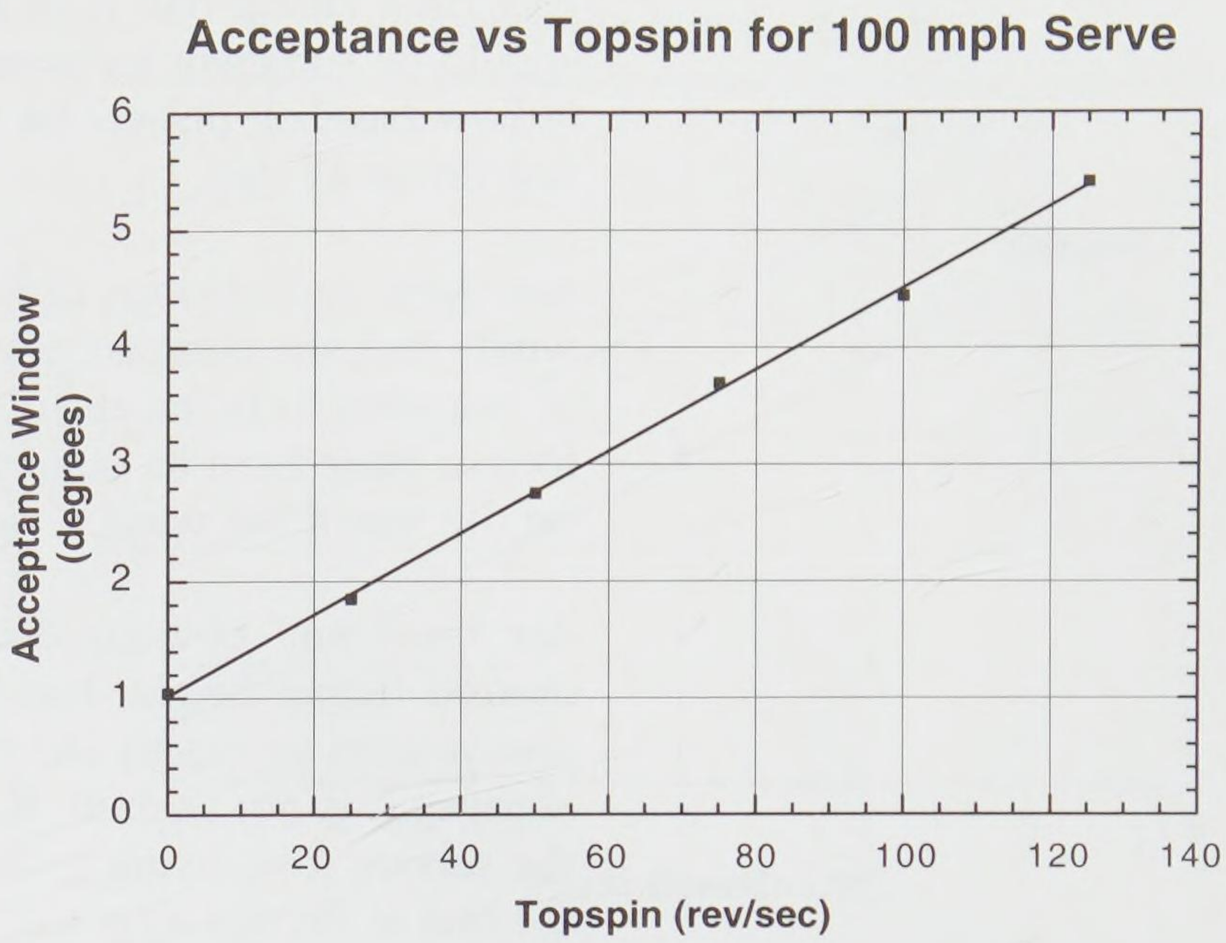


Figure 19.4

Angular window of acceptance for a serve versus topspin on ball for a 100 mph serve hit at 8.5 feet above the ground.

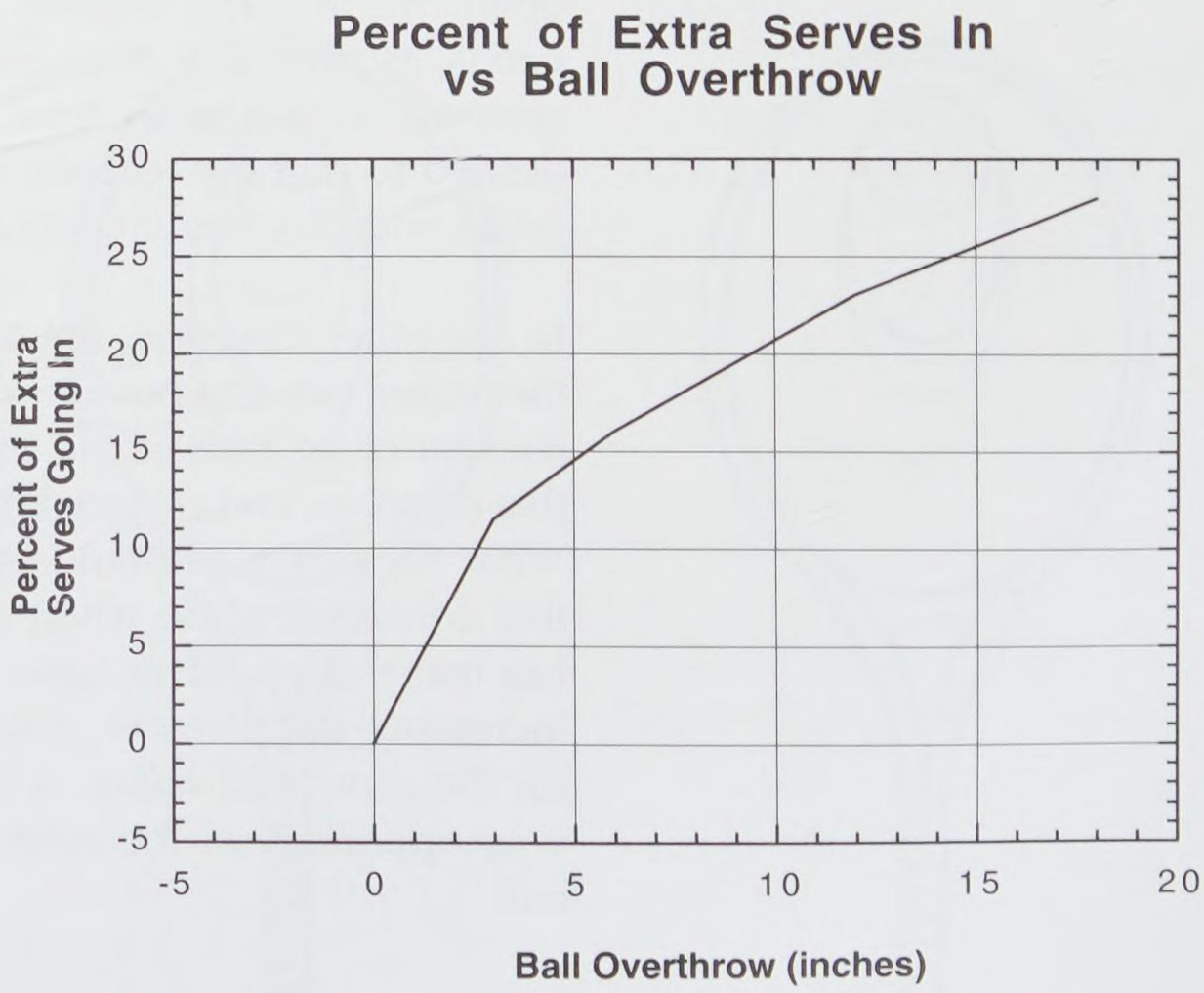


Figure 19.5

Increase in the acceptance window versus ball toss above impact point for 90 mph serves hit 8 feet above ground (5' 4" server).

a foot or so over the anticipated hitting point and see if you can hit the ball on the way down with the edge of the frame. If you can, then your hitting accuracy on a falling ball is as good as the racquet is thick (+/- 0.5 inch), which should be sufficient for most servers. It is interesting to note that this is exactly how baseball players hit "fungos," and an experienced batter can hit fungo line drives all day.

Are there any obvious disadvantages to a high ball toss on the serve? On a windy day, the compact toss is a great benefit, particularly if the wind is gusty as opposed to being steady. A brisk breeze can easily push a high toss by a foot or more from its desired hitting point. It is possible to compensate for this on the toss if the wind is steady, but swirls cannot be anticipated.

Are there any obvious disadvantageous to a toss that reaches exactly the desired hitting height, but no higher? If the toss is perfectly grooved and it always goes to exactly the correct height, it is a fine technique if the additional topspin is not desired. If, however, there is some variation in the height of the service toss, trying to hit at the peak of the toss can lead to problems. If the ball is thrown a bit too high, it can be compensated for with timing (wait for it to come down) or can be hit when it is higher. If the ball is not thrown up quite high enough, the impact point will be too low, which will close down the window, reducing the chances of the serve going in, and possibly reducing the ball speed.

Location on Court

The next parameter to examine is the location of the server on the court. The server can stand as far behind the baseline as desired and can move away from the center line to as far as the sideline. The displacement toward the sideline opens the window slightly on wide serves, and in this respect it is advantageous. However, it does give the ball a longer flight path which allows the receiver more time to react. The result is probably no net benefit unless it is desired to pull the receiver wide, or in doubles where the server may want to begin wide for better court coverage.

In the other direction, the answer is clear. The closer the server is to the net, the larger the acceptance window, but it is not very essential to be within a fraction of an inch of the baseline and run the risk of a foot fault. An inch in this direction makes very little difference in the probability of a serve going in. What the server can (and should) do is to toss the ball well into the court so that contact is made when the ball is a foot or more inside the baseline. This has many direct advantages. It will increase the number of serves that go in by increasing the window opening. It will allow the player to get to net quicker for the serve and volley. It will allow you to hit a serve slightly wider, pulling your opponent off the court. It will give the opponent less time to react to the ball.

A recent article in Tennis Magazine[1] noted that Pete Sampras strikes the ball when it is two feet inside the baseline. Figure 19.6 shows the increase in window at the net as the striking point moves inside the baseline for both a Sampras-like serve and the serve of someone such as 5 foot 2 inch tall Amanda Coetzer, who does not hit the ball quite as hard. Note that it is advantageous to both of them to lean into the court when serving. In addition, the reduction in flight path of 2 feet gives the receiver less time to react or get to the ball. A

110 mph serve, hit from 2 feet inside the baseline has the same flight time to the opposite baseline as a 113 mph serve hit from directly above the server's baseline. This is almost a 3% increase in effective serve speed, and a larger percentage of the serves go in at the same time. It is a win-win situation.

It is also of interest to note that the size of the acceptance window does not change very much between serves hit down the middle (aimed at the TEE) and serves that bounce near the corner of the service box.

Racquet Parameters

For the player who is presently reaching up as high as possible and as far into the court as possible, it is clear that a longer racquet or a higher impact point on the racquet would improve the serving percentage. This will only be advantageous if, at the same time, the ball speed of the serve is not decreased by these changes. This may not have been the case when the classic wood racquets of 30 years ago were used. Racquet technology can now produce a frame that will give maximum power (ball speed) on the serve for impacts close to the tip and allow a longer racquet to be used with none of the disadvantages that an extended old wood racquet would have had.

To move the maximum rebound ball speed impact location higher in the head of the newer frames, manufacturers have done a number of things. They have made the frame much stiffer (with a thicker cross-section of graphite reinforced composite material), and they have moved the center of mass up higher on the head (by reducing the weight in the handle). In addition, the head shape has been modified to make the racquet wider near the tip and a fan-like string pattern is being used. It is now possible to get maximum ball speed on a serve when the impact point is very close to the top of the head. Figure 19.7 illustrates this. This advance in racquet technology can provide both maximum ball speed and the maximum window of acceptance at the same impact location, high on the head.

Making the frame longer, which allows the player to hit the ball when it is higher, will

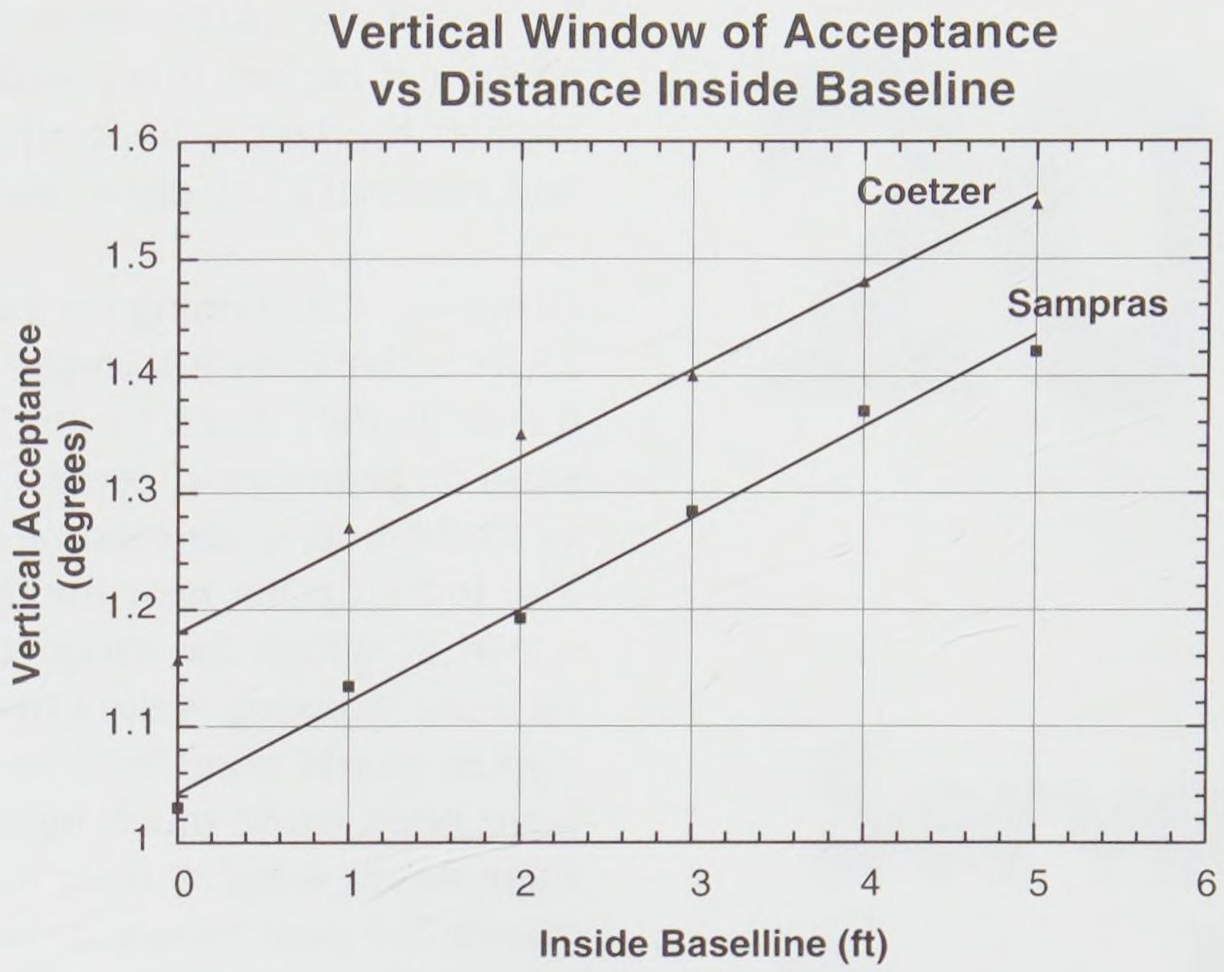


Figure 19.6

Angular window of acceptance for a serve versus impact location distance inside the baseline that ball is hit. Upper curve is at 90 miles/hour and 8 foot high impact (Amanda Coetzer) and lower curve is 120 miles/hour and 9 foot high impact (Pete Sampras).

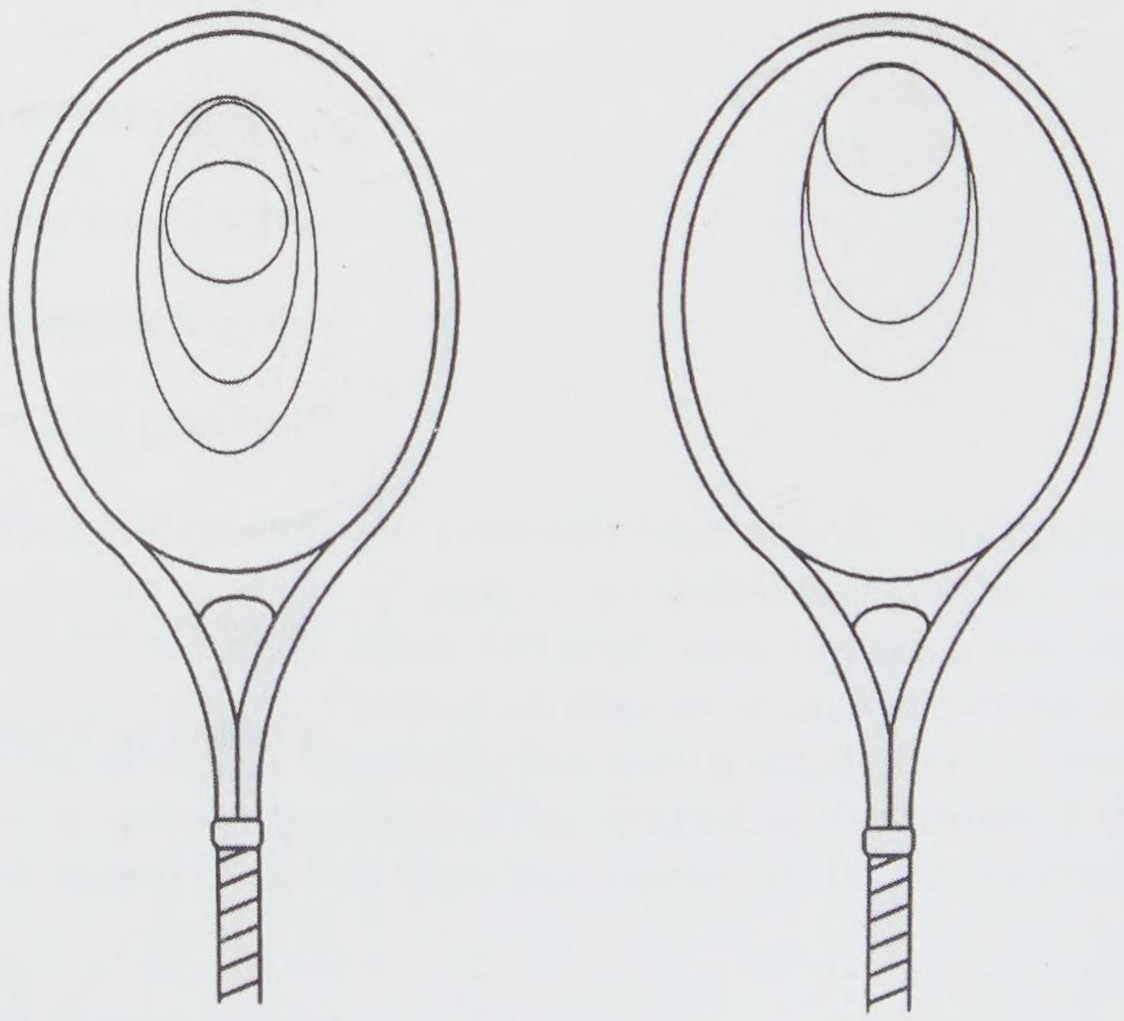


Figure 19.7

Computer generated power contours of the original Prince racquet and a modern, very stiff racquet for the serve.

clearly produce a racquet that will allow more serves to go in. This will only be true if the player takes advantage of the extra length, by hitting the ball when it is higher. It is possible to determine exactly where on the head of the racquet the ball is impacting the strings, by chalking the ball before the toss and observing the chalk mark on the strings after impact.

However, maintaining (or bettering) the playing characteristics of the standard 27 inch frame with a longer racquet was a challenge for the racquet manufacturers. If they were to just scale up the standard, neutral balance (center of mass at geometrical center) racquets by another few inches, or add two inches to the butt of an existing racquet, the result would be a poor serving frame. The impact point near the tip would be farther from the balance point and the speed at which the racquet could be swung might be lower because of the increase in swing weight (moment of inertia about the butt end). Both of these effects would contribute to slower serves. Many of the new, longer racquets have been made much lighter, their balance point moved toward the tip, and their swing weights have been kept similar to the older 27 inch frames. This allows the new frames to be swung with the same angular velocity as the old frames, but the longer lever arm can result in higher ball speeds.

Summary

Table 19.1 gives an estimate of how much the vertical acceptance angle (window) has been increased by each of the suggestions made in this article. These results are based on a 100 mph serve hit initially at a height of 8 feet above the ground by a six-foot tall server, hitting flat-footed, and using a standard (27 inch) frame with an impact point at the center of the racquet head.

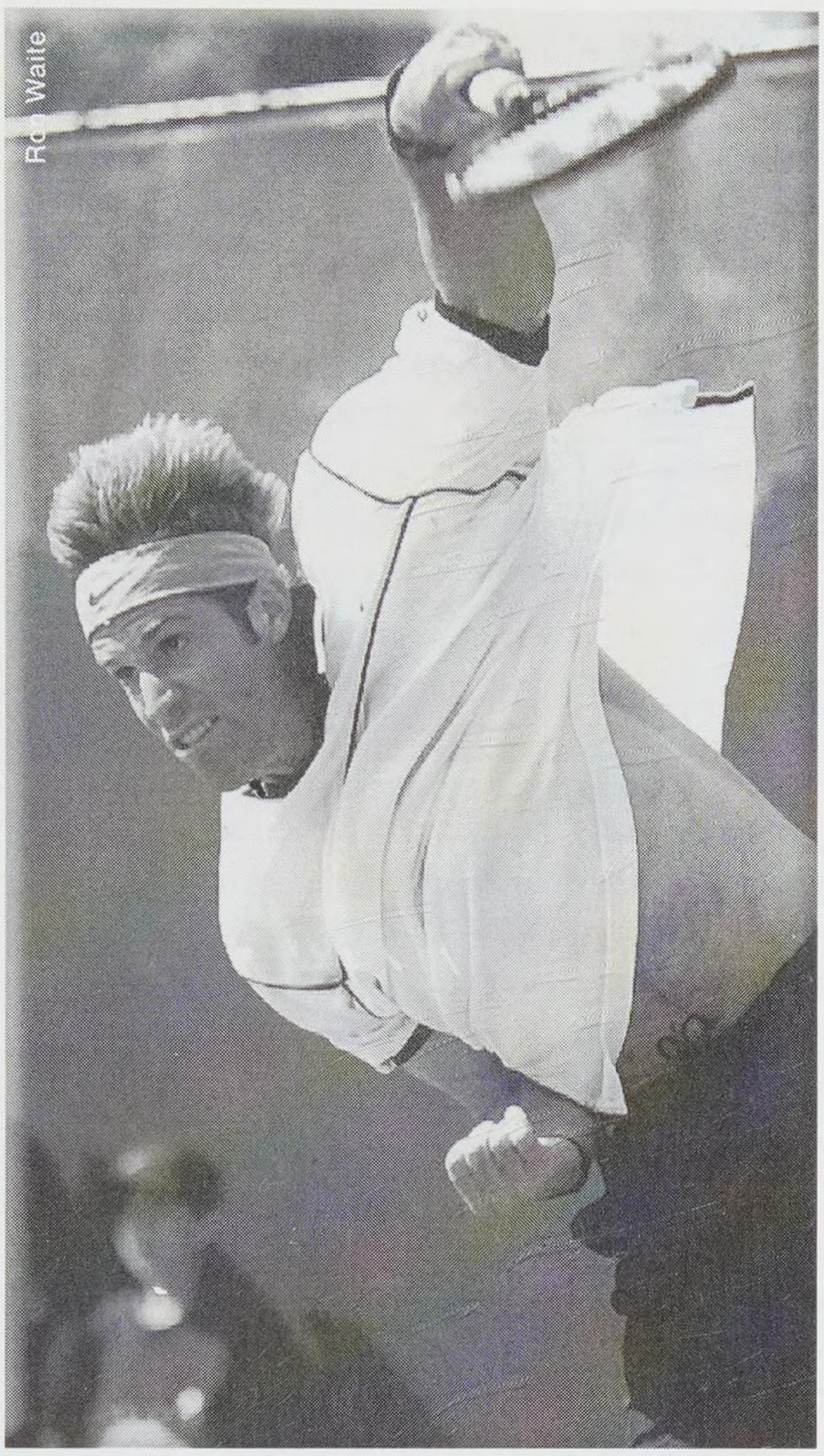
Table 19.1

Increase in serve window for various changes

2 inch longer racquet	10%
Hitting 2 inches higher on head	10%
Stretching up 6 inches	30%
Spin on ball due to higher toss	20%
Reaching into court	10%
Jumping up 4 inches	20%

References

1.) The Sampras Serve, Tennis, pg 45 February 1994
2.) How Would a Physicist Design a Tennis Racquet, H. Brody Physics Today pg 26-31 March 1995



Gred Rusedski, Great Britain

20

Serve Speed

Theory, Motion, and Strategy

By Rod Cross and Howard Brody

Serve Speed vs Racquet Speed

In order to serve a ball at 100 mph, a player must swing the racquet at relatively high speed. Most people would probably guess that the racquet needs to be swung at about 60 mph or so depending on the power of the racquet. In fact, the tip of the racquet needs to be swung at about 100 mph, give or take a few mph depending on the weight and the weight distribution of the racquet. There is an important and surprising lesson in this result. That is, the power of the racquet and the power of the strings has only a small effect on serve speed. The serve speed is always about the same as the tip speed regardless of the racquet or the strings. A different racquet or a different string will change the serve speed by a few mph but almost all the power comes directly from the player's arm.

Figure 20.1 shows a satellite tournament player serving at 162 km/hr (100.7 mph). The action was captured on a digital video camera at 100 frames/sec or one frame every 10 ms (0.01 s). Digital video camera images can be transferred in real time to a computer for analysis, usually at 25 or 30 frames/sec. This is not fast enough to record a high speed serve and to measure everything of interest. The camera used to capture the images in Figure 20.1 was a JVC 9600 which records at 100 frames/sec for smooth slow motion playback. Not many video cameras have this feature. A shutter speed of 1/500 sec or smaller is necessary to avoid blurry images.

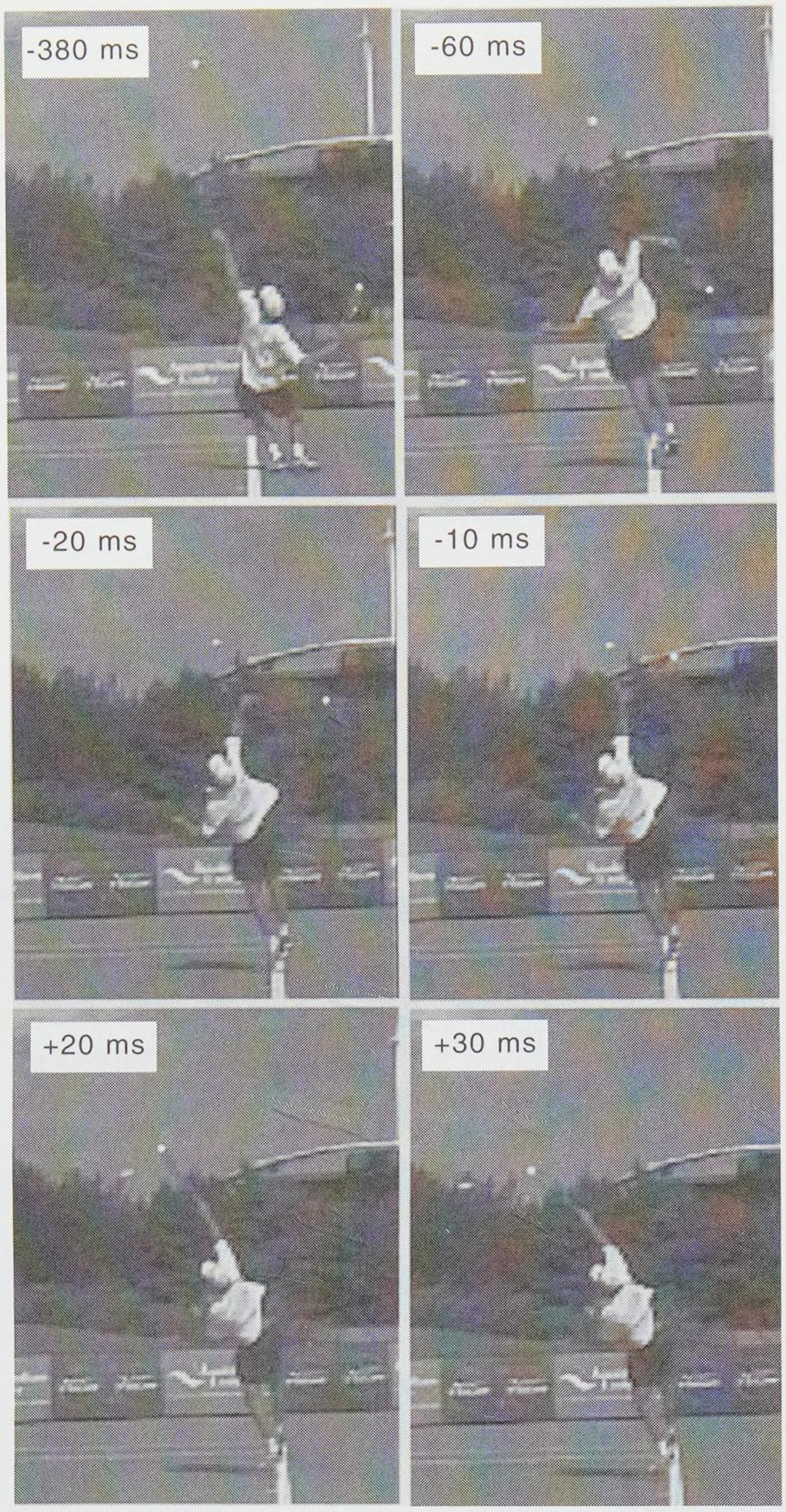
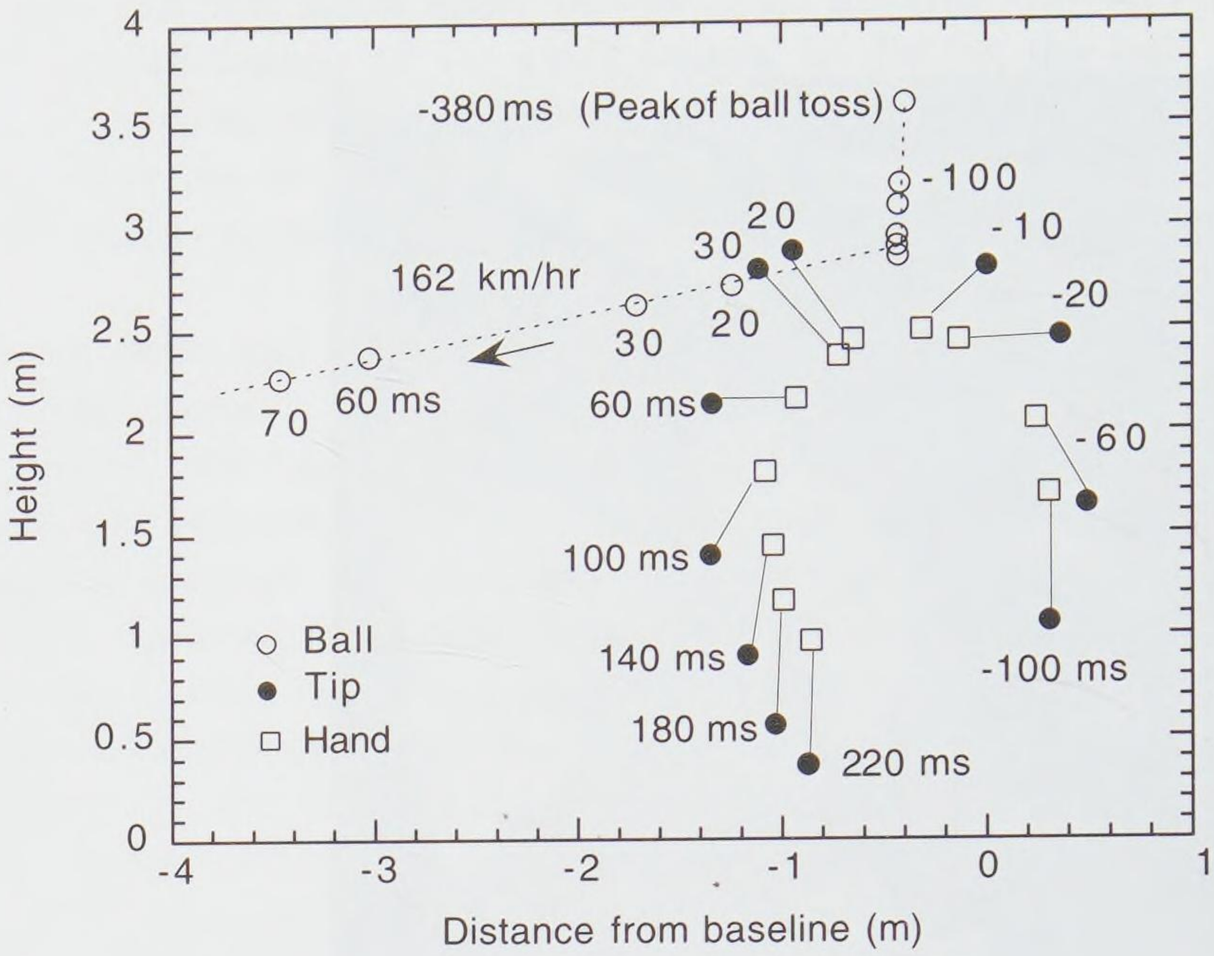


Figure 20.1



Ball speed = 45 m/s = 162 km/hr = 100 mph
Angular velocity of racquet at impact = 80 rad/s
Speed of tip at impact = 43 m/s
Speed of impact point (10 cm from tip) = 35 m/s
Hence ACOR = 0.29 at impact point

Figure 20.2

The picture quality in Figure 20.1 is not brilliant so a white dot has been added to show the tip of the racquet. The positions of the ball, the racquet tip and the player's hand are shown in Figure 20.2 at various times before and after impact. The racquet impacts the ball at time zero. The vertical and horizontal distances are measured from the center mark on the baseline. The distance scales on the video image were calibrated from the height of the player (6' 2"). This was checked against the drop height and speed of the ball during the ball toss, and both methods gave the same calibration. The impact location and time was determined accurately from the intersection of the ball trajectories before and after impact.

The racquet was almost vertical at impact, almost horizontal 20 ms before impact and was at about 45 degrees to the horizontal 10 ms before impact. The racquet therefore rotated by about 45 degrees each 10 ms prior to impact, at an average rotation speed of 4,500 degrees/sec = 80 radians/sec (1 radian = 57 degrees). The tip and the hand were both moving horizontally at impact, at speeds of 43 m/s and 12 m/s respectively, and the ball came off the strings at 45 m/s. The ball did not impact at the tip of the racquet. It impacted about 10 cm from the tip. The racquet speed at the impact point was 35 m/s (78 mph).

Serve Speed Formulas

The speed V_p at any point on the racquet can be calculated if one knows the rotation speed ω (in radians/sec) and the speed at the tip (V_t). The formula is $Vp = V_t - D\omega$ where D is the distance from the tip. In this case, $V_p = 43 - 0.1 \times 80 = 35$ m/s. The racquet slowed down immediately after impact. In Figure 20.1, the rac-

quet rotated by about 45 degrees each 10 ms before impact but it rotated only about 10 degrees each 10 ms immediately after impact. This means that energy was taken out of the racquet and given to the ball.

The serve speed V is given by $V = (1 + e_A)V_p$ where e_A is the apparent coefficient of restitution. e_A is typically about 0.4 in the middle of the strings and about 0.1 near the tip. In this case, we find that $1 + e_A = V/V_p = 45/35 = 1.29$ so $e_A = 0.29$. The best position to serve the ball is about 10 cm from the tip since the racquet travels faster there than at the center of the strings and since the server gains a height advantage by serving closer to the tip. e_A is larger at the center of the strings but V_p is lower, with the result that the serve speed varies only slightly (about 5%) with impact point. The impact point at which the serve speed is a maximum depends on the distribution of racquet weight but is typically about 10 cm from the tip.

Impact Duration

The time spent by the ball on the strings is only about 5 ms. The impact duration can be shortened from say 5 ms to 4.5 ms by increasing the string tension or by an increase in ball stiffness. The impact time is so short that one might think that a difference of 0.5 ms is not very significant. However, Figure 20.1 shows that the racquet travels a relatively long distance and rotates through a large angle during the last 20 ms. The racquet slows down during the impact but it will still travel a respectable distance during the impact. Changing the impact duration by 0.5 ms can therefore make a significant difference to the angle of the ball off the strings. To serve accurately at a different string tension, a player would need to need to change the actual impact time or change the ball toss slightly.

What if the player hits the ball 0.5 ms too early or 0.5 ms too late? Will the ball come off the strings at the wrong angle? If the racquet rotates at 80 rad/s then in 0.5 ms it will rotate through an angle of 80 x 0.0005 x 57 = 2.3 degrees. This is more than enough to change a good serve into a fault. In fact, the whole range of good serve angles is only about 2 or 3 degrees at 160 km/hr. If a ball served 5 degrees down from the horizontal lands on the service line then at 7 degrees the ball will only just clear the net. This seems to indicate that a player must hit the ball at exactly the right time, with less than 0.5 ms room for error. The problem is, no one can judge time that accurately. For example, if you try to hit an object at exactly the same time with your left hand and your right hand, then measurements show that you would be lucky to hit within 5 ms.

Fortunately, the serve problem is not one of timing but one of positioning. If you toss the ball in the right spot, then the racquet will hit the ball at that spot almost regardless of the exact timing. The ball in Figure 20.2 dropped from a maximum height of 3.6 m and it was dropping at 3.7 m/s when it was hit. If it was hit say 1 ms later, then it would have dropped an extra 3.7 mm. The angle at which the racquet strikes the ball depends mainly on the ball toss position rather than the exact timing. If the ball is hit 1 ms later than intended, the ball will be hit a few mm lower on the strings, but the angle will be the same if the ball toss and arm swing are the same.

Arm Rotation

During the last 20 ms prior to impact, the hand and the racquet both rotate at 80 rad/s. It might appear that this is all due to deliberate wrist rotation on the part of the player. However, every joint is involved, including the elbow, shoulder, hips,

knees and ankles. Each joint adds rotation speed to the previous one. For example, if the only body segment that rotated was the hand (about an axis through the wrist) then the rotation speed of the racquet would be only about 10 radians/sec. If the forearm is also rotating at say 40 radians/sec about an axis running along the upper arm then the combined rotation speed would be 40 + 10 = 50 radians/sec. The upper arm rotates about an axis through the shoulder, adding further rotation speed to the racquet.

Detailed studies of body segment rotation in tennis are described by Elliot et al. (1995) and by Bahamonde (2000). An interesting feature is that the sequence of rotations starts from the bottom and works its way to the top. For example, most of the hip and shoulder rotation occurs from -380 ms to -60 ms, most of the elbow rotation occurs from -60 ms to - 20 ms and most of the wrist rotation occurs from -20 ms to impact time. These features are all clearly evident in Figure 20.1. Similar results are found in other sports. For example in the javelin throw, the hip, shoulder, elbow and javelin reach their peak velocities in that order and all within about 100 ms of each other. This is described in biomechanics books as the kinetic link principle. A similar sequence of events is observed if one swings two or more rods joined end to end by pivot joints. If the rods are swung correctly, the last rod in the chain will rotate last and it will rotate the fastest.

Serve Speed Strategy

Many tennis players have the ability to hit their first serve hard, very hard, or really smash the ball — hitting as hard as they can. There is a small problem with hitting as hard as you can — the ball usually does not go in as often as you would like. This poor first serve percentage is due to two separate effects. As you swing harder you may lose some control over your racquet, and the higher the ball speed, the smaller is the "acceptance window" for a ball going in. However, when your really hard serve does goes in, you are more likely to win the point (Table 20.1).

What strategy should you adopt? Smash the ball as hard as you can? Play it safe and get your serve in as often as is possible? Hit the ball hard, but not as hard as you can? The answer is not obvious, and depends on a number of factors.

Table 20.1

Percent first serves in	Percent win if goes in	Total win percentage on first serves
50	90	45.00
55	85	46.75
60	80	48.00
65	75	48.75

Let us assume that the winning percentage on your first serve depends on how hard you serve and that the percent of serves going in also depends on how hard you serve. Let us take some typical examples.

On this basis, you might advocate the softer serve (which goes in more often) since it would have the highest winning percentage of points on the first serve. This would be absolutely the wrong strategy. Under these assumptions, you should hit your first serve as hard as you can and you will win more total points, because you also get a second serve. If you get in 95% of your second serves and you win 55% of the points when that second serve goes in, then (100 - first serve in) x .95 x 0.55 must be added to the first serve numbers. The result of this is shown in Table 20.2.

In this case, it pays to hit the first serve hard, as you will win a larger percentage of the points. These numbers may not hold for every player (if you

never get your first serve in when you really smash it, or if your second serve is so weak that you never win a point on it, then the other answer is obvious), but under the assumptions made in Tables 20.1 and 20.2, hitting the first serve really hard is the best strategy.

Table 20.2

Percent first serves in	Percent win if goes in	Win percentage on First serve	Win percentage on Second serve	Total
50	90	45.00	26.1	71.1
55	85	46.75	23.5	70.3
60	80	48.00	20.9	68.9
65	75	48.75	18.3	67.1

Table 20.3

Percent of first serves in	Winning percent when serving	Percent of games won	Chances of winning 6 games
50	71.1	91.5	58.9
55	70.3	90.4	54.7
60	68.9	88.7	48.7
65	67.1	86.2	41.1

This range of winning percentages seems very narrow (67.1 to 71.1), but it has some effect on the chances of winning a service game and a very big effect on the chances of not being broken during an entire set, as is shown in Table 20.3.

Case Study #1: Second Serve Speed

By Howard Brody

The final point of the Wimbledon Championship Match between Sampras and Agassi ended on a "second" serve ace. You are seeing more and more second serve aces, as well as serve speeds in the 110 to 120 mph range for the second delivery. Is this foolishness and bravado by the players or does it make real sense and win more points?

Let us examine Pete's serving statistics in the Championship Match. Sampras got in 60% of his first serves and he won 87.5% of the points when his first serve went in. Multiplying these two numbers together (60% x 87.5% = 52.5%) gives you the probability of Sampras winning a point if the rules of tennis allowed only one serve and Pete hit his "first" serve every time.

On his second serve, Sampras won 18 of 37 points for a 48.6% winning rate. Had Pete used his "first" serve instead of his "second" serve after a single fault, he would have won MORE points (52.5% versus 48.6%). That's why Sampras will occasionally go for the ace or the big one after missing the first serve, rather than use his conservative serve, particularly against an excellent returner like Agassi. The odds are actually in his favor to play aggressively rather than conservatively.

There are people who advocate changing the rules of tennis and allowing only one serve per point as a way of slowing down the game and reducing the dominance of the serve. If that rule was in effect at the Wimbledon Championship Match, Sampras might have served his big one every time rather than using his second serve. He would have ended up winning more points, which would increase rather than decrease the importance of the serve. In addition, he would miss his first serve (equivalent to a double fault) about 50 times in order to win the 67 service game points that he needs to win the match. If 50 of the 127 points served by Sampras ended in a fault, interest in tennis might deteriorate quickly.

There are other matches that might be examined to see how the strategy of using only first serves would work.
If Rafter had used his "big" serve for his second serve as well as his first serve, he would have won the point on 29 rather than 22 second serves. This is an additional 7 points.

Rafter - Agassi Australian 2001, semifinal.

Rafter, first serve average speed 161 km/hour
96 in/153 points = 63%
79 won/96 in = 82%
points won 79/153 = 51.6%

Rafter second serve average speed 147 km/hour
points won 22/57 = 38.6%

Sampras - Martin Australian 2001, round of 16

Sampras, first serve average speed 170 km/hour
76 in/123 points = 62%
63 won/76 in = 83%
points won 63/123 = 51.2%

Sampras second serve average speed 158 km/hour
points won 18/47 = 38.3%

If Sampras had used his "big" serve for his second serve as well as his first serve, he would have won the point on 24 rather than 18 second serves. This is an additional 6 points.

Case Study 2: Serve Speed

By Rod Cross

All tennis players serve their first serve faster than their second serve. A fast serve is less likely to go in, but if it does go in it is more difficult to return. An interesting and surprising consequence is that most players win a greater fraction of their second serves than their first serves, because a larger fraction of first serves are faults. At least, this is the case at all four Grand Slam events where statistics are readily available on the web. In that case, would it not be better to serve a slow first serve and a fast second serve? And would it be even better to serve both serves slowly?

We can answer these questions by looking at a few examples. Suppose that a player serves 100 first serves during a match, and suppose 50 are faults. The player therefore needs 50 second serves and may serve 5 double faults. The first serve % is 50% (half the first serves went in) and the second serve percent is 90%. Now suppose that the player wins 35 of the 50 good first serves (winning percentage on first serve = 100 x 35/50 = 70%) and 25 of the 45 good second serves (winning percentage on second serve = 100 x 25/45 = 55.5%). These figures are typical of any match and are not unusual. Out of the100 points played, the server won 35 points on the first serve and 25 points on the second serve. The chance of winning a point on serve is therefore 35 + 25 = 60%, meaning that the server is more likely to win the game than the receiver.

	FAST FIRST SERVE					SLOW SECOND SERVE					TOTAL	
Total 1st Serves	In %	In	Win %	Won	Total 2nd Serves	In %	In	Win %	Won		Total Won	Total %
100	50	50	70	35	50	90	45	55.5	25		60	60%

In the above example, the server won 35 first serves out of 100, and 25 second serves out of 50, so the chance of winning a second serve is actually greater than the chance of winning a first serve. So, lets suppose the player reverses the serve order and serves a slow first serve first and a fast second serve. This time, the player will serve 90 good first serves and 10 first serve faults and the first serve percentage increases to 90%. Using the same averages as the first example, the player will win 50 points on the first serve. Out of the 10 second serves, the player will serve 5 faults and will win 3 or 4 good second serves (since 70% of 5 is 3.5). Out of the 100 points played, the server wins 50 points on the first serve and, at most, 4 points on the second serve. The chance of winning a point on serve will therefore drop to 50 + 4 = 54%.

	SLOW FIRST SERVE					FAST SECOND SERVE					TOTAL	
Total 1st Serves	In %	In	Win %	Won	Total 2nd Serves	In %	In	Win %	Won		Total Won	Total %
100	90	90	55.5	50	10	50	5	70	4		54	54%

Serving a slow first serve doesn't help since the chance of winning a good first serve is too low, especially when most of the first serves are going in. It would actually be better to serve a slow second serve if the first serve is slow. Out of the 10 second serves, the player would then win 5 instead of 3 or 4. The chance of winning a point on serve, using two slow serves, is then 50 + 5 = 55%.

	SLOW FIRST SERVE					SLOW SECOND SERVE					TOTAL	
Total1st Serves	In %	In	Win %	Won	Total 2nd Serves	In %	In	Win %	Won		Total Won	Total %
100	90	90	55.5	50	10	90	9	55.5	5		55	55%

How about serving two fast serves instead of a fast and a slow serve? In that case, the player would win 35 points out of 100 on the first serve, and would need 50 second serves as before. Applying the same ratio, the player would then win 17 or 18 of the 50 second serves. The chance of winning a point on serve would drop to 35 + 18 = 53%, which is even worse than serving a slow first serve.

	FAST FIRST SERVE					FAST SECOND SERVE					TOTAL	
Total 1st Serves	In %	In	Win %	Won	Total 2nd Serves	In %	In	Win %	Won	Total Won	Total %
100	50	50	70	35	50	50	25	70	18	50	53%

Can one increase the chance of winning a first serve by serving a bit slower or a bit faster? The problem here is that if you serve slower you will get more serves in, but your opponent is more likely to win the point. If you serve faster, you are more likely to serve a fault. It seems that there is nothing you can do to improve your chances apart from practicing more to increase your first serve percentage. This is in fact borne out by Grand Slam statistics. Players with a high average first serve speed generally have a lower first serve percentage than players with a low first serve speed. As a result, slow servers and fast servers win essentially the same fraction of points on their first serve when averaged over all players. Put another way, the players who win the most matches are not necessarily the fastest servers. The players who do win the most matches not only serve relatively fast, but also have a high first serve percentage and an ability to hit a good return when a fast ball is served by their opponent.

Summary

Case studies 1 and 2 seem to contradict each other. In case study 1, the players won a bigger fraction of their first serves (51%) than their second serves (38%) and would therefore have achieved a better result by serving two fast serves. In case study 2, the player won a bigger fraction of his second serves (50% counting all second serves) than his first serves (35%), in which case the best result is achieved with a fast first serve and a slow second serve. In case 2, serving two fast serves gives the worst possible result.

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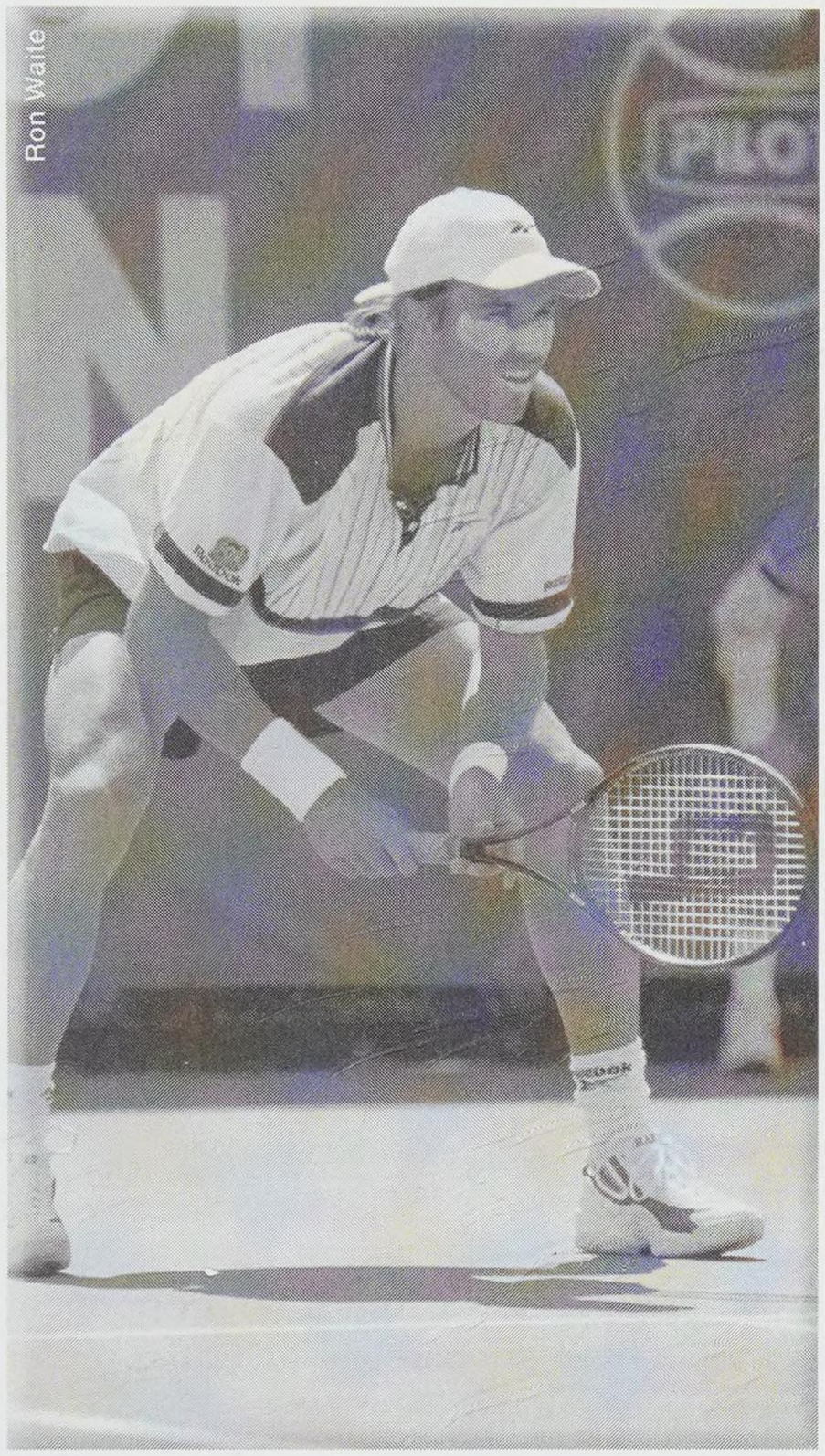


Patrick Rafter, Australia

21

Receiving Serve

Where Should You Stand?

By Howard Brody

Where should you stand when you receive serve? On the baseline? Inside the baseline? Well behind the baseline? It depends on the type of serve you are returning, whether you are playing singles or doubles, whether the server can swing the serve wide, and whether the server is coming to the net behind the shot.

I have calculated the time you have to respond to a flat, down-the-middle serve as a function of where you position yourself on the court (Table 21.1).

Assume your opponent hits a serve down the middle that leaves the racquet at 110 mph. If you stand five feet behind the baseline to receive that serve, you will have as much time as if you were on the baseline and your opponent is only hitting 102-mph serves. Moving five feet behind the baseline gives you 10% more time to react and get to the ball.

If you move in so that you are five feet inside the baseline, you have effectively transformed your opponent's 110-mph serve into a 120-mph blast, when you consider the time that you have to respond.

Since it is 78 feet from baseline to baseline, why does five feet make such a large difference (10%) in the time you have to return the ball?

Table 21.1

Serve Speed	Position Relative to Baseline	Time (sec)
120 mph	0	0.59
110 mph	-5 ft (inside)	0.59
110 mph	0	0.65
110 mph	+5 ft (behind)	0.71
102 mph	0	0.71

The reason is that the ball slows down quite a bit due to air resistance and the bounce. By the time a 110-mph serve gets to you at the other baseline, it has slowed down to about 55 mph; therefore, each foot added or subtracted at the slow end of the trajectory gives you proportionally more (or less) time than the actual distance itself might lead you to anticipate.

Based on this information, should you then routinely stand five feet behind the baseline when returning serve? If your opponent is booming in big, flat serves down the middle, and is not coming to the net behind them, it seems like a good idea. If, on the other hand, your opponent often slices one wide or kicks up high, then you may not be happy with your deep position.

To return the top spin serve from very deep, you may almost have to hit an overhead, and a wide slice may pull you into the next court. If your opponent is charging the net, your return from well behind the baseline gives the server more time to get to your return and hit a very good volley.

At the other extreme, if your opponent is plopping in serves at 70 mph, then where you choose to stand depends on what you are trying to do. If your principal concern is to get the serve back, then you clearly want to stay at or behind the baseline. If, on the other hand, you want to be aggressive and attack the serve, you should move in and be well inside of the baseline.

In doubles, there always is a player facing you at the net when you return serve, and often the server is moving up as well. How does that change your choice of position on the return?

Again, assuming it's a 110-mph serve, the speed at which your return leaves your racquet depends on the incoming ball speed (55 mph by the time it gets to you) and your racquet head speed. If, instead of taking a full swing, you just punch or chip at the ball with a racquet head speed of about 30 to 55 mph, the return ball speed off of your racquet will be about 70 mph.

If the opponent is crowding the net (less than six feet behind it), then you have an interesting option. When you move in and hit your return from five feet inside the baseline right at that opponent, you might handcuff the player at the net. The time that your opponent at the net has to return your shot depends on how hard you hit the ball and where you are standing when you hit it.

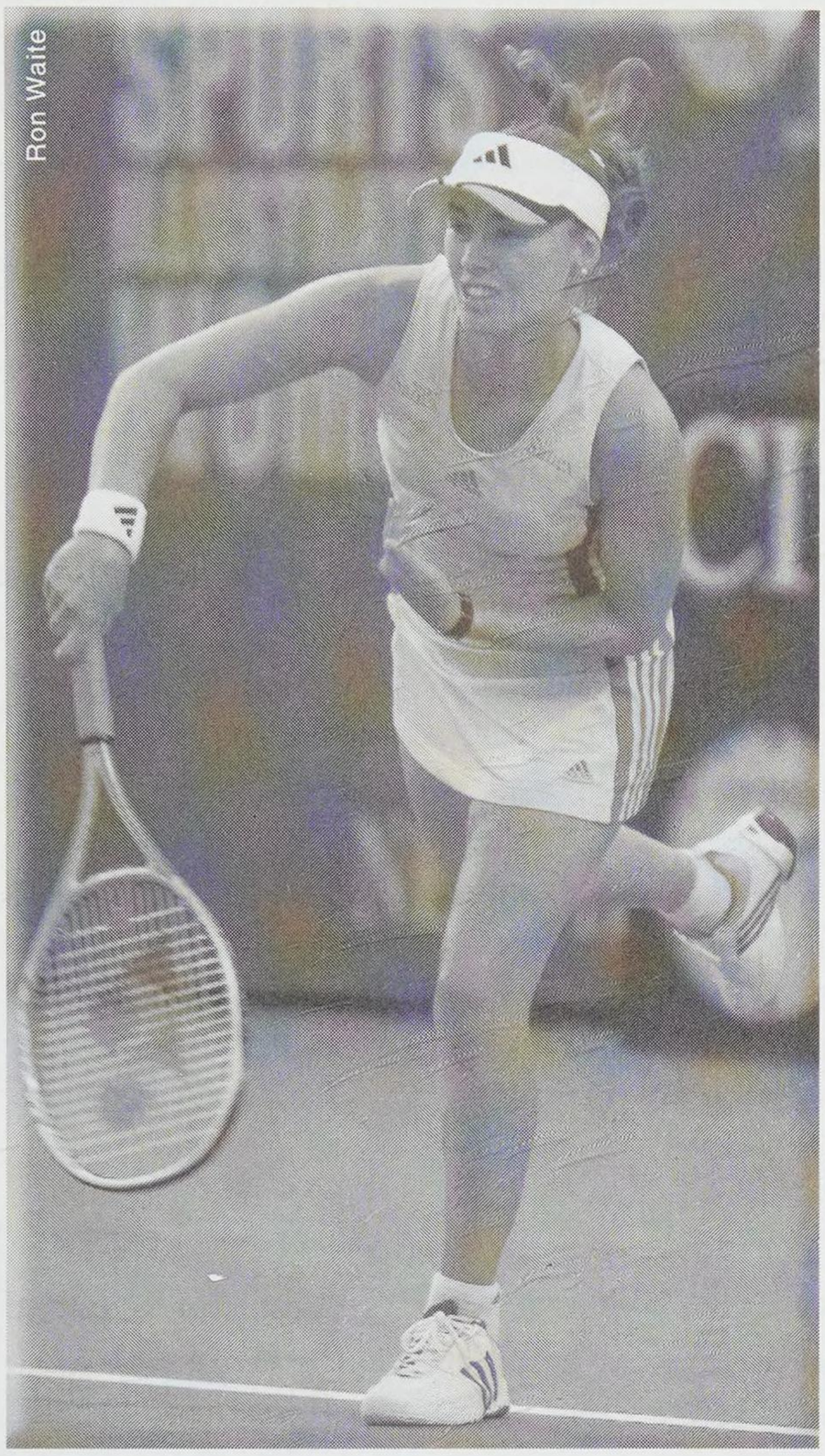
Assuming your return leaves your racquet at 70 mph, the result will be the following times for the player at the net (Table 21.2).

Table 21.2

5 ft inside the baseline	On the baseline	5 ft behind the baseline
0.42 sec.	0.49 sec.	0.56 sec.

Some of the best professional athletes can just barely react (catch a ball, block a puck, etc.) in about 0.35 seconds, so if you step inside the baseline and punch your return at that opponent who is crowding the net, you will probably win the point (and force that player to retreat on future shots).

Another way of looking at these numbers is to realize that the time you have to react to a 150-mph serve (baseline to baseline) is 0.49 seconds. If the person at the net tries to return a shot, but has only 0.42 seconds of time, the odds are that you will win the point. Just think about how you would react to a 150-mph serve!



Martina Hingis, Switzerland

22

Strokes and Spin

Factors in Hitting and Receiving Spin Shots

By Howard Brody

Topspin vs Backspin

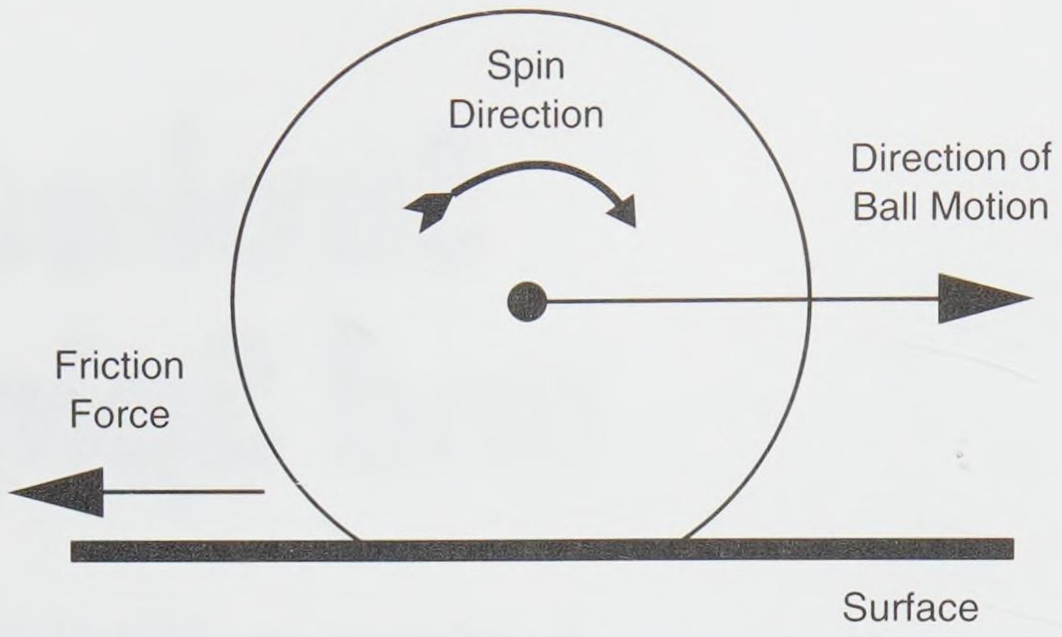
Over the years, we have been told by many tennis experts that topspin provides a much better shot than a flat shot or one with backspin on it. Then why is it many players still like to hit shots with backspin, using a chopping motion for the stroke, or hit flat shots with very little spin on the ball? Why is it that most tennis coaches teach the flat stoke to the beginner rather than start them with topspin?

When a computer is used to calculate ball trajectories, it is very clear that topspin shots have a much higher probability of clearing the net and landing in the court than do flat shots or shots with underspin. In fact, it has been shown by this author (in the book Tennis Science for Tennis Players), that for a ball leaving your racquet at 60 miles per hour, the window or vertical acceptance angle (the variation in exactly what angle the ball can leave your racquet and still be a good shot) was two and a half times larger for a topspin shot compared to a ball hit with backspin. This means that if all else is the same, you are two and a half times more likely to make an error with the ball going into the net, or long, if you chop at the ball rather than hit it with topspin. Then why do players still slice at the ball instead of hitting with topspin?

The answer is that everything else is not the same. The variation or error in the vertical angle of the racquet (hence the ball leaving your racquet) is different when you attempt to hit a flat shot or one with backspin on it

Figure 22.1

Friction Force on a Bouncing Ball. The friction force to the left will cause a torque on the ball giving it a clockwise spin, which is topspin for a ball moving to the right.



as compared to attempting to hit with topspin. The reasons for this are due to physics and the mechanics of stroke production.

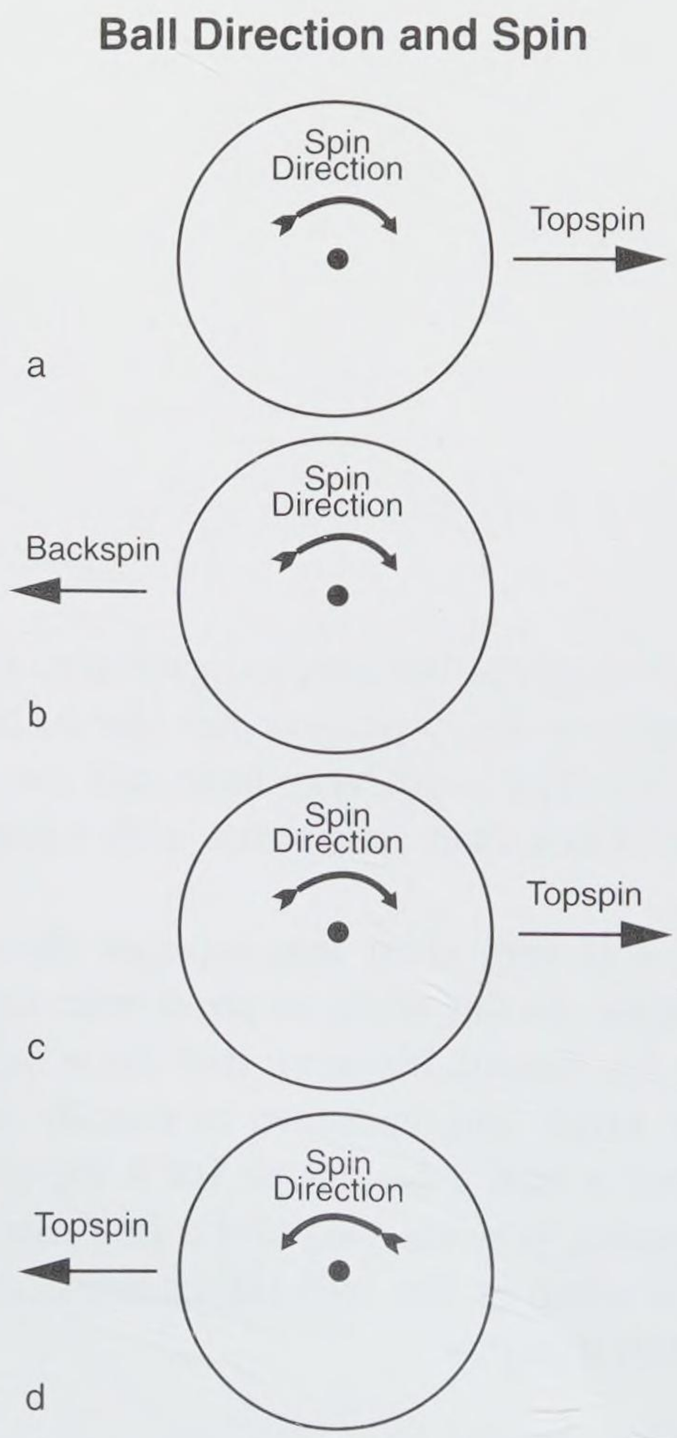
Since ground strokes (by definition) take place after the ball has bounced, the spin the ball acquires due to the bounce is important. After the bounce, the ball is usually coming at you with considerable topspin even, if the shot was hit as a normal, flat drive with no initial spin. This is because of the friction force on the ball as it bounces. This friction force produces a torque on the ball that causes it to have topspin (Figure 22.1).

If you want to return that shot with considerable topspin of your own, not only must you turn the direction of the ball's velocity around (it is coming toward you and after you hit it, it is moving away from you), but you also must reverse its spin direction by applying a large angular impulse. (A topspin ball coming toward you has its spin in the opposite direction than a topspin ball moving away from you.)

If you desire to return the shot with underspin, you only have to reverse the ball's direction because the ball is already spinning in the way that you want it to. (The topspin on a ball coming toward you has the same absolute spin direction as the backspin on a ball moving away from you.) Figure 22.2 shows the direction of the spin for balls moving to the right and to the left. If you assume the ball moving to the right is coming toward you after the bounce, then it is clear that the ball's actual spin direction must be reversed in order for it to have topspin after you hit it.

Figure 22.2

Ball Direction and Spin. (a) and (c) are balls moving to the right with topspin. (b) and (d) are balls moving to the left with backspin and topspin respectively. Note that the actual spin of the ball must be reversed to get topspin on a ball moving to the left.



This has been run through a computer program that was written to determine ball velocities and spins for various hitting speeds and racquet angles. Under these conditions (a groundstroke return), to get comparable ball speeds on your return, you must swing the racquet much harder to produce a topspin shot than to get underspin or hit the ball flat. At first glance, you would expect the racquet speed to be the same, but the fact that the ball coming toward you has topspin (due to its bounce) breaks the symmetry and means you must swing considerably harder to get your return to have topspin.

Assume the ball is coming toward you at 60 feet/sec (41 mph), and you attempt to return it

with underspin and a racquet head speed of 45 feet/sec (31 mph). With that incoming ball speed, the ball will leave your racquet with a speed of about 90 feet/sec (60 miles/hour). If you want your shot to have topspin, to get the same outgoing ball speed, it will require a racquet head speed of 70 feet/sec (48 mph). You really have to swing much harder to get the same speed on your topspin shot as on your backspin shot, and this probably will cause you to spray the ball around quite a bit. It is not easy to swing a racquet at 70 feet/sec and maintain the degree of control that you would get when you only have to move the racquet head at 45 feet/sec. A top player can do this, but a beginner or intermediate player cannot. To hit the ball harder requires greater racquet head speed, hence better preparation. To slice at the ball requires little racquet head speed, therefore little racquet preparation. It is a great stroke for an emergency. To a beginner, every shot is an emergency.

Topspin shots, because of their trajectory in the air, give you a larger margin of error in your stroke, allowing you to miss-hit the ball (in angle) a bit. However, the argument presented in this note shows that the inherent errors associated with production of topspin shots may be considerably larger than those associated with backspin or flat shots because you must swing the racquet so much harder. If your timing is superb and your strokes really well grooved, topspin may be fine. For the elite, high performance player, topspin is the stroke of choice. For the beginner and the occasional player, a flat shot or underspin probably will lead to better depth control, less errors, and fewer points lost.

Watch That Spin

Most players adjust their strokes for the speed of the incoming ball. If their opponent hits a really hard shot, they know that they can return a ball with good pace without swinging their racquet very hard. When their opponent is hitting softly, most players realize that they must swing out if they want their shots to have good pace. Not only is it difficult to swing out when your opponent is pounding the ball, but if you successfully connect, the ball may come off your racquet like a rocket. In such a case, it is unlikely to stay in the court. An experienced player usually makes these adjustments without thinking about it.

What many players do not do, is adjust their swing and strokes to compensate for the spin their opponent puts on the ball. Just as you can often anticipate the speed of your opponent's shot from the racquet speed you observe, you should watch your opponent's racquet trajectory (low to high, high to low, or level) and adjust your swing and court position to compensate for it.

When your opponent hits a flat shot (not much spin on the ball as it leaves the racquet), your eyes track the ball's initial trajectory and your experience at tennis allows you to anticipate the bounce location. If you have been playing tennis for years, you know just where to move so that the ball will be at a comfortable hitting position for you (in baseball talk, in your "strike zone"). When you see your opponent hit with a very pronounced low to high motion of the racquet, you can anticipate a topspin shot that will dip in the air, bounce shorter, rebound higher, and may not slow down as much when it bounces. Your tennis experience with flat shots is such, that when a topspin shot does not slow down very much on the bounce, you may (incorrectly) perceive that it has gained speed. When you see your opponent slice down at the ball (or

come under it), you know that the ball will have a flatter trajectory, bounce a little deeper in the court and then, if it was hit with good pace, it may skid or slide, staying low. If it was hit without great pace, the sliced shot may float a bit and then "sit-up" when it bounces.

However, the problem most players are not aware of concerns the spin of the ball AFTER the bounce. For a typical groundstroke that leaves the racquet at 60 miles an hour, the spin after bouncing depends on the initial spin given to the ball by the opponent's racquet. A ball with a backspin of 30 revolutions per second will skid off of the ground after the bounce and end up with essentially no spin. A shot hit flat (no initial spin) will have a forward or topspin of 40 revolutions per second after bouncing. A 60 mph shot hit with a topspin of 30 revolutions per second will end up with a forward or topspin of 70 revolutions per second after the bounce. This is all on a typical mid-pace tennis court. On a slow clay or fast grass, the spins after the bounce may be different.

Topspin (after the bounce) will interact with your racquet strings and tend to make the ball come off of the racquet at an increased angle. As an example, if you swing your racquet perfectly level and have the face perpendicular to the ground (neither open or closed), when you hit a spinless ball that is moving horizontally, the ball will rebound perfectly horizontally also. If you try to return your opponent's flat shot (having 40 revolutions per second topspin after bouncing) with the same flat swing, the ball will leave your racquet at about 7 degrees above horizontal. This is because the spin on the ball will bite into your strings and give the ball some upward velocity. It turns out that the 7 degree "lift" the ball acquires from this interaction with the strings is just the correct amount for the ball to clear the net and land in the court.

If you attempt to return your opponent's slice (very little spin after the bounce) with exactly the same stroke and racquet orientation as you did for your opponent's flat shot, the ball will leave your racquet moving horizontally and it will end up in the net. For your shot to clear the net, you must give the ball extra lift and have it leave your racquet with some upward angle. You don't want to slice down at the ball unless you also give the ball a great deal of lift. Be very careful if you want to try to return a slice with a slice, or the ball will end up in the net.

If you take the same flat swing at a shot that your opponent has put topspin on, the 70 revolution per second topspin the ball has after the bounce will dig into your strings and make your return fly up into the air. Here is a good opportunity to slice down at the ball.

REMEMBER. When your opponent hits with spin, not only does it change the ball trajectory while in the air (which you will observe) and the ball speed and angle as it bounces (which you can usually anticipate), but it also will change the way the ball interacts with your racquet. So watch your opponent's swing pattern carefully and adjust your strokes and racquet head angle to counteract the after-the-bounce spins.

A shot hit at 60 mph with top, back, and no spin is returned perfectly flat. The vertical angle of the return is given in Table 22.1.

If all three types of shots are moving near 30 mph as they cross the baseline and all three types of shots are returned exactly the same way, only the return of the flat shot will land in the court.

Initial spin (rps)	Spin after bounce (rps)	Ball angle (degrees) for flat hit
top 30	top 70	14
none	top 40	8
back 30	none	0

Table 22.1

A shot hit at 60 mph with top, back, and no spin is returned perfectly flat. The vertical angle of the return is given in the last column.

If you decide to chop or slice back your return (instead of hitting it flat), you will get the results in Table 22.2.

Initial spin (rps)	Spin after bounce (rps)	Ball angle (degrees) due to your chop
top 30	top 70	11.0
none	top 40	2.5
back 30	none	-6.3

Table 22.2

Results if you decide to chop or slice back your return (instead of hitting it flat).

If you return your opponent's topspin or backspin shot with EXACTLY the same swing as you would return a flat shot, the ball will end in up the net or go long. It is clear that you must adjust your swing or racquet head angle to compensate for the spin your opponent puts on the ball.

Slicing the Slice

Over the years, you may have heard that it is difficult to return a sliced ground stroke with a slice. That it is much wiser to hit flat or even use some topspin when your opponent has sliced (put backspin on) a shot to you. Is this true – and if it is, what is the reason?

When a flat or topspin shot bounces, it acquires topspin due to the court friction acting on the ball. A moderate drive of 60 mph, with little or no spin, will gain about 40 revolutions/sec of topspin after the ball bounces. If a ground stroke is hit at 60 mph with a moderate topspin of 30 revolutions/sec, after bouncing it will have acquired 65 revolutions/sec of topspin. These shots will be relatively easy to return with backspin (sliced), since they are spinning in the correct way already (see diagram). All your racquet has to do is turn the ball's direction around – the spin will take care of itself.

When a 60 mph drive with moderate backspin bounces, it ends up with essentially NO spin and stays low after the bounce. If you try to slice back such a shot, not only must you turn the ball's direction around, but you must also supply the energy needed to get the ball to spin. If you want to slice back a slice,

you must hit the ball harder (swing your racquet with greater speed). In addition, since sliced shots bounce lower, you will have to lift your return higher to clear the net. To make matters more difficult, the spin on flat and topspin shots after the bounce tend to bite into your strings and lift the ball to a larger angle. Since sliced shots have little or no spin after the bounce, to get your return to go over the net, you must lift it even more.

If you want to return your opponent's sliced backhand with a slice of your own, you must hit the ball harder and lift it higher over the net than you would if you were returning a flat shot by your opponent.

The Approach Shot — Why Keep It Low and Deep?

The books, the articles, and the TV announcers all tell you to hit your approach shot deep when you want to come to the net. Some add the comment that you should also keep the ball low. They go so far as to recommend a slice approach shot rather than topspin to keep the ball as low as possible. Is this advice correct?

Keeping the ball low has two immediate and obvious advantages. A low shot, such as a slice, because of its trajectory, gives your opponent slightly less time to get to the ball compared to a higher topspin shot that bounces in the same location. The more you can hurry your opponent, the less likely that you will be passed or handcuffed by a strong return. This timing effect is actually of less importance than the second reason for keeping the approach shot low.

The higher the ball is when your opponent hits it, the better are the chances that the return will go in. Computer analysis shows that your opponent can hit a high bouncing ball much harder than a low skimming ball, and have the same chance of the shot going in. When a ball is hit while it is quite low, there is much less margin for error in the shot, particularly if the ball is being hit hard. You have more than twice the chance of getting a 70 mph groundstroke to go in if you hit it when it is four feet off the ground as opposed to 1 foot off the ground.

The chances of you being passed or handcuffed when you are at net depend on how hard your opponent can hit the ball and how fast the ball gets to you. A high-bouncing approach will allow your opponent to tee off on a shot and still have a good chance of the ball going in. You will have trouble returning this type of shot. If your opponent has to hit the ball when it is quite low, a hard hit will have a small probability of both clearing the net and bouncing inside the baseline. This means your opponent will either have to ease up on the shot or attempt a risky, poor percentage shot. Therefore, keeping your approach shot low increases your chances of winning the point.

But what about depth? This is a matter of ball speed and reflexes. The baseline is 39 feet from the net, and probably between 45-50 feet from you when you are at the net. This means that there is approximately half a second between the time a hard shot is hit and when it gets to you at net. Is this enough time to react to a ball, determine where it is going, move the racquet, and volley the ball into the court where you want it to go?

Yes, if you have reasonable reflexes. Plagenhoff has studied high-speed films of tennis players, hockey goalies, etc., and he has discovered that if there is not at least 0.37 seconds between the hit and the block, the ball (or puck) will not be blocked unless you are lucky or guess right (it's called anticipation). This means that if you allow your opponent to hit the ball hard from within the baseline, it will

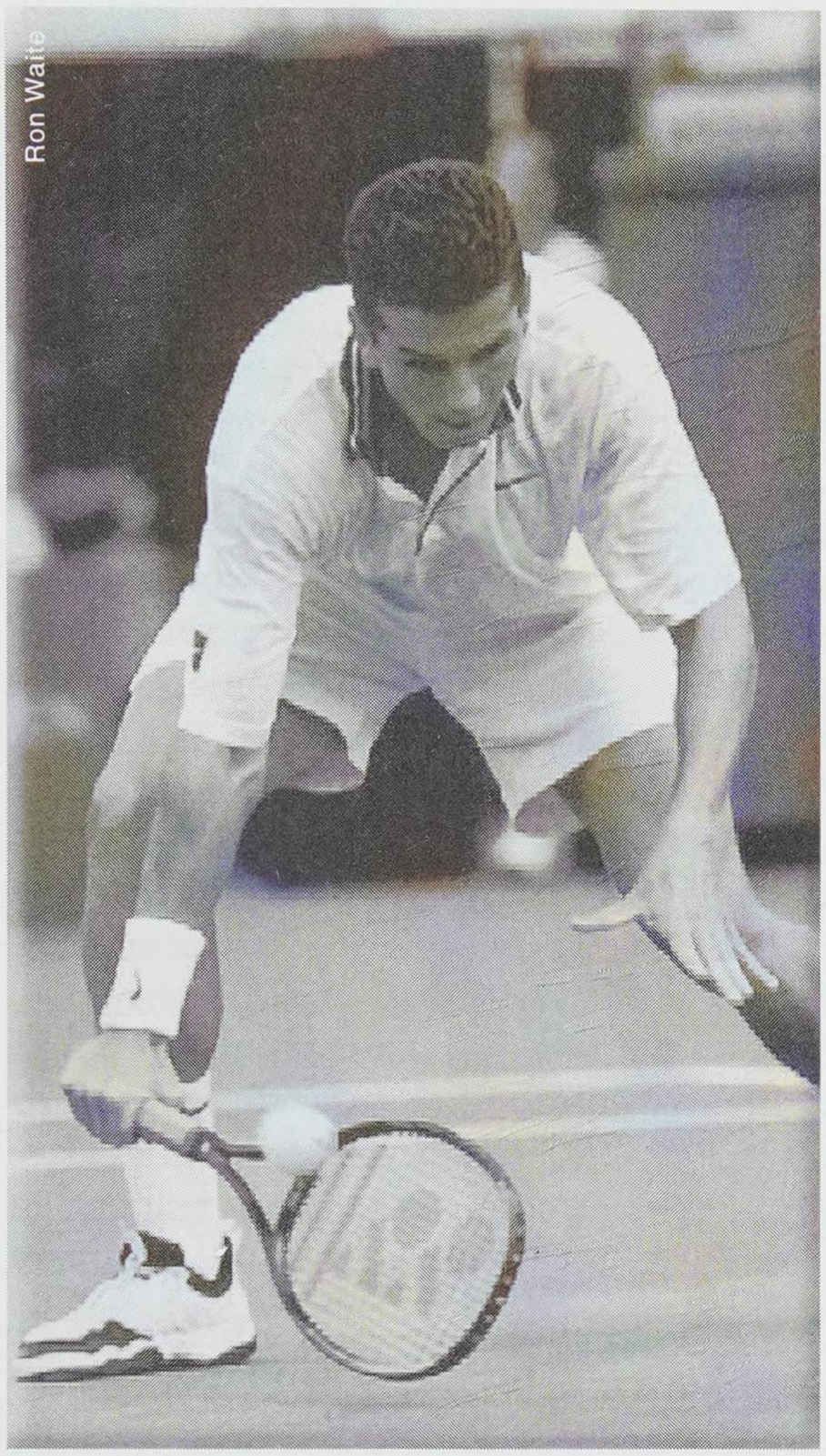
be marginal as to whether you will be able to react and get to it, even if it is within your reach.

By forcing your opponent back behind the baseline, you have bought yourself time to react to the return and pounce on the ball. Therefore, it will take a much better shot to pass you or handcuff you if you force your opponent to hit the ball from behind the baseline.

This leads to an interesting question. You are generally advised to close in tight to the net. However, if you find yourself at net and have hit a short volley, should you retreat a little to buy yourself some time for your return? The analysis above says "Yes." If your opponent can get to it with some ease. This is illustrated by Table 22.3 of time versus positions on the court. This table shows the amount of time you have for well-hit (70 mph) shots hit from 10 feet behind the baseline, on the baseline, and 10 feet inside the baseline for various positions you can have at net. Since very few of us have the 0.37 second reflexes of a hockey goalie, a retreat is clearly recommended.

Your Position Behind Net (feet)	Hit From 10' Behind Baseline (seconds)	Hit From On Baseline (seconds)	Hit From 10' Inside Baseline (seconds)
2	.56	.44	.32
7	.63	.50	.38
12	.70	.56	.44
17	.77	.62	.50

Table 22.3
Time you have to return a shot when you are at net (for a ball speed leaving opponent's racquet at 70 mph).



Richard Krajicek, Netherlands

23

The Drop Shot

Hitting and Returning Drop Shots

By Howard Brody

Hitting a Drop Shot

This short chapter is an attempt to clear up one common misconception about how to hit and how to think about the drop shot and in what way that shot differs in a critical way from a standard ground-stroke.

When normal groundstrokes are played, your first concern is to get the ball over the net, and then you usually want the ball to bounce deep in your opponent's court. On the drop shot, your first concern is, again, to have the ball clear the net, but now you want the ball to bounce as short as possible – hopefully having a second bounce inside the service box. This difference in the resultant shot can be accomplished by visualizing the net and court in a different manner.

For the normal groundstroke, you often visualize a window at the net, and your shot must pass through this window in order to both clear the net and not bounce long. (Too much clearance above the net will often result in a shot bouncing beyond the baseline.) Since, on a groundstroke, you are usually hitting the ball from your own baseline at a height that is not too different from the height of the net (3 feet), you must lift the ball up with your stroke so that it will clear the net. Most of you probably visualize the ball's trajectory having a maximum height (its apex) at the location of the net. This trajectory will produce a shot that bounces fairly deep into your opponent's court, which is what you want.

For a drop shot, you hit the ball with less pace, and usually with considerable backspin or underspin. Why is this? Both hitting with less pace and with considerable backspin or underspin will produce a ball trajectory that will not go very far after its bounce. This is what you want. The more speed a ball has, the farther it will go between its first and second bounce, so you clearly want less pace. The more backspin on the ball, the more the ball will slow up when it bounces, reducing the distance between first and second bounce.

However, the most important thing with a drop shot is not to have the peak of the ball's trajectory located at the net. A peak of the ball's trajectory near the net will result in a very poor drop shot since the ball will bounce fairly deep into the court. For a good drop shot, you want the peak of the trajectory (the highest point the ball reaches) approximately halfway between you and the net. If you are on the baseline (which is a poor place to hit a drop shot from), the peak of the trajectory should be about 20 feet from you. If you are 10 feet inside the baseline when you hit the ball (a good place to hit a drop shot from), then the trajectory peak should be 15 feet from you (and 15 feet from the net).

If you can visualize in your mind an imaginary wall or barrier 5 or 6 feet high, that is halfway to the net, a good drop shot will just clear that obstruction. However, if as your shot clears the imaginary wall, it is still going up, your shot will bounce well into the court rather than bounce close to the net. If, as your shot clears the imaginary wall, it is coming down, your shot will end in the net. If as your shot clears the imaginary wall, it is at its peak (and moving essentially horizontally at that point), your shot will clear the net and bounce very short.

Figure 23.1 shows the trajectory of balls hit (with backspin) from 10 feet and 20 feet into the court. For each hitting location, one trajectory shows the results when there is a peak at the location of the net, the other shows the result when the peak is half way to the net. Several things are apparent from these diagrams.

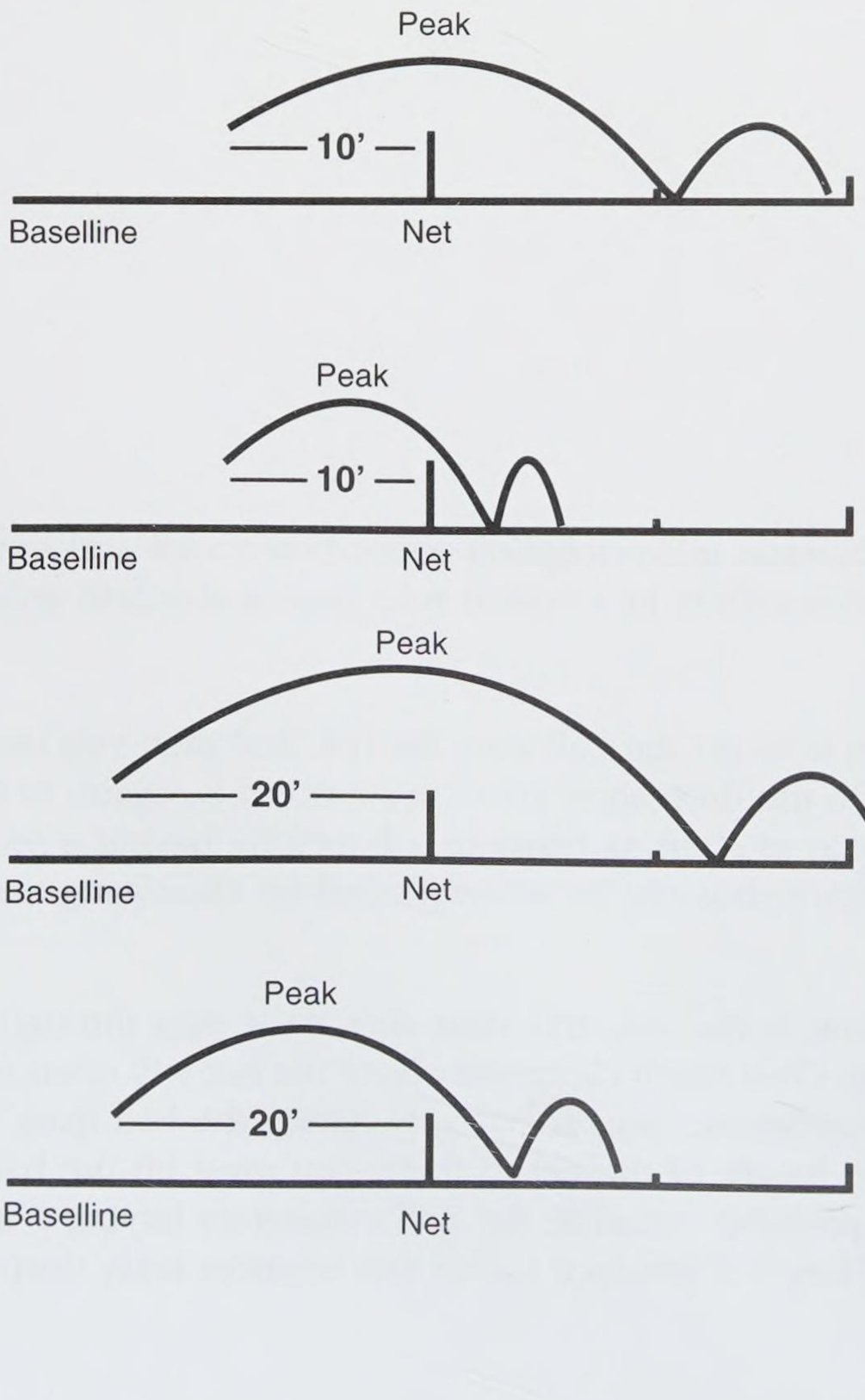
1. The closer you are to the net when you hit the ball, the shorter your drop shot can be.

2. A ball trajectory that peaks half way to the net produces a much better drop shot than a shot whose trajectory peaks at the net.

An obvious trajectory that one might think of would have the ball peak near the net and also cross the net only inches above the tape. This will result in a shot that bounces very long (probably out of the court) because to produce this trajectory you have to hit the ball very hard. Where the typical shots shown in the figures correspond to balls hit at 20 to 30

Figure 23.1

Backspin Drop Shots From 10 and 20 Feet From Baselilne



mph, to produce a ball trajectory that peaks near the net and just clears the tape would require an initial ball speed of over 80 miles/hour. That is not what is wanted for a drop shot.

Returning a Drop Shot

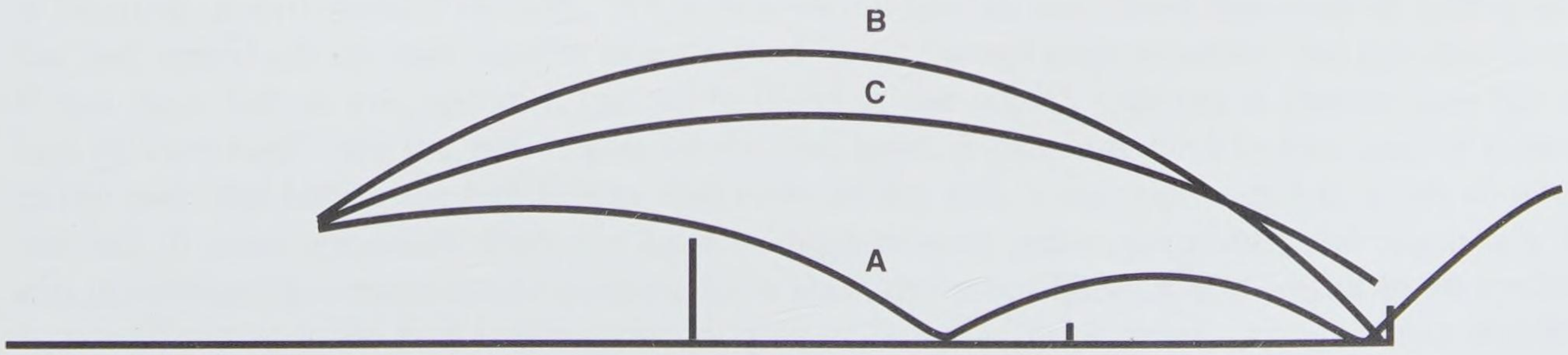
Your opponent has hit a nice drop shot and you charge in to get to it before the second bounce. Hitting it on the dead run, you give the ball a gentle flick, and your return sails long over the baseline. Why does this happen? You have been practicing that gentle flick from the area around the service line for years, and in practice it always clears the net and lands well inside the baseline. Now in a match, it sails long. Did you choke in a pressure situation? No! You hit it exactly the same way as you did when you were standing on the service line and practicing all those times. Only this time you were not standing still, but you were running forward, and your forward speed must be added to the racquet head speed that you get from the flick.

When you are running to get to a ball, your speed is about 8 to 10 mph (miles/hour), so your racquet will be moving at 8 to 10 mph faster than if you hit the same shot while you have your feet planted. This will add about 10 to 14 mph to the ball speed, which may be enough to send it out over the baseline. Why does adding 10 mph to your racquet speed add 14 mph to the recoiling ball speed off your racquet? That is because the speed of a ball off your racquet is given by the formula V_{out} = ACOR x V_{in} + (1+ ACOR)V_r. A typical value of ACOR is about 0.4, so the term (1+ACOR) has a value of 1.4. If you add 10 mph to V_r, then V_{out} will increase by 14 mph.

To analyze this matter further, look at Figure 23.2 of ball trajectories for a flat shot hit from near the service line. There are three curves in this figure, the lowest one showing the minimum angle (about 6 degrees) needed for a 40 mph ball to just clear the net. Any 40 mph shot leaving the racquet at a smaller angle will hit the net. The upper curve shows the maximum angle (about 19 degrees) for a 40 mph ball to just land on the baseline. Any ball leaving the racquet at a larger angle will go long over the baseline. For a 40 mph shot to be good, it must leave the racquet at an angle between the angles shown by those two lines.

Trajectories for Returning A Drop Shot **Figure 23.2**

	A	B	C
Initial velocity	40.0 mph	40.0 mph	50.0 mph
Initial angle	6.3°	18.9°	12.0°
Initial spin	0	0	0



Let us take a typical case. In practice, you hit your shots so that the ball leaves the racquet at 40 mph. This means that the angle must be between 6 degrees and 19 degrees in order for it to be good. To play safe and give yourself a margin of error over the net and inside the baseline, you try to hit the shot so that the ball leaves the racquet somewhere between 11 and 14 degrees. In this angular range, the ball easily clears the net and bounces well inside the baseline. Now look at what happens when you add 10 mph to your ball speed. If you again have the ball leave your racquet at the same 11 to 14 degrees, it will sail well out because it is moving at 50 mph. Looking at the figure, you can see that at a 50 mph ball speed (the middle trajectory), a shot leaving the racquet at 12 degrees will go long.

What should you do about this when you opponent hits that short drop shot? If you can get there early and stop before you hit the ball, there is no problem. Your years of practice will let you get the shot in. If you must hit the ball while you are in full flight, either hit it much easier than you did in practice (reduce your racquet head speed on your flick), or aim your shot lower so that the ball clears the net by a much smaller margin than you did in practice. Either of these will work, but it takes the mental discipline to remember to do it in the middle of a hectic point when you are on a dead run.

Daniela Hantuchova, Slovakia

24

Time and Strokes

How Time Is Key in Tennis Strategy

By Howard Brody

TIME is the crucial element in a tennis match and most players do not seem to realize it. Almost everything you do when you are trying to win a point is based on an attempt to control time. You always try to reduce the time your opponent has to get to a ball or hit a ball. You usually attempt to give yourself as much time as you need when you are going to hit the ball. In each of the cases listed below, you will see how your attempt to control time has determined your choice of what strategy to use and how you play. You do these things to win points, without ever realizing that what you are doing is attempting to control time.

Why do you try to hit the ball hard in tennis in order to win more points?

You know that the harder you hit the ball, the smaller are your chances of that shot landing in the court. This is because of two distinct reasons. When you swing harder, you have less control over your racquet, and when the ball speed off of your racquet is increased, your "acceptance window" for the shot being good is reduced. If you hit a ball at low speed, it can go in (land in the court), whether it passes over the net, close to the net top, or very high over the net. If you hit the ball hard, it can only land in the court if it passes close to the top of the net. But hitting the ball harder also reduces the time your opponent has to get to your shot and to return the ball. If your opponent does not have enough time to get to your shot, the result is a winner for you. The less time your opponent has to prepare for a shot, the more likely it is to result in an error or a weak shot. On a groundstroke, if the ball leaves your racquet at 50 miles/hour, your opponent has almost 1.5 seconds to get

to it and return it. If the ball leaves your racquet at 70 mph, your opponent has about a full second to get to it. This difference of half a second is quite significant and it may not allow your opponent time to set up and hit a good shot.

Why do you pounce on short shots your opponent has hit? Why would you come in and hit a ball on the rise, even though it is a much tougher shot to hit and it does not produce an appreciably higher ball speed off of your own racquet?

Moving in and taking the ball early reduces the time your opponent has — both the ball round-trip time between his/her swing and the return and also the time between your hit and the ball getting to the other baseline. The total time is important, because if it is too short, your opponent may not have had time to get back into position. Remember, when you hit a winner, it means your opponent did not have sufficient time to get to it. If the ball leaves your racquet at 60 mph, just moving your hitting point from 5 feet behind the baseline to 5 feet inside the baseline will reduce the time your opponent has from 1.28 seconds to 1.04 seconds. This is a quarter of a second less that your opponent has to play the shot.

Why do you try to get to net to win points?

Getting to the net allows you a larger angular range for your returns and it greatly reduces the amount of time your opponent has to get to your return. Those are the reasons why you can hit so many more winners when you volley from inside the service line, as compared to hitting groundstrokes from the baseline. When you are at net, it essentially reduces the time your opponent has by half, compared to the time your opponent has for a groundstroke hit from the baseline.

Why do you try to hit your shots deep in the court (bounce near the baseline on groundstrokes, the service line on serves), when doing so increases your chances of making an error (your ball going long)?

Hitting shots deep reduces the time your opponent has to hit the ball, since the ball slows down when it bounces. When a ball bounces short, it takes that ball an extra 0.1 second to get to the baseline, compared to a ball that has bounced near the baseline. When your shot bounces near the baseline, unless your opponent retreats, it gives your opponent less time to hit the ball after the bounce. As an example, if the ball leaves your racquet at 60 mph, for every 4.5 feet closer to the baseline it bounces, your opponent has 1/10 of a second less time after the bounce to react. If your opponent does retreat well behind the baseline to obtain more time, you then get much more time to get to his/her return. It is very difficult to hit a winner from well behind the baseline — the ball just takes too much time in its flight. Hitting the ball deep also gives you more time to get to, and to react to, your opponent's return of that shot.

Why don't drop shots work well when you hit them from behind the baseline?

Drop shots don't work well when you are hitting from the baseline region because your opponent has more time to get to them, compared to the time your opponent has when you hit a drop volley or a drop shot from well within the baseline. When hitting from behind the baseline, it is also harder to try

to keep your ball's trajectory short, because when you are hitting from that position, you must give the ball more forward velocity to make it go over the net.

Why should your approach shot (and volley) be deep?

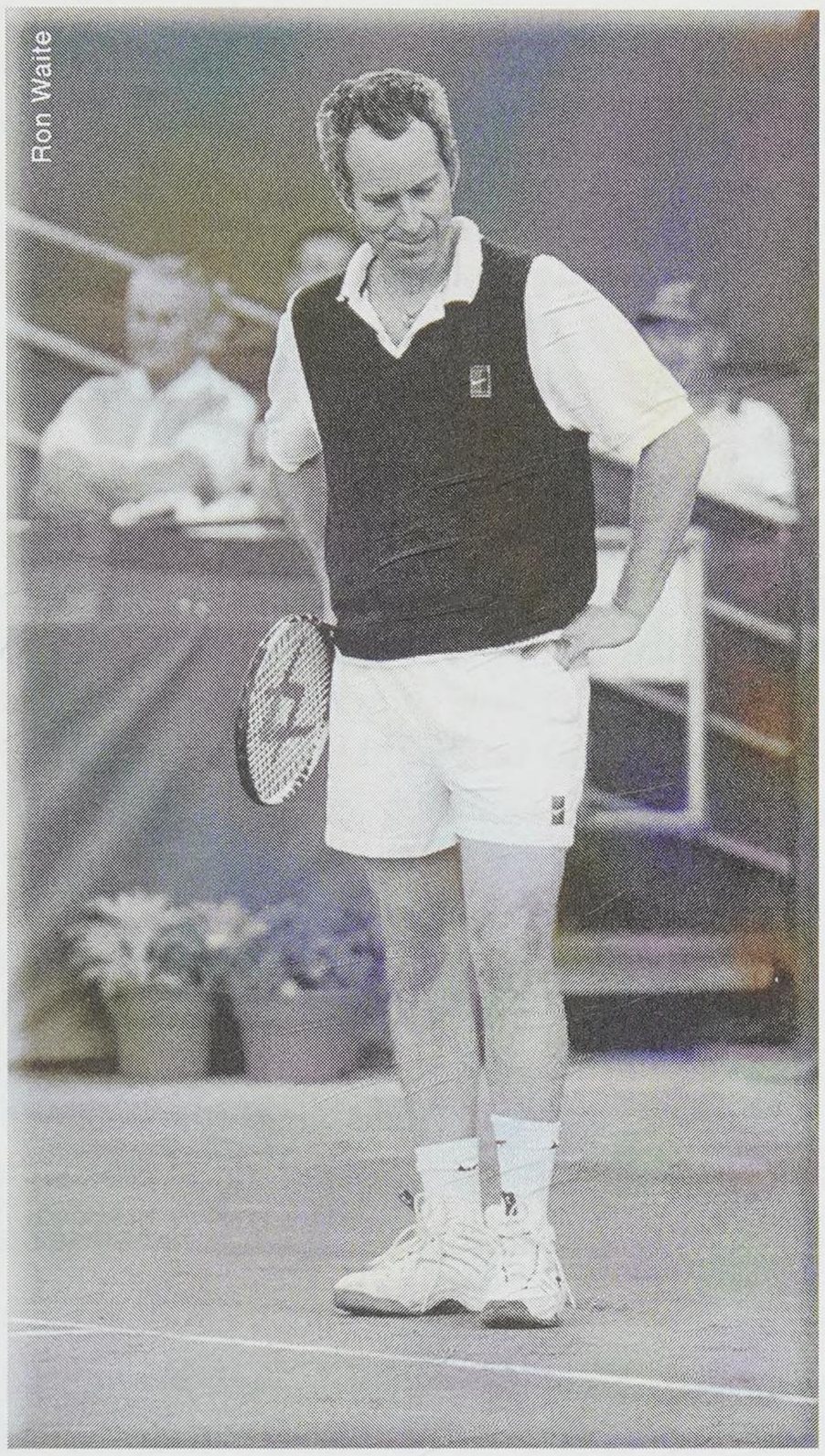
Approach shots and volleys should always be hit deep because it forces your opponent to hit from behind the baseline, which gives you more time to react and cover more of the net. If you can force your opponent well behind the baseline, you will be able to crowd the net and not be passed — you will have time to cover both down-the-line and crosscourt shots. If your opponent lobs, it gives you more time to get back under the ball.

Why is a topspin lob more effective than a backspin or flat lob?

You can hit a topspin lob with much more forward speed (and still have it go in) compared to a flat or backspin lob. This gives your opponent at net less time to get back and cover it. On the bounce, a topspin lob tends to not lose much speed, where a normal lob will lose 40% of its forward speed on the bounce. Your opponent will not have the time to run down a topspin lob after the bounce if it gets over his/her head.

Why is there a premium on getting the first serve in when playing doubles?

Many people have a weak second serve that allows the receiver to move well inside the baseline when returning it. This can turn the server's partner, who is at net, into a "sitting duck." Hitting, even a mediocre shot, at the net player when you are well inside the baseline gives that player so little time to react that he/she is handcuffed and will often return it with a weak volley, if at all. To counter this, the net player can retreat back to the service line to regain some time, but then most of the advantage of being at the net is lost.



John McEnroe, United States

25

Error Reduction

Reducing Unforced Errors

By Howard Brody

If you are a typical tennis player, you will lose many more points by making errors than you will lose because of winners hit by your opponent. If your rate of errors can be reduced, it should lead to an increase in the number of points that you win. There are several ways in which your error rate can be reduced, and some of them will be discussed here.

An improvement in stroke mechanics, lots of practice, and better concentration are obvious ways to reduce your error rate, but these are not the subject of this chapter. Appropriate shot selection and a different racquet or stringing can also lead to a reduction in your error rate, if you are willing to follow advice based on scientific analysis.

Lateral Errors

Errors come about because the ball has hit the net or gone long (an error of depth), or because the ball has gone wide (a lateral error). If you aim your shots so as to have the ball go down the middle, you have an allowable error of almost ten degrees to the right or to the left before the ball goes wide, and this is a sizable margin for error. If your shot bounces well short of the baseline, then your margin for side to side error is even greater. The problem is that most of you don't want to play safe and hit every shot down the middle. You want to go for the corners or go down the sideline. You may want to pass an opponent at the net. You want to go

for an occasional winner or at least make your opponent run for the ball once in a while. When you do that, you are inviting lateral errors to occur.

However, you can reduce the number of errors from shots that go wide, even when you go for corners or side lines by remembering this advice. Don't change the ball angle! If a shot is coming to you cross court, return it cross court. If a shot is hit down-the-line, return it down-the-line. When you change the ball angle by attempting to return a cross court shot down-the-line or return a down-the-line shot cross court, you are asking for error troubles. The reason for this comes out of the physics of the ball-racquet interaction.

When your return does not change the ball angle, the ball's impact direction is perpendicular to the face of the racquet at contact. It will then leave your racquet in a direction perpendicular to the face of the racquet. The ball will go out in the direction of the racquet motion whether you swing hard or soft. This is not the case when you attempt to change the ball angle. In this case, the direction that the outgoing ball makes relative to your racquet face depends on how hard you swing your racquet. This is illustrated in Figure 25.1. The higher the relative ball-racquet speed, the closer the ball will be to perpendicular to the racquet face as it leaves the racquet. If your swing is slow, then the ball will leave your racquet at a larger angle. Therefore, when your return changes the ball angle, the chances of you making an error and the ball going wide, depend on how hard you swing. When you do not change ball angle, since the ball direction does not depend on how hard you swing, lateral errors should be reduced. It is your choice.

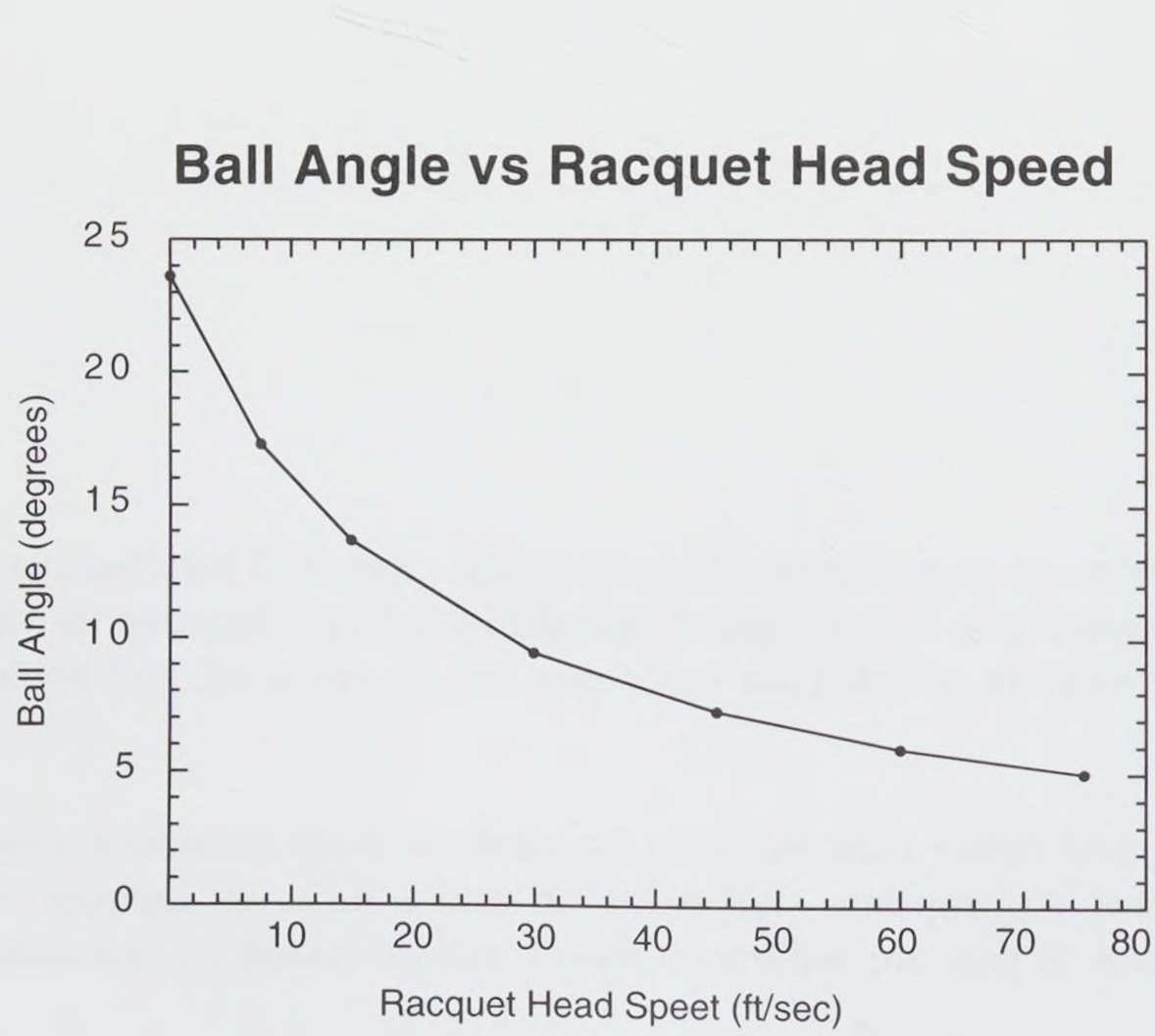


Figure 25.1

Ball angle leaving the racquet versus racquet head speed. This is for a ball incident at 20 degrees and 60 feet/sec for a typical groundstroke. The harder the ball is hit, the closer is the resultant ball direction to perpendicular to the string plane.

This advice also holds when you volley. If you punch out at your return, the ball will go close to where you aim it. If, on the other hand, you just try to block the ball on your volley, and you are changing angles, the ball will slide off at a large angle, possibly going wide.

However, if you return every shot back to where it came from, your opponent will quickly catch on, and will be there waiting for it. Knowing the facts about the ball-racquet interaction, you can reduce the errors that might occur even when you change the ball angle. If you are not going to hit the ball hard, then aim the ball a little closer to the center of the court. If you are really going to swing out, then you can aim your shot a bit closer to the sideline or the corner with confidence.

The famous statement that the angle of reflection equals the angle of incidence holds for light reflecting from a plane mirror, not tennis balls rebounding from a racquet.

Often, in a match, when you find yourself well ahead of your opponent, you may ease

up a bit, and not hit your shots quite so hard. You do this to reduce the number of errors you are likely to make. This can lead to a problem. If you continue to aim the ball the same way, only not swing as hard, balls that previously went down-the-line will now end in the alley.

A similar problem can be the result of you changing your game plan in the middle of a match. You become concerned about the final outcome, so instead of hitting out and playing your regular game, you decide to play it safer and ease up on your strokes. Again, balls that previously went down-the-line will now end in the alley. You end up making more, not less errors. People will claim that you "choked," when actually all you did was not understand the laws of physics.

A second cause of lateral errors is timing errors. If you strike the ball too early or too late in your swing, the ball will not end up going in the direction you want it to. One way to reduce this sort of error is to follow through your swing in the direction that you want the ball to go. When you swing this way, it looks as if you are trying to keep the ball on the strings longer. However, what you are actually doing is keeping the racquet head in the proper lateral angular orientation for a longer period of time (using less wrist in your swing), and therefore making the shot less sensitive to timing errors. In baseball, when you miss-time a pitch, you get a foul ball, and you get another swing. In tennis, when you miss-time a shot, you may lose the point.

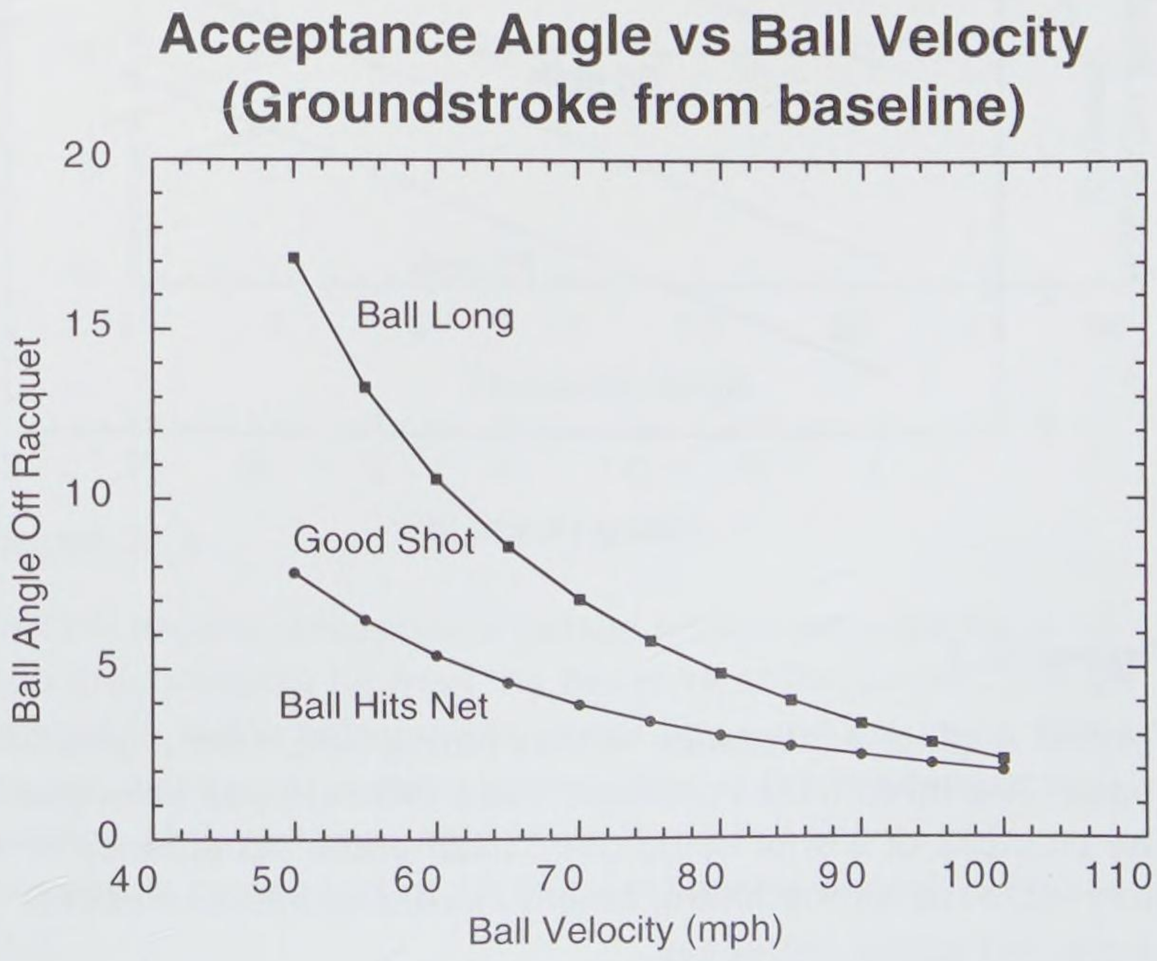
Errors of Depth

Where the side-to-side errors in tennis are basically a matter of court geometry, the vertical or depth errors depend on many factors, and your margin for error is often an order of magnitude smaller than the side-to-side error margin. This means, that on occasion, you will be trying to thread a vertical needle with your shot. In order for a shot to be good, it must be hit through a window — either a virtual window in vertical angle as the ball leaves the racquet that will allow it to clear the net and bounce in the court, or an effective window just above the net, the upper edge of which corresponds to a ball trajectory which bounces right on the baseline. This is shown in Figure 25.2. For a given ball speed, if your ball passes over the net above the top of this window, it will bounce beyond the baseline.

Window Size and Velocity. The size of this window decreases markedly as the ball speed increases, but increases as topspin is added. Since a groundstroke is hit at a height of about 3 feet, and the net is about 3 feet high, it is gravity that pulls the ball down into the court, otherwise the ball would sail over the baseline. When the ball is not hit hard, after

Figure 25.2

Vertical acceptance angle leaving the racquet versus ball velocity. The lower curve is for a ball just clipping the top of then net and the upper curve is for a ball landing on the baseline. The distance between the two curves is the angular acceptance for the shot being good as a function of the ball speed.



it crosses the net, gravity has more time to pull it into the court before it gets to the baseline. Therefore, the vertical window increases in size if you don't hit the ball as hard. The larger the window, the less likely you are to make an error because of the ball failing to go through it. If the window is large compared to the variation in your strokes from shot-to-shot, you will be known as a steady player who does not beat him- or her-self. If you select a small window (by hitting the ball too hard, for example), then you will have great difficulty in winning, and most points will end early with you making an error, even if you have reasonable control when you stroke the ball hard. The choice of window size is selected by you, and therefore, your error rate also is effectively selected by you. If you want to hit the ball hard you will make more errors, both because the window is smaller and because it is more difficult to control the racquet on a harder swing. Practice and correct stroke mechanics will reduce the variation of your shots from stroke to stroke, but they will not change the vertical window size. Only shot selection can do that.

The fact that shots that have higher velocity have smaller windows (and lead to more errors) may lead to an unanticipated source of errors. When a ball is coming at you with a very high velocity, if you stroke it back with your normal swing, your resulting shot will be a higher speed ball. This is because the speed of the struck ball is given by the formula:

$$v_{(struck)} = e_A \cdot v_{(incident)} + (1+e_A) \cdot V_{(racquet)} \qquad (25.1)$$

where the parameter e_A is about 0.40. This means that, if the ball coming toward you is hit harder, unless you reduce your swing speed somewhat, your return shot also will have a higher speed. The result will be a reduced window for your shot to be good, and an increase in your error rate. If you don't want this to happen, you should slow down your swing a little when your opponent is blasting the ball at you. As a bonus, this also might give you better racquet head control.

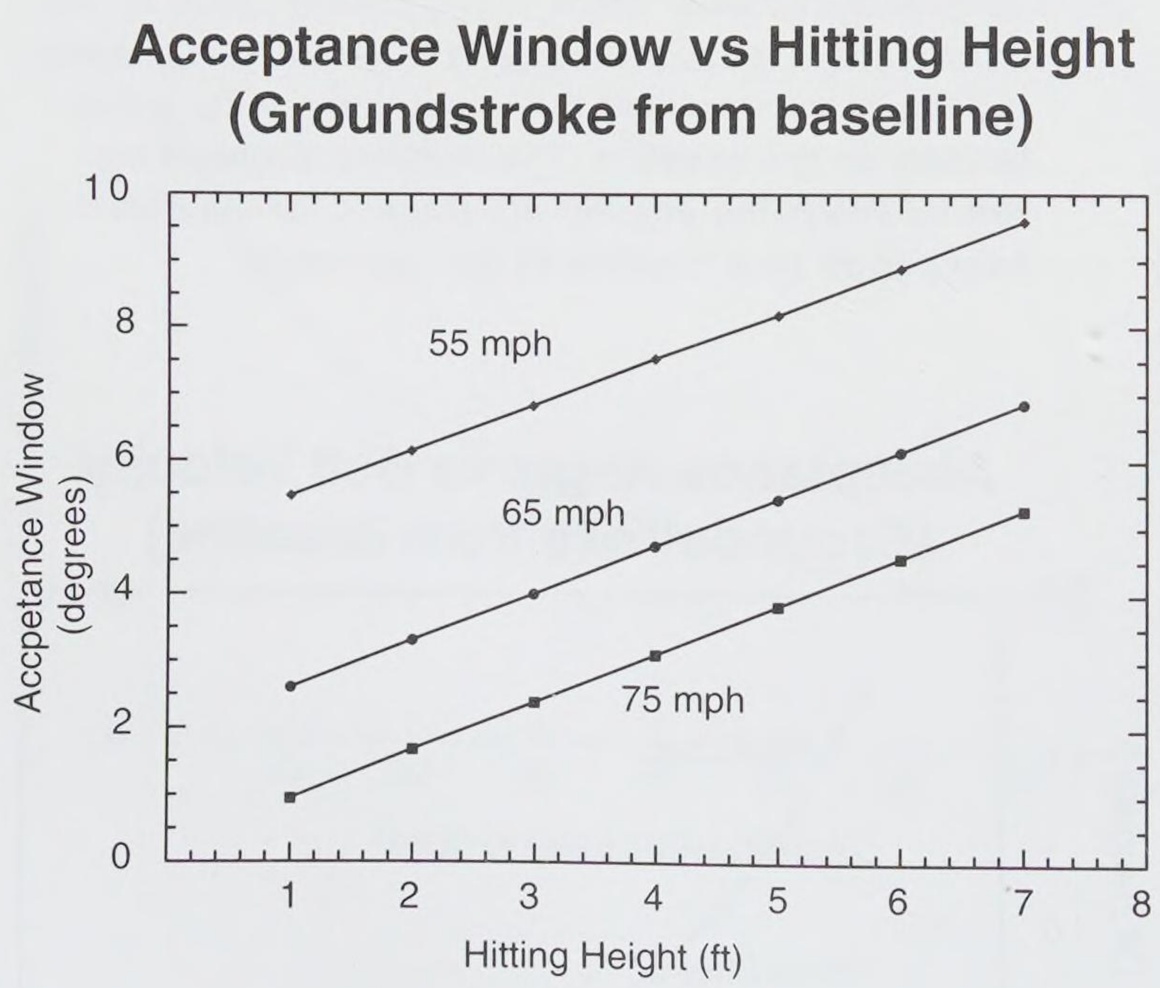


Figure 25.3

Vertical angular acceptance versus hitting height for a groundstroke. The three lines represent three different ball velocities. The chances of a shot being good (size of the acceptance window) increase as the hitting height increases and decreases as the ball speed increases.

Window Size and Hitting Height. You also can increase the size of the window on any shot (which reduces the probability of making an error) by hitting the ball when it is high, as opposed to striking it when it is down around your ankles. The incoming ball trajectory does not always give you that option, and many players do not have the control or power to hit shots at shoulder height. But if you can hit the ball when it is higher, vertical errors will be reduced. Figure 25.3 is an illustration of the increase in the vertical acceptance window with hitting height and the decrease of window with ball speed.

Window Size and Spin.

Topspin. You can increase the small window that comes from hard shots by putting

topspin on the ball. Topspin causes the air to provide a downward force, in addition to gravity, that causes the ball to dive into the court. But topspin also has some penalties associated with it. Many players have almost no control of their shots when they attempt to use topspin, so even though the window size increases, their shots spray all over (and often out of) the court. One of the reasons for this may be the following. When a ball bounces, it normally comes off of the court with considerable topspin due to the friction force between the ball and the court surface. When you try to return such a shot with your own topspin, not only must you turn the ball's direction around, you also must turn its spin around. In order to do this, you must swing your racquet 1.5 times faster than if you want your return to be flat (with no spin on the ball). If you are already swinging your racquet fairly hard (which is why the window is small), the loss of control due to a 50% increase in swing speed may be a major cause of the increase in your errors.

Backspin. Backspin (under spin or chop) shots have a reduced window because the ball tends to float long after clearing the net. (The interaction of backspin with the air produces an upward force on the ball that counteracts gravity.) Yet, many intermediate level players seem to be able to play a steady game chopping away at the ball and getting it to go through the small window they have set for themselves. This is because they do not have to hit the ball very hard to return it, due to the fact that the incoming ball has acquired top-spin because of its bounce. All they have to do is turn the ball's direction around, since the ball is already spinning the way they want it to. Not having to spank the ball really hard means that the player can use a compact stroke, get away with poor racquet preparation, and still have good racquet head control. This often reduces their shot to shot variation sufficiently to more than compensate for the reduced window size.

Window Size and Shot Selection. Another option you have is the down-the-line versus cross court decision when you are hitting from the corner. This has already been discussed with respect to change of angle, and now it will be analyzed with respect to errors in depth. The vertical window for a successful shot is always greater for a shot hit cross court rather than down-the-line. This is because the court is much deeper (the baseline further away) when you hit at an angle, along the hypotenuse of a triangle. You are effectively hitting into a bigger court when you go cross court, compared to down-the-line. The harder you hit the ball, the greater is the advantage of cross court over down-the-line. If your shot lacks pace, you will not make many more depth errors going down the line. If, however, you really swing out at the ball, then the cross court shot will result in far fewer errors. Figures 25.4, 25.5, and 25.6 show how the window of acceptance depends on the angle at which the ball is hit and the ball velocity.

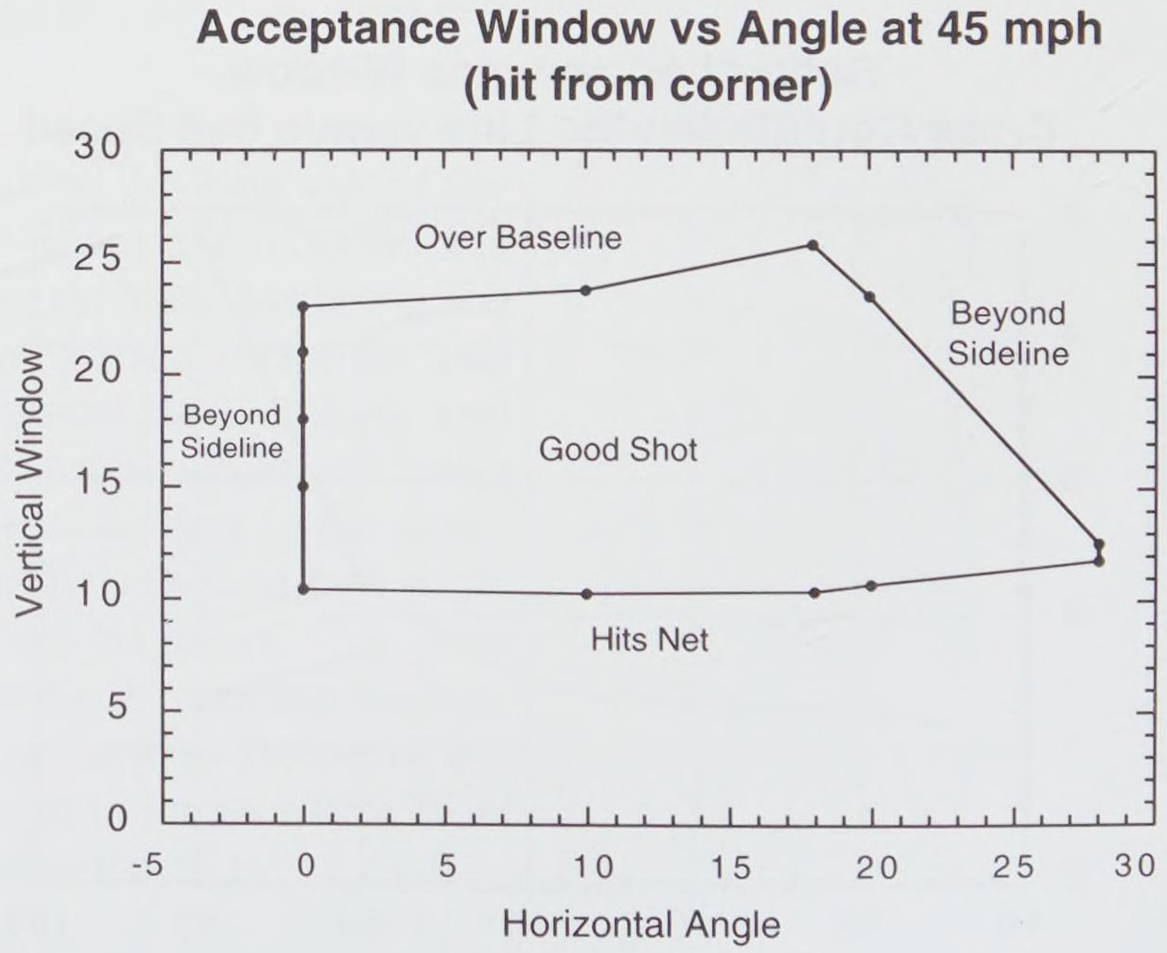


Figure 25.4

Vertical angular acceptance versus horizontal angle for a 45 mph groundstroke hit from the baseline at the corner. The difference in angle between the upper lines and the lower line is the actual vertical angular acceptance of the window as a function of the hitting angle (from down-the-line at zero degrees to crosscourt at 19 degrees) and is shown in Figure 25.6.

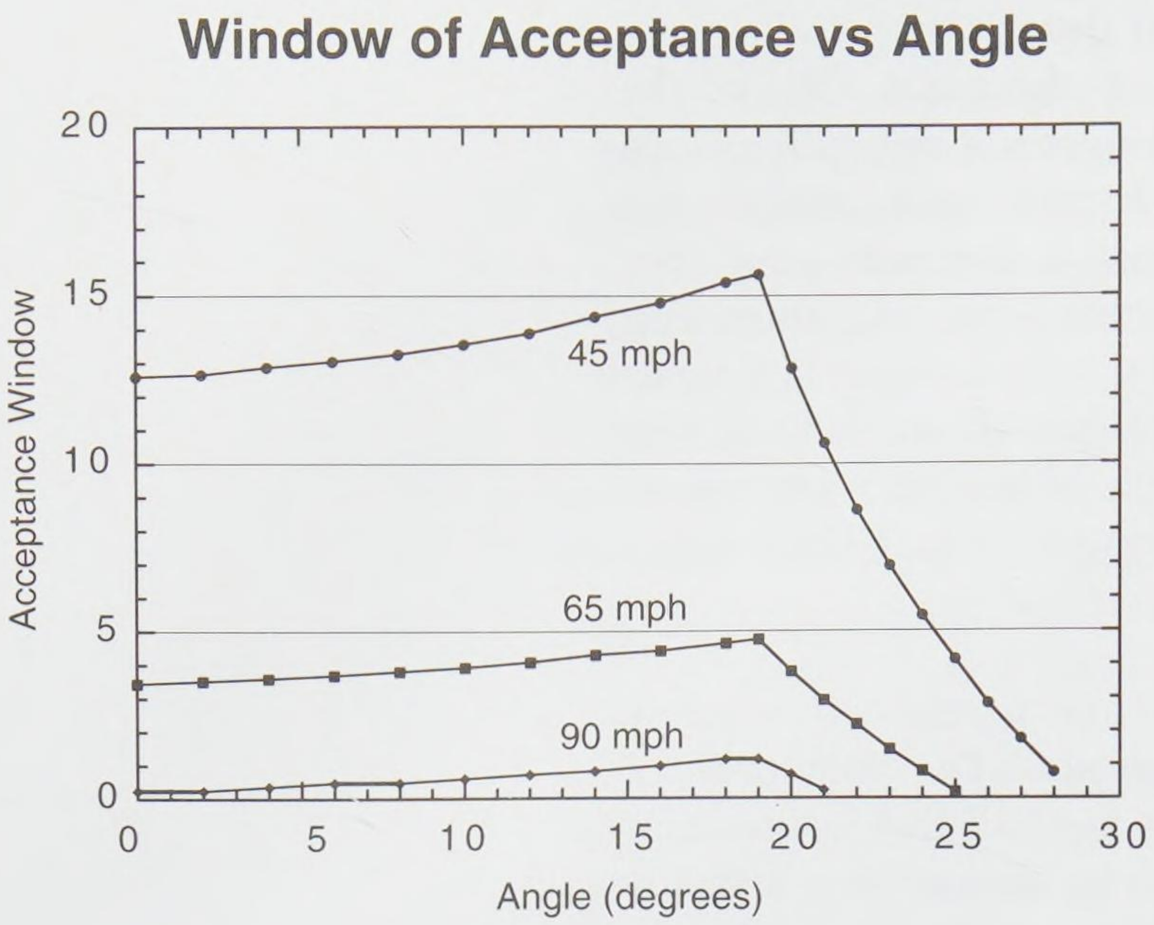


Figure 25.5

Window of vertical acceptance for a groundstroke hit from the corner for three different ball velocities.

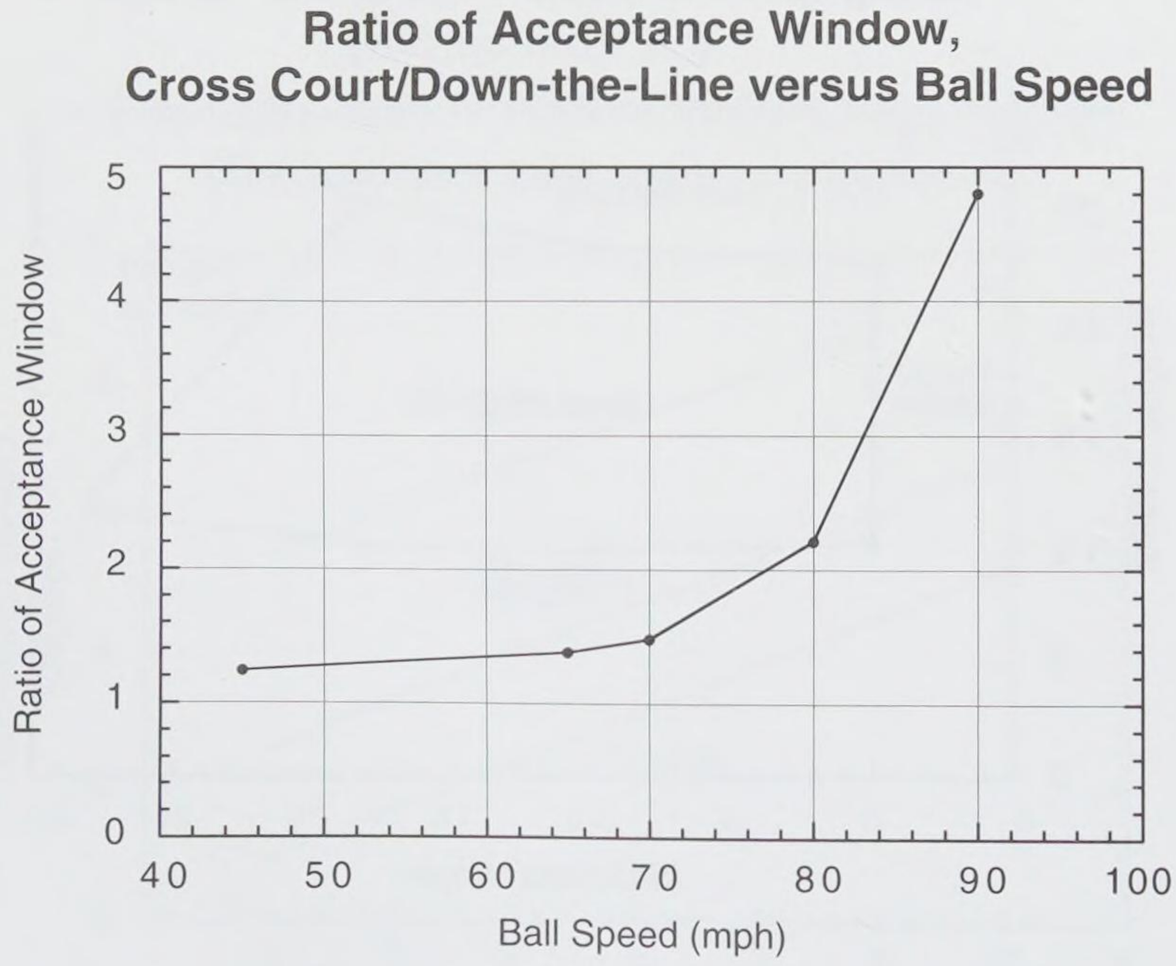


Figure 25.6

Ratio of vertical angular acceptance window for crosscourt versus down-the-line as a function of ball speed for a groundstroke hit from the corner. There is a small advantage to hitting cross court at low ball speed, but at high ball speeds, it is very important to hit cross court rather than down-the-line.

You often hear TV commentators claim that when hitting from the corner, you should go crosscourt rather than down-the-line since the net is higher down-the-line, and therefore, you must lift the ball higher to clear the net. This is not true. Since the distance from the corner to the center strap is greater than the distance to the net down-the line, you must lift the ball to the same vertical angle to clear the net for both cases. It is the greater depth of court behind the net that increases the window cross court, not the variation in net height. The opposite effect occurs when you are hitting from the center of the baseline. Hitting down the middle has a lower net, shorter distance to the net, and shorter court behind the net. This is almost exactly balanced by a shot toward the corner which goes over a slightly higher net, has a greater distance to the net, and a bigger court to hit into behind the net. The vertical acceptance window for a shot down the middle is almost the same as one into the corner.

Equipment and Error Reduction

String Tension, Dwell Time and Control. Often you hear the advice "keep the ball on your racquet longer for better control." It's easy to keep the ball on the strings longer – just reduce the string tension and the ball will spend more time in contact with the strings. (It is a well documented scientific result that looser strings lead to longer ball dwell times.) But will this lead to better control? Probably not. You have often heard the advice "string tighter for more control, looser for power." Both of these pieces of advice cannot be correct since they contradict each other. What is correct?

The racquet frame and the stringing in that frame can influence the number of errors that you make. For a long time the literature of tennis has stated that tighter strings lead to better control, and better control means fewer errors. However, nowhere in the literature can you find an experiment, data, or a scientifically valid argument explaining why this is so. (An article appearing in the Winter 1999 edition of the International Sports Journal was the first to show a measurement of the correlation between string tension and control.) It has long been known that lower

string tension does produce greater ball speed (power), but the arguments about the correlation between tension and control are usually anecdotal.

Spin Shots and Control. There is a second reason why tighter strings can lead to fewer errors. Often when you are hitting a heavy spin shot, the ball will clip the frame and you will get a complete miss-hit. By stringing the racquet at a higher tension, the chances of such a miss-hit will be reduced. On a heavy spin shot, the strings meet the ball at a large angle, as opposed to nearly normal on a flat shot. The ball then slides across the strings before rebounding off. The distance the ball slides along the strings depends on the angle of the racquet, the racquet head speed, and the dwell time of the ball on the strings. The shorter the dwell time (the tighter the strings), the shorter is the distance that the ball slides along the strings, and the less likely is the ball to run into the frame. Therefore, increasing the string tension will reduce the probability of a frame miss-hit and increase the acceptance window of ball impact locations on the head of the racquet. As a general rule, when hitting shots that are designed to have a lot of spin, you don't want the initial ball impact location to be at the center of the head (along the long axis). Since the ball is going to slide or roll along the strings for a certain length, you want an initial impact point that is a distance away from the axis, which will allow the ball to cross the axis while rolling or sliding, and then depart the strings on the other side of the axis. This will also reduce the probability of striking the frame.

Headsize, Polar Moment, Off-Center Hits and Control. In normal play, you attempt to hit the ball at a specific location on the strings, possibly the center of the head. If you miss that location, you may make an error. This is because the ball comes off the racquet at an unanticipated angle or with a speed higher or lower than desired. The result of this slight miss-hit is a ball trajectory that often ends in the net or over the baseline.

When the ball hits the strings at a location that is not along the long axis of the racquet (toward the side of the head), the frame will twist and the rebound will come off with lower speed than the same shot hitting on axis. Assuming the long axis of the racquet is parallel to the ground, a ball impact above the axis will result in the racquet twisting, a rebound angle greater than desired, and also less ball speed. This combination of effects (larger vertical angle and lower ball speed) will often lead to ball trajectories that still end landing in the court. If, on the other hand, the ball strikes below the axis, the rebound ball angle will be lower than desired and the ball speed again will be lower. This combination often leads to shots that end in the net. (An error reducing strategy would be to attempt to always hit the ball slightly above the axis. However, the unanticipated consequence of such a strategy might result in tennis elbow from the constant twisting of the racquet due to off-axis ball impacts.)

The fractional reduction of rebound ball speed depends on how far off axis the impact is and a physical property of the racquet called the moment of inertia about the long axis. The larger this moment of the racquet, the less the rebound ball speed will be effected by off axis hits and the more stable the racquet will be against twisting. Therefore, by going to a racquet with a larger polar or roll moment of inertia (the moment about the long axis), you will make fewer errors due to the miss-hits, some of which are almost inevitable. As a general rule, racquets with a wider head (as opposed to widebody, which is thicker) will have a larger moment of inertia and be more stable. A frame that is 25% wider (10 inches instead of 8 inches) can result in a 50% greater

moment. This means that using a racquet with a wider head usually will result in fewer errors due to this type of miss-hit. Howard Head knew what he was doing when he designed the original Prince oversize racquet.

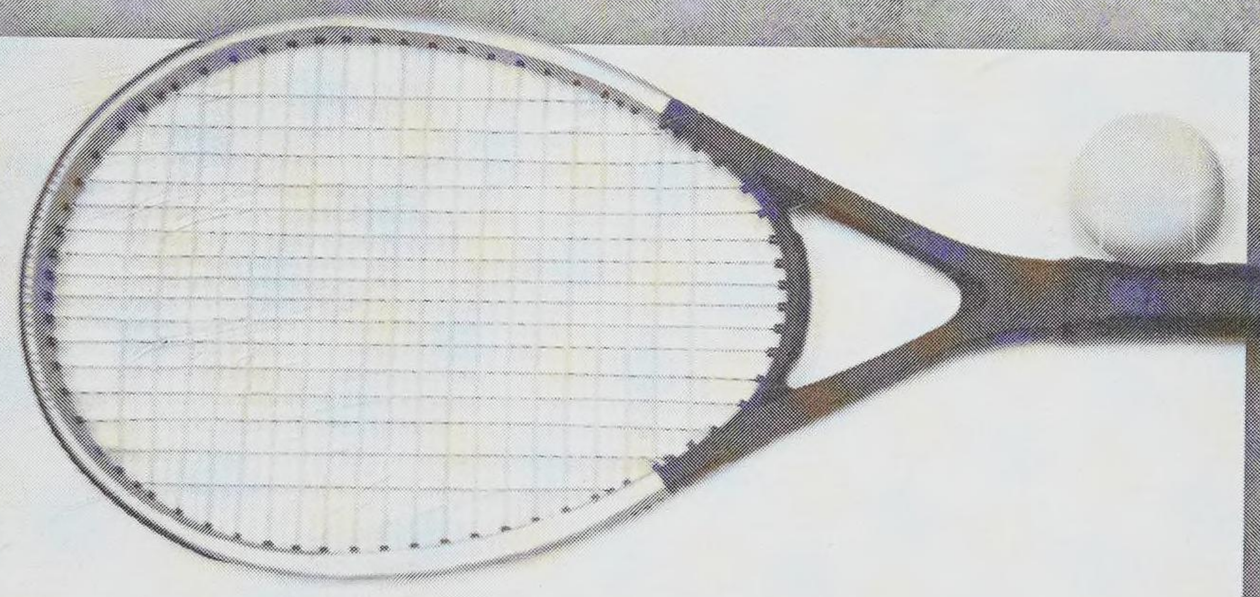
Racquet Balance, Sweetspot and Control. If the ball hits the racquet along the long axis of the racquet, but at different locations along that axis (closer to the tip or to the throat), errors of depth can again occur. For many racquets and classic ground stroke swings, the ball rebound speed decreases as the ball impact point moves from the throat area toward the racquet tip, all other parameters held constant. If you are attempting to strike the ball at the center of the head and trying to hit a deep shot, ball impacts that occur closer to the throat are likely to go long because the ball will come off of the racquet with too great a speed. This can be compensated for by consciously hitting your normal (central impact) shots so they bounce short. Now the balls that inadvertently impact closer to the throat will land in the court. This is a very conservative strategy, and is comparable to hitting every ball softly to insure a large window and few errors.

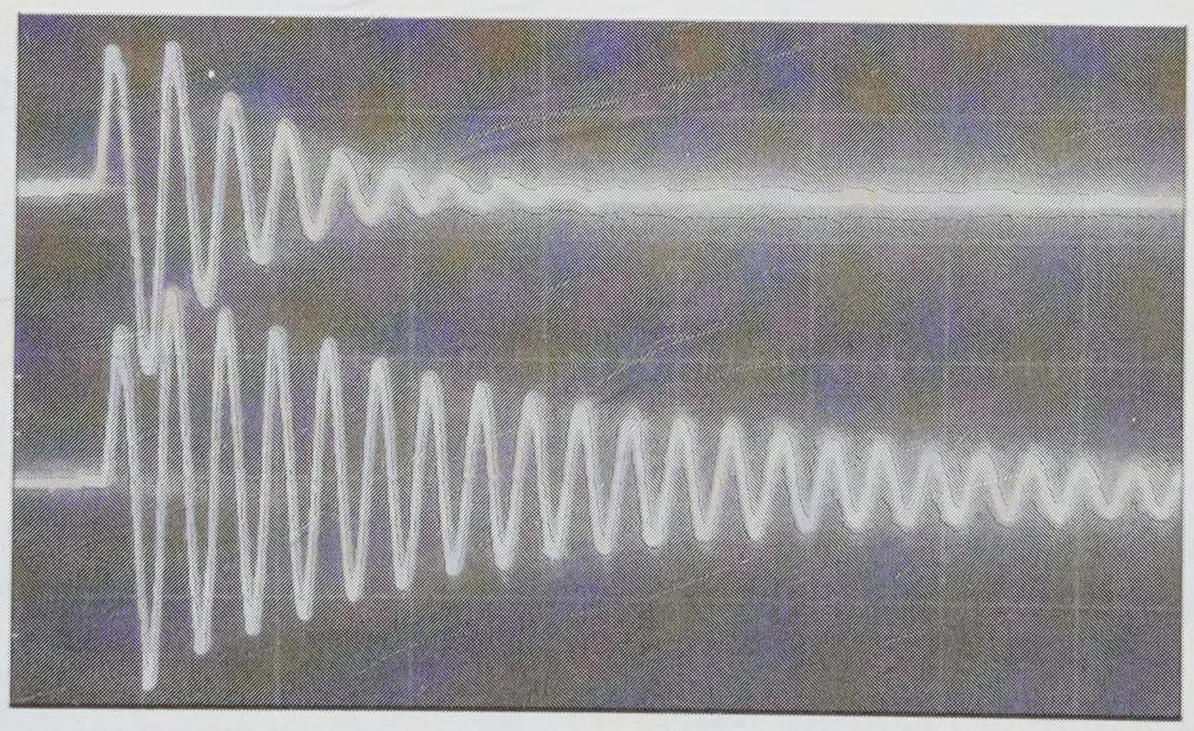
Some of the modern, very stiff, handle-light frames now available, have succeeded in moving the power region further up into the head of the racquet. When one of these frames is combined with a flexible wrist swing (not the classic long, flowing stroke), the effective maximum power point of the racquet can move up so that it is at or near the center of the head. This is because the center of the head is moving faster than the throat on these type of swings. Calculations show that when the power maximum is at the center of the head, the variation of rebound ball speed as the ball impact point moves away from the center, is greatly reduced, compared to the classic racquet used with a classic stroke. In other words, the "power spot" has been enlarged and made more uniform. Clearly, this will reduce errors of depth.

In addition, there is a built-in error correcting mechanism that occurs when the maximum power point is at the center of the racquet head. Any shot that misses the center will come off the racquet with slightly less rebound speed than a shot that hits at the center. If you then attempt to hit your shots so that balls that strike the racquet at the center of the head land deep in the court, depth errors (balls going over the baseline) due to variation in impact location will automatically be reduced. Any shot that hits the strings slightly closer to the throat or slightly closer to the tip than the center of the head, will come off of the racquet with slightly less speed, and therefore will land a bit shorter. These types of slight miss-hits will not result in balls that land long, and therefore depth errors are reduced. Because this effect is working in your favor, you can hit your shots deep, go for the baseline, and play a more aggressive game

PART THREE

STRINGS





Justine Henin, Belgium

26

Overview of Strings

Introduction to String Behavior

By Howard Brody

Introduction

The strings in a tennis racquet perform a number of functions. Their principal purpose is to absorb the kinetic energy from the relative ball-racquet velocity, turn it into potential energy (stored energy of elastic deformation), and then give that energy back to the ball in the form of an outgoing ball velocity or kinetic energy. Since the string plane is the only thing that touches the ball, it is also the agent for putting spin on the ball and for controlling the outgoing direction and speed of the ball.

Air Resistance

A racquet is strung with thin strings, rather than having a fabric or a membrane covering the head of the frame because of air resistance. Even with the relatively thin strings taking up just a small fraction of the area, there is some difference in air resistance between a strung and unstrung racquet. To get a feeling for what air resistance does, perform the following experiment. Cover the face of your racquet with a sheet of paper and swing it, then remove the paper and swing it again. Note the difference in the way the racquet feels. You might also see if you can detect the difference in feeling between a racquet swung with the face perpendicular to the ground (a normal swing) and the same racquet swung with the face parallel to the ground (knifing through the air). It is interesting to ask whether you as a player, can tell the difference in air resistance between a racquet strung with 1.4

mm thick nylon and one strung with 1.3 mm thick nylon? Probably not, but a scientist who had a well-instrumented wind tunnel might notice a difference.

The String Plane Deformation

While players and racquet stringers talk about the tension in the strings, it is actually the string plane deformation that determines many of the playing characteristics of the racquet. Racquets strung at the same tension with the same type of string can play very differently, depending on the head size and the string pattern or string density. A measurement of string plane deformation takes all of these factors into account, yet tension (specified in Newtons, pounds or kilograms) is still the standard measure for specifying to a racquet stringer how you want the racquet strung (because that is what the stringing machine is calibrated in). Don't ask your stringer for a specific string plane deformation, because the odds are that you will not get it, unless you have a very experienced stringer who checks string plane deformation on every job he does. Tension is an acceptable parameter to work with if you always use the same racquet model (same head size and string spacing or pattern); but to compare the string performance between two different racquets, string plane deformation should be used. For small deflections of the string plane, string plane deformation does not depend on the type of string used, so nylon, gut, poly, etc., when strung in the same racquet at the same tension, will give about the same string plane deformation.

String plane deformation can be measured by at least two different types of commercially available devices. The one most commonly used is an instrument (such as an RDC) that pushes on the string plane with a known, static force in a direction perpendicular to the face and measures the distance the string plane deforms or deflects. A second type of instrument places a mass on the strings at the center of the head and measures the resonant frequency of the mass-string system. The frequency of this oscillation depends on the square root of the string plane stiffness. Both of these types of devices are usually calibrated so they produce a number that is closely related to the tension in the strings, once the head size and string spacing are taken into account.

- String plane deformation determines most of the playing characteristics of the racquet that are due to stringing.

- For a given racquet, an increase in string tension will produce a decrease in string plane deformation. (The tighter the strings, the more you have to push on the string plane to get a certain deformation.)

- As a general rule, if all else is held constant, a frame with a larger head will have more string plane deformation.

- As a general rule, if all else is held constant, a frame with a larger spacing between strings (a more open string pattern) will have more string plane deformation.

- A racquet with a greater string plane deformation will play softer.

The Ping of the String

When the ball hits the strings, or when you pluck or strum the strings, there is a sound, almost musical in character, if there is no vibration dampener installed

in the string bed. For most racquets, the strings will vibrate at about 500 to 600 Hertz (cycles per second) when displaced from their equilibrium position, and that is what you hear. Many people tap their racquet's strings to assure themselves that they are still at the correct tension. This is because the frequency that the string plane vibrates at is directly proportional to the square root of the string tension. In fact, the frequency at which the entire string plane vibrates is close to the frequency of vibration of a single string that makes up that plane at that tension. You cannot compare tension between two different size racquets by listening to the ping. At the same tension, the bigger head will have a lower pitch. (The frequency goes as the inverse of the string length, so a longer string will have a lower frequency than a shorter string at the same tension.) Between two identical racquets at the same tension, you will get a different sound if different gauge strings are used. If you do have two identical frames, strung with the same string, then you can compare them, and the lower pitch will be caused by a lower tension. For those of you with "perfect pitch," you will be able to tell just how much the tension in your racquet has decreased in time.

If you place a string dampener in the string bed near the throat of your racquet, it will damp out the vibrations of the string (not the frame) very quickly and most of the pitch information will be lost. You will hear a thud, rather than a ping. The Rules of Tennis do not allow you to place a string dampener within the pattern of crossed strings.

- There is no evidence that string vibrations can cause any harm to the player.

- There is no evidence that string vibrations have any effect on power or control.

Dwell Time

The string plane deformation also determines the dwell or contact time of the ball on the strings. The more the string plane deforms for a given load, the longer a ball will spend in contact with those strings. This means that lower string tension produces longer dwell times and string planes with less deflection produce shorter dwell times.

It is counter-intuitive, but the harder you hit the ball, the shorter the dwell time. You somehow have a picture in your mind of a harder hit ball sinking deeper into the string bed, and therefore taking longer to come out. What actually happens, is that the harder you hit the ball, the stiffer the strings get, as does the string plane. Even though the ball sinks in more, it also snaps back sooner due to the increase in string plane stiffness.

Many types of strings get stiffer or less elastic as they are stretched, so when you hit the ball harder, the impact may feel harsher, because of the shorter dwell time and the increase in string stiffness. This will contribute to the "boardy" feeling that some synthetic strings produce.

There is an old adage in tennis that says "Keep the ball on your strings longer for better control." As far as I know, there is no scientific justification for this statement in the literature of tennis, or anywhere else. In fact, there is another old adage that says, "Tight strings give better control." When you go to tighter strings, the time the ball spends on the strings DECREASES (shorter dwell time), so these two old adages disagree with each other. There is nothing that you can do with your swing that will increase the 0.004 seconds (4 milliseconds) of

ball dwell time on the strings, except not hit the ball as hard, which will probably improve control by itself. What can be done, and should be done, is to follow through the stroke in the direction you want the ball to go. Then slight errors in timing will not translate into errors in ball rebound angle. This follow through in the ball rebound direction will improve control and give the appearance that you are keeping the ball on the strings longer.

When the new Type 3 ball (6% larger in diameter) was introduced, it was tested extensively in both the lab and on the courts. Players claimed that the larger ball felt heavier in play than the standard Type 2 ball, even though it was actually slightly lighter. Lab tests showed that the larger ball was just a bit softer, leading to a slightly longer dwell time. Since a ball that is actually heavier will have a slightly longer dwell time, it is possible that this is the effect that causes players to claim the bigger ball feels heavier.

Power

A tennis ball is designed (and required by Rule 3 of the Rules of Tennis) to lose energy when it deforms. When you drop a tennis ball from a height of 100 inches onto a hard surface, the Rules of Tennis state that it must rebound to a height of at least 53 inches and not more than 58 inches. This means that 42 to 47 percent of the energy the ball had when it hit the ground is lost – dissipated in heat, sound, or other mechanisms. Another way to characterize the interaction is through velocity. The way to calculate the velocity return in a collision is to take the square root of the rebound height to drop height ratio. This is called the Coefficient of Restitution (COR), and it is the ratio of the rebound velocity to the incident velocity. As an example, for a rebound height of 55 inches, the ratio is 55/100 = 0.55 and the COR = 0.74, which is the square root of 0.55.

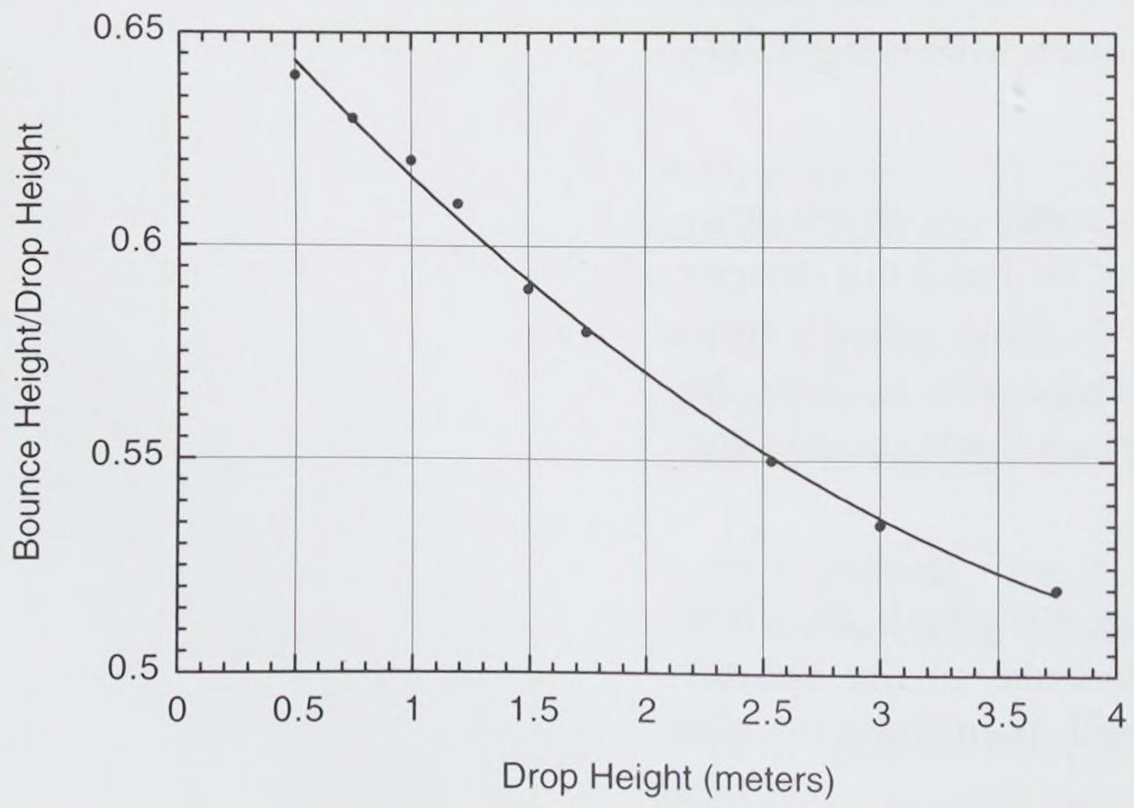


Figure 26.1

Bounce Height/Drop Height. The ratio of rebound height to drop height versus drop height for a tennis ball on a hard (concrete) surface.

In collisions where the ball deforms more (a higher velocity collision with the hard surface), the fraction of the energy lost increases, which means the COR decreases. If the ball does not deform as much (for example, it is dropped from a lower height than 100 inches), the percent of energy loss will decrease and you have a higher COR. This is displayed graphically in Figure 26.1, which shows the ratio of rebound height to drop height (fractional energy return) for various drop heights. If you could keep the ball from deforming when it collides with something, then the energy loss in the ball might be reduced and the ball will rebound with a higher velocity (or it will be "livelier"). If a ball is dropped from 100 inches, not onto a hard surface, but onto the strings of a racquet (with its head firmly clamped) the ball will rebound to a height between 75 and 80 inches. In such a collision, the strings deform, absorb some of the ball's incoming energy, and then return it to the ball very efficiently (with essentially no loss of energy in the strings). The strings act

like a trampoline, absorbing some of the incoming energy, and then returning it to the ball. The more energy the strings absorb, the less is left in the form of ball deformation. The strings give back 95% of the energy they absorb while the ball gives back only about half of the energy it stores. Clearly, you will get more "power" if you arrange it for the efficient strings to absorb more of the incoming energy and have the dissipative ball store less energy (in the form of deformation). The looser the strings (low tension), the larger is the fraction of energy stored by the strings, and therefore, a larger fraction of the incoming energy will be fed back to the rebounding ball. A calculation of this effect is shown in Figure 26.2. However, there is a limit to this, since you cannot play tennis with a butterfly net. When the strings begin to move within the string plane, rubbing each other as they move, energy is lost.

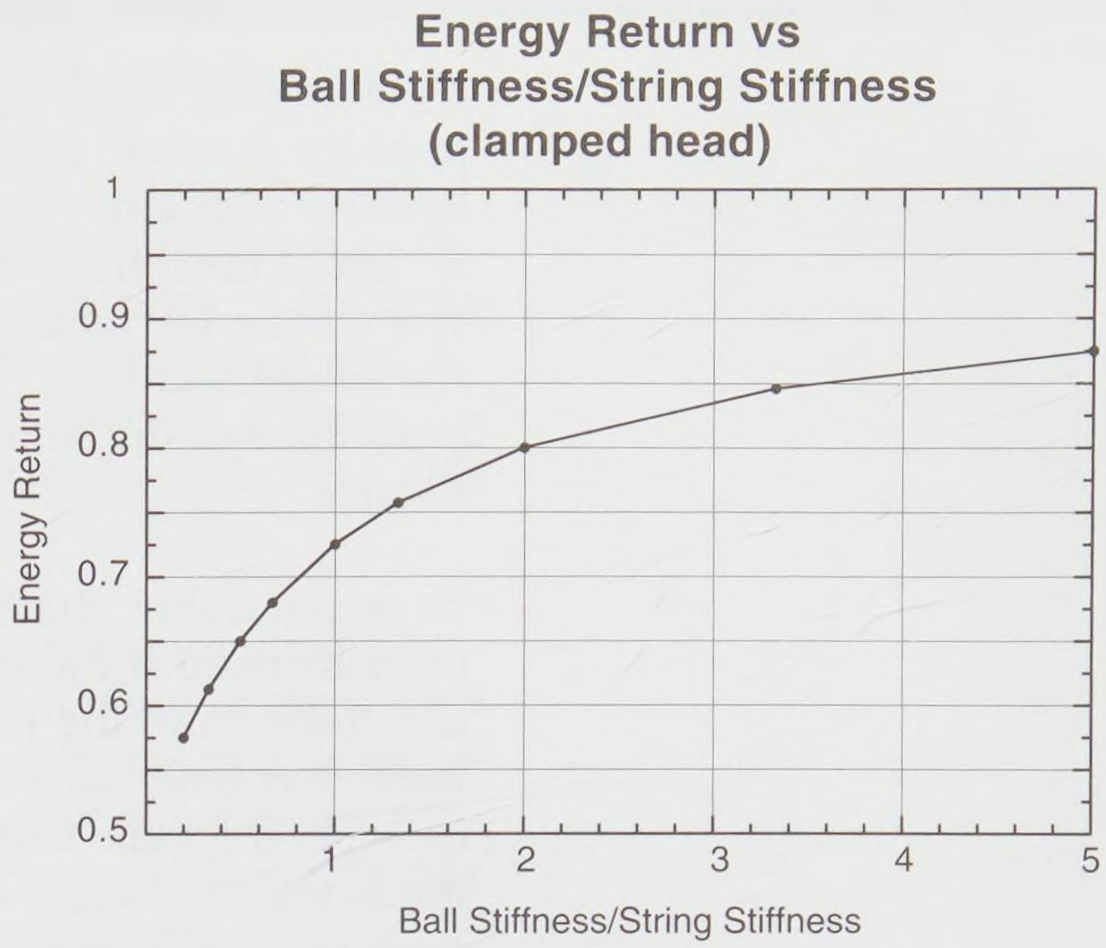


Figure 26.2

Calculation of Fractional Energy Return versus Relative Ball/String Stiffness. As the strings become softer, they absorb then return more of the energy the ball is bringing in.

How do we know that strings are about 95% efficient in returning energy? If you clamp the head of a racquet so the string plane is horizontal and drop a non-deformable (hence non-energy dissipating) ball on it, it will rebound to about 95% of its drop height. This has been done with a wood Bocce ball and also with a steel ball. The drop heights were such that the energies involved were comparable to a typical ball-string impact. These results were also fairly independent of whether the strings were old (in the racquet for years) or new and also independent of the type of string. If you were to do the same experiment with a tennis ball (which is deformable and dissipates energy when deformed), the rebound height would be about 80% of the drop height, compared to a 55 to 60% return when a tennis ball is dropped on a hard surface. This does depend somewhat on the string tension.

- Tight strings (high tension, stiff string plane) will return a little less energy to the outgoing ball.

- Loose strings (low tension, softer string plane) will provide a little more power, but strings that are too loose will dissipate energy.

- For a given tension in the strings, longer strings (a bigger racquet head) will produce a softer string plane and more power.

- For a given tension, an open string pattern (more space between the strings) will produce a softer string plane and more power.

Are there limits to how low you can go in tension and still get a bit more power (rebounding ball speed) from your strings? As long as the deformation of the strings is perpendicular to the string plane, the power should increase. If the strings begin to move within the string plane, rubbing against each other, then it is possible to not get back out of them the 95% of the energy you might otherwise expect. (You can't play tennis with a butterfly net).

The Feel of the Strings

If you reduce the string tension (reducing the string plane stiffness) you should produce a more comfortable, softer feeling racquet. Primarily, this is not because with lower tension you get more power, so you don't have to swing as hard. This extra power is a small effect compared to the true reason. When you reduce the tension in the strings, the dwell (contact) time of the ball on the strings increases. The increase in dwell time leads to a reduction in the peak force of the strings on the ball, which makes the impact feel more comfortable.

To return a shot, the ball's momentum must be turned around, and a change in momentum is caused by the impulse of the strings on the ball. An impulse is a force times the length of time the force acts (it's actually the integral of force over time), so if the time increases (longer dwell time) the force decreases when the same impulse is needed to return the shot. Another way of looking at this is to think about the shock being spread over a longer time, so its peak value is reduced.

As a general rule, to have the strings play the same, the string tension should be scaled with string length. When you go to a racquet with a larger head, you should increase the tension in the strings in proportion to the increase in string length, if you want a comparable feel – the racquets to play the same.

Elasticity

All tennis strings are elastic – they stretch, absorbing energy, then return it to the ball. Some strings require more force to elongate a certain amount. The ratio of the incremental force required to increase the string length a certain percentage is called the modulus of elasticity. The smaller this number, the softer are the strings and the softer they play. Steel strings (piano wire) or Kevlar strings have a very high modulus of elasticity, and even though they give back essentially all the energy they absorb, they hardly stretch at all when impacted by a tennis ball. Nylon and gut have a low modulus of elasticity, so they deform when a ball impacts and they play well. Nylon and gut differ in elastic modulus when they get to high tension. Both may stretch about the same amount in being brought to 60 pounds tension, but synthetics get stiffer (higher modulus) above that tension, where gut does not. Since when the ball hits, the strings in the region around the impact area stretch a great deal, their tension increases a lot. The synthetic materials become stiffer, and you do not get as much "pocketing" of the ball in the string plane or the same soft feel that gut gives.

Chapter 30 explains how to measure the elastic modulus for various strings.

Gauge

The thickness of a string (its diameter) is specified by a gauge number, where a higher gauge number means a thinner string. Where almost all tennis measurements and specifications are often given in English units (tension in pounds, string length in feet, etc), the string diameter is traditionally given in millimeters (mm) rather than inches. So this means that a 16-gauge string can be about 1.30 mm in diameter and a 17-gauge string can be about 1.25 mm in diameter, UNSTRETCHED. When under tension, the string stretches and becomes slight-

ly thinner. (The amount the diameter changes for a change in length varies according to the material. The ratio of how much the diameter changes for a fractional change in length is called Poisson's ratio and it runs from about 0.1 to 0.3 for typical materials.)

As a general rule, the tensile strength of a given string is proportional to the string's cross sectional area or its diameter squared. You may think of the string as made of fibers, the number of which goes as the area of the string and the strength also goes as the number of fibers. Hence, the strength goes as the diameter squared. However, since strings often have a unique construction, this is just a rule of thumb.

The elasticity of a string is proportional to the inverse of the string's cross sectional area. The thinner the string, the more it will stretch under a given load and the more elastic it will be. Since elasticity is a desirable trait in a string, as another general rule, thinner strings play better. This again assumes the construction is the same, and the thinner string has not been produced by just stretching a somewhat thicker string down to a thinner diameter.

The better playing string (control, comfort, power) is usually the string which is thinnest, so it may break sooner. If breaking strings is a problem, then a thicker string will often be advantageous.

Wear and Breakage

How come some recreational players seem to be able to play with the same set of strings for years, while other players cannot get through a match without breaking a string at least once every set or two? There are a number of factors here that depend as much on the style of play as on the inherent string properties. If a player does not hit the ball hard and does not use spin on the shots, a set of strings may not break for a long time. On the other hand, a player who hits the ball hard can easily break strings, particularly mains near the top of the frame if they are worn or notched.

When a ball impacts on the strings near the frame, the strings tend to stretch asymmetrically, and the percentage of string elongation on the frame side of the impact is much greater than a typical impact near the center of the head. This increases the tension in that short section of string a great deal, possibly well beyond the tensile strength limit of that string, so it breaks. A good example of this is a very hard serve, struck near the tip of the racquet or on a misshit near 3 or 9 o'clock. The strings on the short side of the impact location are fixed in place by the frame. The ball may penetrate the string plane by a considerable amount, stretching the short lengths of string beyond their limit.

Another cause of premature string breakage is due to a particular type of wear. If a player hits with spin, the mains saw back and forth on the crosses, notching the mains. This notching reduces the cross sectional area of the string capable of carrying a load, making it effectively a thinner, weaker string at the notch. When sufficient string has been sawed away, the remaining string cannot support the incremental tension induced by the impact of balls, so it fails. If the courts have loose granules that get picked up by the ball, they may get into the strings and then they tend to act as an abrasive in the sawing process, which accelerates the breakage. Testing of string in wear chambers (where strung racquets are hit by balls repeatedly) show that higher string tension may

increase string life, compared to lower string tension and this seems counter-intuitive. This result is because at higher tension there is less string movement, hence less sawing, less wear, and less notching.

String Patterns

Racquet frames come with many different types of stringing patterns, ranging from 45-degree strings to three-way stringing to a rectangular grid or fan pattern of strings with various spacings. In each case, the manufacturer (or inventor) is attempting to accomplish something. We will analyze only the rectangular grid patters, since they are by far, the most commonly found patterns.

If the head of the racquet were rectangular or square, as opposed to what it actually is — oval, round, or teardrop in shape - then all the main strings and all the cross strings would be the same length. Since the string behavior is a function of string tension divided by string length, this would make the analysis much easier. However, since for most racquets, the strings near the periphery of the head are shorter than the ones that are in the middle, they play somewhat stiffer and with a bit less power. In addition, a string tends to play stiffer near its end, as opposed to in its middle. This is a second effect that tends to stiffen the string plane around its edges. To compensate for these effects, the racquet designer goes to a non-uniform string pattern, with more space between the peripheral strings than between the central strings.

Control

There is an old adage in tennis that says, "String tight for control, loose for power." Is this true, just a perception that players have, or another of the tennis myths that are not correct when put to the test? If you look at some of the tennis magazines of 20 to 30 years ago, you would find just the opposite advice. The logic on power went something like this: most big hitters string their racquets rather tightly, so it follows that tighter strings give more power. The logic on control went something like this: to control the ball better you want it to spend more time on your strings, and lower tension leads to longer dwell times. Hence the prevailing (and erroneous) argument was: string looser for control, tighter for power. However, we still see advice that tells the player to keep the ball on the strings longer for better control.

We now think we know better. Looser strings give you just a little more zip on the ball and this has been confirmed in both laboratory testing and in theoretical models. Where it is relatively easy to measure rebound ball speed in the lab, control is another matter, and most (but not all) of the data is anecdotal. One prevalent theory is that when a ball strikes the racquet off of the principal axis, the racquet tends to rotate about that axis, and the ball comes off of the strings at an errant angle. The looser the strings, the longer the ball dwell time, the more the racquet will rotate while the ball is in contact, and the larger will be the ball angle relative to the desired ball direction. This scenario has been verified in lab testing by firing balls off center at racquets with different string tensions. There was a correlation between string tension and the errant angle the ball came out at.

The fact that the ball speed will be in error for off-axis impacts tends to compound the problem.

There is a second effect that can cause a ball to come off of a racquet's strings at an angle.

If a ball impacts on the strings at some distance from the center of the head, the deformation of the strings is asymmetric. This asymmetry in string deformation should cause the ball to rebound at an angle that depends on a number of factors. The higher the tension (the less the string plane deformation) the smaller is this effect and the farther the impact is from the center, the bigger is the effect. It is the opposite direction from the direction the ball would go if the rotation of the racquet is considered. This effect tends to make the ball rebound back toward the center (a focussing effect), while the rotation effect tends to make the ball rebound away from the center.

Since there is good experimental evidence for the change in rebound angle due to the racquet rotation, this asymmetry of string deformation effect must be small compared to that effect.

Rebound Angle

When a ball impacts at the center of the head, normal (perpendicular) to the string plane, and the racquet is also moving normal to the string plane, the ball's exit or rebound angle will be approximately normal to the string plane. When a ball impacts at the center of the head at some angle to the normal to the string plane, the ball will rebound at some angle relative to that normal. As a general rule, the angle of rebound and angle of incidence are NOT equal to each other, as they are when light hits a mirror. You can calculate the rebound angle, to a good approximation, by breaking up the problem into a ball velocity normal to the string plane and parallel to the string plane. You then solve for the normal and parallel velocities after the impact, and then combine them to get the rebounding angle. When you actually carry out the calculation, for most ball impact angles, the ball goes into the rolling mode while on the strings, so the final parallel component of ball velocity is about 0.60 times the initial parallel component of velocity. The normal or perpendicular component of ball velocity is given by the formula:

$$v_{out} = e_A \cdot v_{in} + (1 + e_A) \cdot V_r \qquad (26.1)$$

where:

e_A (apparent coefficient of restitution) is the ratio of ball rebound speed to incident rebound speed for a free racquet as obtained in a lab test.

V_r is the speed of the racquet head at the point of ball impact.

V_{in} is the incident ball speed (normal).

V_{out} is the rebounding ball speed (normal).

The value obtained for v_{out} and $0.60v_{(parallel)}$ are added as vectors and the resultant ball velocity and angle are obtained.

Typical values of ACOR run from 0.5 (near the throat) to 0.20 (near the tip), and typical racquet head velocities are often similar to incident ball velocities. However, when the ball is hit hard, the ball tends to come off at an angle close to the normal to the string plane. When the ball is not hit hard, the exit or rebounding angle is close in magnitude to the incident angle. If the racquet is not moving, then the rebounding angle (relative to the normal) can be greater than the incident angle. This effect can be a cause of errors in ball placement, since the angle of rebound from the racquet depends on how hard the ball is hit.

Figure 26.3 shows the angle of the ball off of the racquet as a function of the racquet head speed for an incident ball of 60 feet/sec at an angle of 20 degrees.

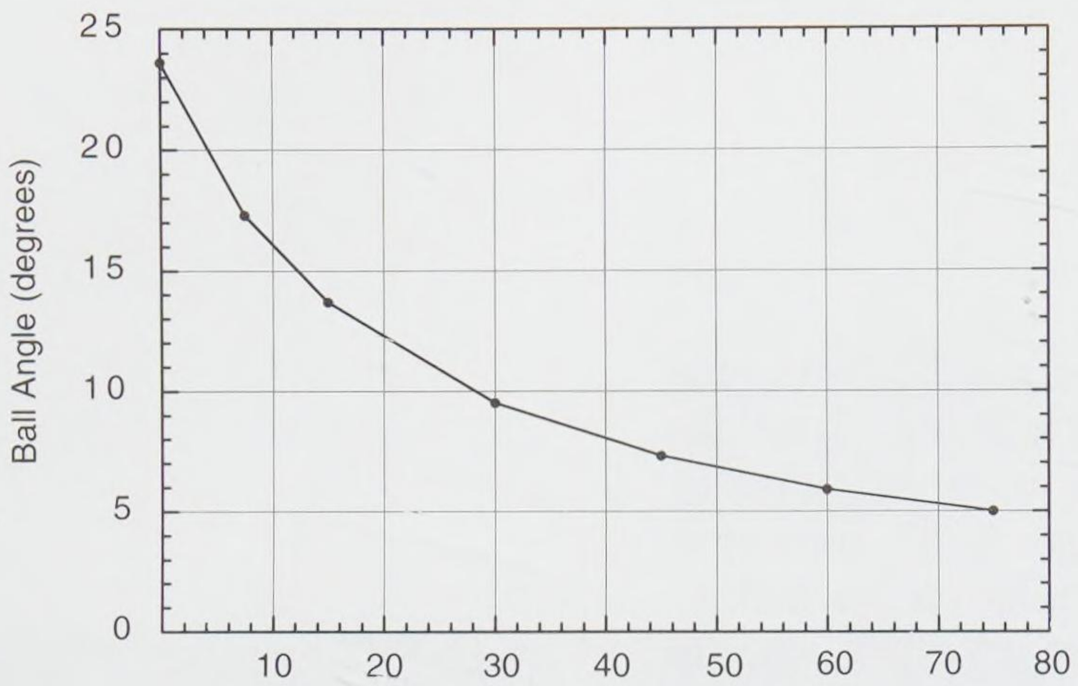


Figure 26.3

Ball Angle off of the Strings for Different Racquet Head Velocities. The ball is incident at 60 feet/second, at an angle of 20 degrees to the direction of the racquet normal.

When the ball is not incident normal to the string plane, it will come off the strings with a side-spin (spinning about a vertical axis). It is possible to calculate the amount of spin on the ball by taking the final parallel ball velocity component (about 60% of the incident parallel ball speed) and dividing it by the ball radius. This will give you the answer in radians/sec. To convert this to revolutions/sec, divide by 2π or 6.28.

Grommets

Grommets and grommet strips are placed in the frame of a racquet to protect both the strings and the frame itself. The grommets and grommet strips are made from a very tough material which prevents the strings from breaking through the thin-walled composite frame and also prevents string breakage by softening the bends and sharp edges where the strings go though the frame.

Recently, the size and shape of the grommets has been changed by many manufacturers, to make them bigger, rather than have them fit closely around the string. What is the purpose of having enlarged grommets with extra clearance, particularly in the string plane deformation direction (perpendicular to the plane)?

It is felt by many that longer strings play better, feel better, and give more power than shorter strings. However, the ITF in Rule 4 says, "The frame of the racquet shall not exceed 12.5 inches (31.75 cm) in overall width. The hitting surface shall not exceed 15.5 inches (39.37 cm) in overall length and 11.5 inches (29.21 cm) in overall width." This seems to put a clear limit on string length (15.5 and 11.5 inches) until the manufacturers and racquet designers realized that they could get around it – but only slightly around it. If they could suspend the string from the outside of the frame, rather than the inside of the hoop, they could effectively get an extra inch of string length in the crosses and at least that much extra in the mains. This meant a redesign of the grommets so that the string was free to move on the inner part of the frame, hence larger grommet holes are used.

This also reduces the number of broken strings due to ball impacts very close to the frame. With the old, smaller grommet holes, the string could not deform or deflect beyond the inner part of the hoop, so there were large stresses in the string when the ball hit near the frame. With the larger grommet holes, the string is not constrained by the inner part of the hoop, but the outer part, so the stress on the string is reduced and there is less breakage.

Tim Henman, Great Britain

27

String Chemistry

Where Do Baby Strings Come From?

By Crawford Lindsey

String is different than most of the solid objects you encounter in your day-to-day life — especially metals. The primary difference in their properties lies in how their atoms are arranged. In metal, the atoms are arranged in a crystalline lattice — every atom has the same neighbors in the same location as every other atom. And every atom is bonded with all its neighbors (Figure 27.1).

String is made of different stuff. Instead of a uniform geometric structure, string is made up of an aggregation of lots of molecular chains in various degrees of entanglement or alignment with their neighbors. And the bond between neighboring chains is not as formal as between lattice members in metals (Figure 27.2).

If you apply a force to the metal, it will be distributed instantaneously to all the atoms which will deform their positions until an equal force is built up to oppose the external force. In the string, the force will not be

A Hexagonal Crystal Structure of a Metal

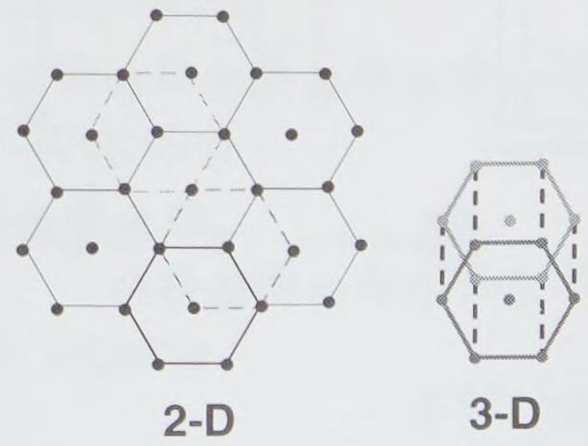


Figure 27.1

In a crystal structure, every atom has the same arrangement of atoms around it as does every other.

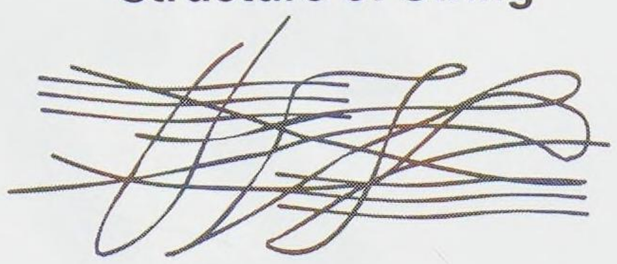


Figure 27.2

String is made of long chain molecules known as polymers. These molecules may be twisted, straight, entangled, oriented and unoriented. A more uniform orientation means a stiffer and stronger string.

as efficiently transmitted. Note the different ways these molecule chains can respond to a force: they can straighten, stretch, rotate, bend, slip past one another, and break.

These chains are known as polymer chains.

Polymers

What is a polymer? "Poly" means many and "mer" means unit. So polymer means many units. "Mer" is short for monomer, or single unit. These units are groups of atoms (molecules) arranged in a pattern that is then repeated end-to-end, over and over. So that's it, a polymer is a molecule arranged in a long chain of atoms. These chains can be several thousand units long and are thus sometimes referred to as macromolecules or giant molecules. Though even at that, polymer chains are still microscopic.

String is a plastic. Like all plastics, it has its beginnings in the ground as oil and its chemical raw materials are by-products of oil refining. These chemicals go to resin manufacturers (these may be the oil companies or the likes of Dupont, ICI) who convert them into resins. This is where the chemicals become polymers. This is where atoms and molecules combine to become nylon, PEEK, polyester or Kevlar®. Polymer resin (usually in the form of flakes, granules or pellets) is then processed (heated into liquid) and shaped (extruded) into the filaments that make up strings. This too is done by the Duponts of the world, though larger monofilaments are sometimes extruded by the string manufacturers.

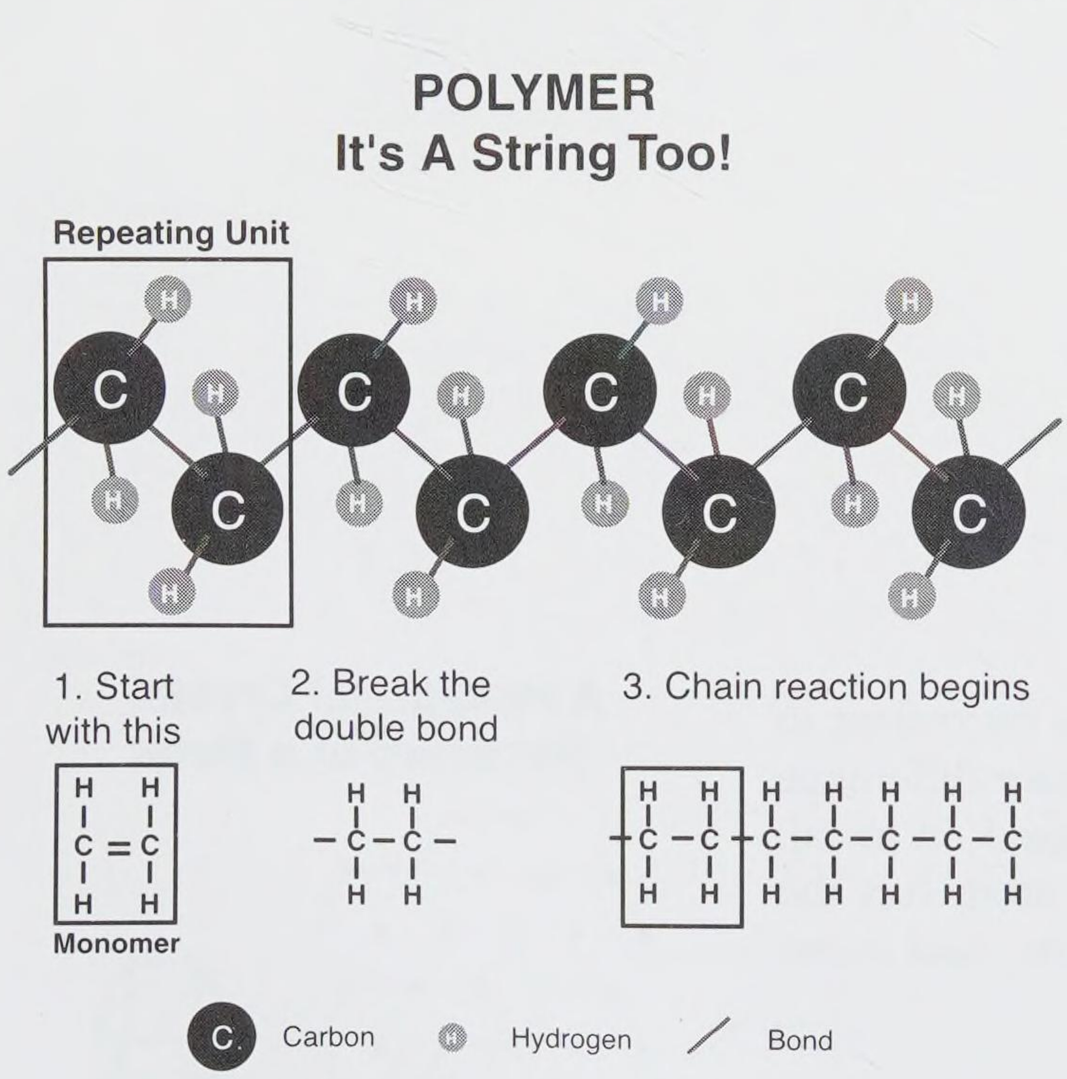


Figure 27.3

"Polymerization" = Fancy word for making long chain molecules. When the double bond in the starting monomer is broken, a bonding site becomes available on each carbon, setting up a chain reaction of polymerization — stringing together units into molecular sized strings. They're called polymer chain molecules.

So let's meet a polymer. Figure 27.3 presents the simplest shape for a polymer (polyethylene). There are a few things to notice about this thing to make us feel more comfortable and familiar with it. First, it is a thing. Polymer molecules have shape, length, and weight. They are put together with parts, just like anything else in the world. These parts, likewise, have different relative shapes, sizes, and weights. Some ingredients are bigger and stronger than others.

There is also an arrangement of parts. The parts have to be put together in an exact order or you no longer have the same thing. And just like any other building materials, you can't be trying to fit square pegs in round holes in order to stick these parts together. As we will see, only certain combinations of atoms can bond together.

It's no different than building a jungle gym in the backyard, and we just have to think in the same way. Don't get scared by formulas. They just tell us how many 2 x 4s and planks we need. And when two pieces fit together we nail them

(bond) together. To build the whole structure (all the molecules in the string), we build the molecules to specification and then fit all the molecules together in the right arrangement and relationship to each other and bond them to each other.

The molecule has some parts. In the polyethylene diagram, the circles are atoms. The chain of carbon atoms is the backbone (carbon is the backbone of virtually all polymers). The hydrogen atoms off to the side are known as pendant or side atoms. It's easy to see that if the backbone is strong and stiff, so will be the molecule. It's also evident that the number, size, and shape of the side groups will determine how closely you can fit molecules together, how flexible they will be, the amount of entanglement and how easily they can move around relative to their neighbors. Add them all up and you have an idea what the string will be like. A stiff string, for example, will have a stiff backbone or have molecules that fit together and are oriented into a stiff structure, or both.

The properties of a string come down to the size, shape, length, arrangement, and bonding of molecules.

Bonding

The properties of the string depend on how all the atoms and molecules are bonded which influences and is influenced by size and fit of atoms and molecules.

Bonding is electric. Chemistry is sort of electric tinker toys — building by matching positive and negative attractions together. It's all about attraction and repulsion and reaching an equilibrium where the energy between all the parts is at the lowest possible balance point. This is the most stable condition. When everything is in its lowest energy state, it takes the most energy to break apart the bonds. If it's not in its lowest energy state, there is either extra attraction or repulsion forcing the atom to behave one way or another — i.e., unstable — and you only need to add a little extra to nudge it into doing something.

Here's the key: bonds between atoms on the polymer chain are stronger than bonds between two separate chains. It's like a plate of spaghetti. If you pull on one strand, first the loose end straightens, then stretches slightly, and then it begins to slip its way out of the plate (Figure 27.4). This happens because the bonds along the spaghetti are stronger than those between two separate pieces. At some point, the amount of

Chain Tinkering
Factors affecting string properties

1. The chemical ingredients determine the polymer.
2. Average length and weight of the polymer chains, called the degree of polymerization. As the chain gets longer, it also gets heavier. The molecules are not all of the same length, so the average is important. The longer and more uniform the molecules are, the stiffer the string.
3. The amount of chain entanglement which in turn depends on length and shape. More entanglement means less slippage.
4. Arrangement and location of side groups affects how molecules can fit together.
5. Regularity of pattern of molecules relative to each other (crystallinity). More crystallinity creates more "structure."
6. Closeness of packing chains together for stronger bonds.
7. Orientation of the chains in the direction of applied stress. The more molecules that are aligned parallel to the axis of the string, the stiffer will be the string.
8. Types and strength of bonds between atoms within a molecule string and between molecules.

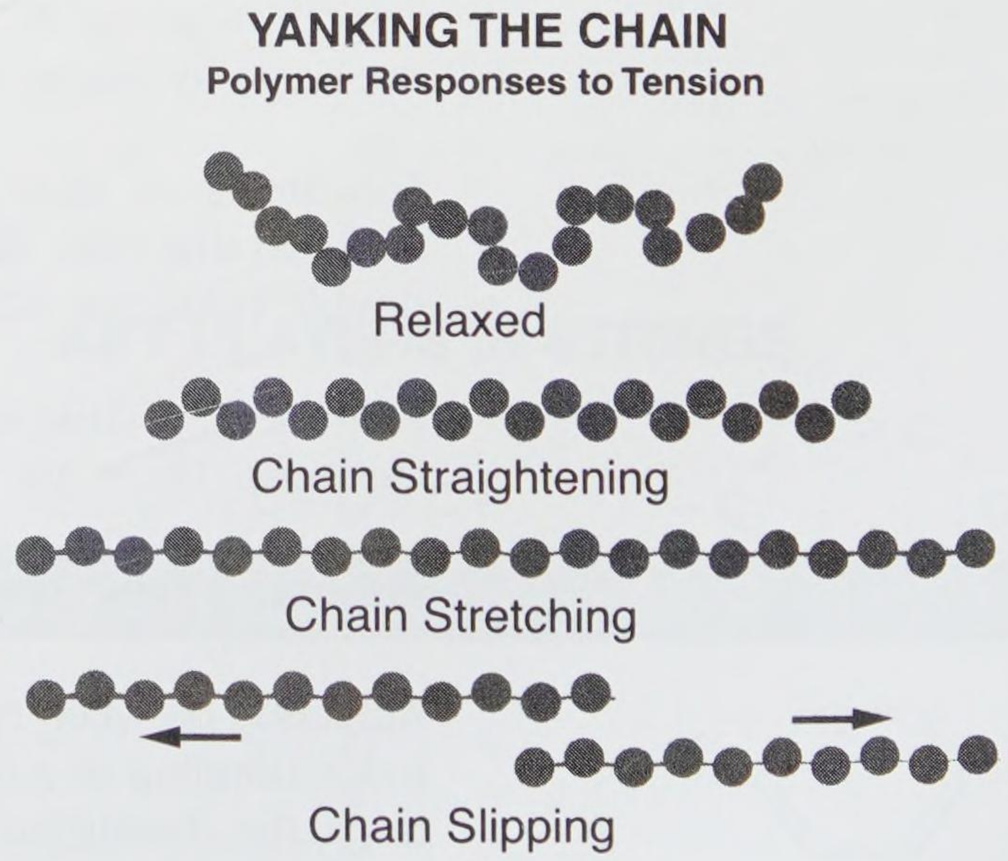


Figure 27.4

Chain straightening and stretching are elastic phenomenon. Chain slipping is plastic involving permanent deformation.

BONDS
Making Baby Strings

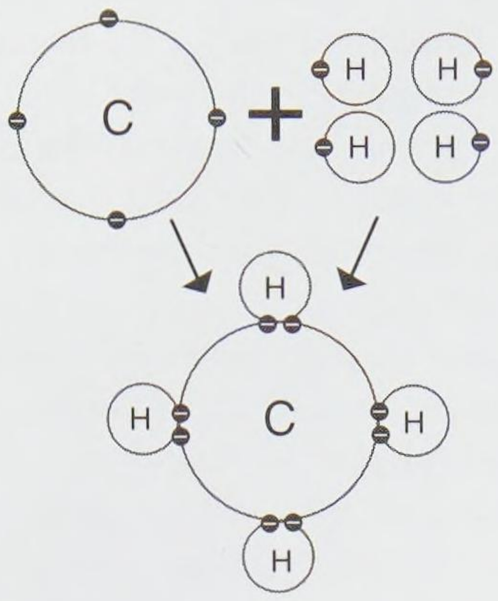


Figure 27.5

In string, molecular chains are held together by atoms sharing electrons. This is known as a covalent bond. Carbon needs 4 electrons to complete its outer shell, hydrogen one. Each bond represents two electrons — one from each atom. By sharing, each atom completes the other, thus forming a very strong bond.

entanglement may grab the spaghetti, and further pulling stretches and finally breaks the piece. This is similar to what happens in a string when it is stretched. The string is the mound of spaghetti and the molecules each individual noodle. It's up to the chemist to control how much of what happens when.

Bonding on a Chain

Understanding why the bonds along the polymer chain are stronger than those between chains is helpful in visualizing why and how a string behaves. The bonding behavior of atoms depends on the behavior of the electrons in its outer shell. Briefly, the atom is composed of a nucleus made up of positively charged protons and an equal number of neutral neutrons. An equal number of negatively charged electrons orbit around the nucleus in shells. There are a maximum of 7 shells and a maximum of 32 electrons in a shell. The inner shell has a maximum of 2 and the outer shell 8 electrons. The number of electrons in the outer shell is what determines an atom's bonding behavior. The most stable, lowest energy state for an atom of any element (except hydrogen and helium) is to have 8 electrons in the outer shell. The goal of an atom's life is to complete its outer shell. In a polymer, this is done by sharing electrons (Figure 27.5).

This sharing bond is known as a covalent bond. "Co" means sharing, and "valence" refers to the number of electrons an atom must share to complete its outer shell. Carbon always has 4 sharing bonds, hydrogen 1, nitrogen 3, and oxygen 2. So in diagrams, you'll always see that many lines (bonds) joining the respective atoms. If you don't, a shorthand notation is being used that leaves out some hydrogen atoms.

Sometimes an atom (especially carbon) shares multiple bonds with another atom. In that case, there are two lines between the atoms. In the instance of a double bond, the atoms would be sharing 4 electrons — two from each atom.

So these molecular-sized strings are made up of atoms along the chain sharing electrons. These are very strong bonds.

Bonding Between Polymer Chains

Attraction between two separate polymer chains is a different story. It's not so much bonding as it is a little hanky panky on the side. When atoms share electrons, the sharing isn't always equal. Depending on the relative sizes of the two atoms and their respective tendencies to attract electrons, the electrons from one atom may tend to spend more time around the other. So even though the bonding arrangement as a whole is electrically neutral, it is not equally neutral all over its surface. Sort of a miniature magnet is set up with a positive and negative end. Figures 27.6 and 27.7 show how the positive and negative ends of bonds on separate chains attract each other.

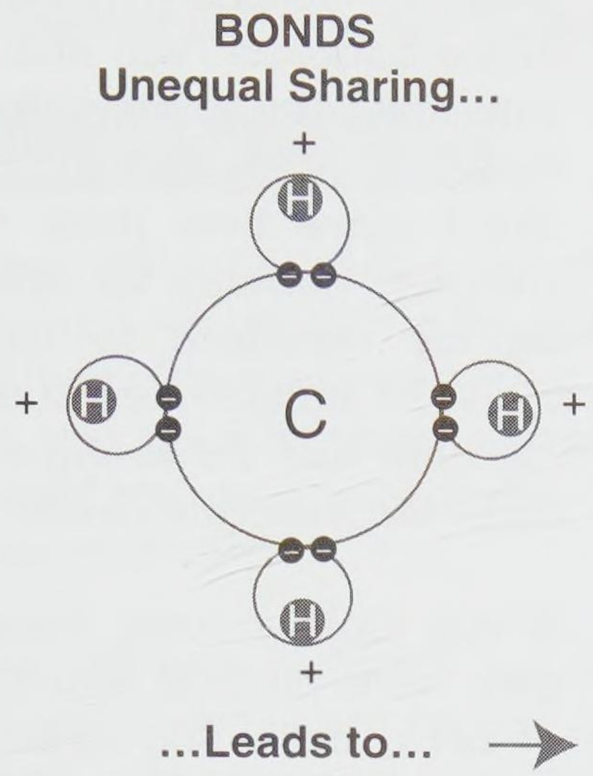


Figure 27.6

Hydrogen's shared electrons spend most of their time hanging out in the space between its nucleus and the carbon atom. This leaves the hydrogen nucleus with its positive proton sticking out looking for a good time – the negative end of a polar bond on another chain.

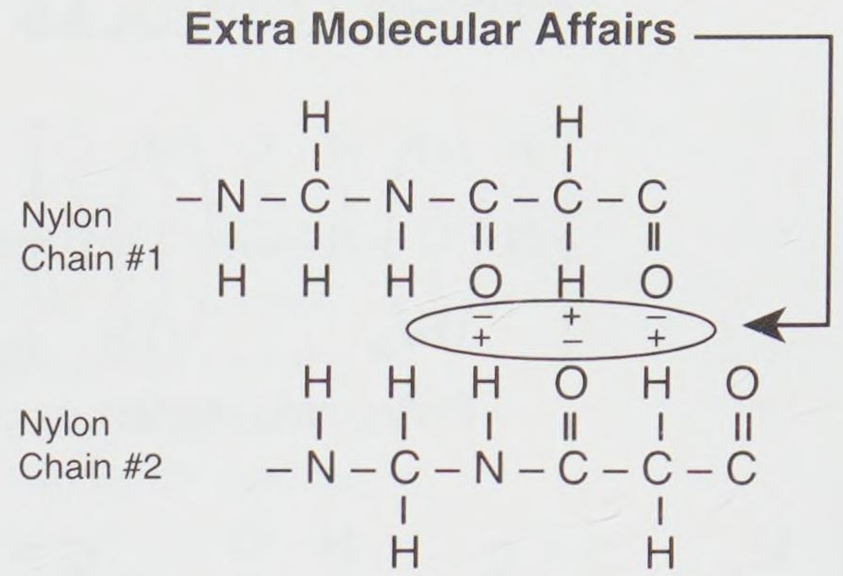


Figure 27.7

Polymer chain atoms are promiscuous. This may not be moral, but it makes for good string. These secondary forces hold chains in place.

Hydrogen, for example, only has 1 electron, so when it is in a covalent bond it becomes very polar as its electron hangs out closer to the atom it is bonded with, leaving an "exposed" positive back-end. These kind of forces are known as secondary forces and this type of polar hydrogen bond is the strongest of these. So chains with a lot of hydrogen atoms as side groups tend to have stronger secondary bonds keeping the chains in alignment. This is true of nylon strings.

String Materials

Okay, so we want to start bonding a bunch of stuff together to make a string. To do this, just visualize taking our simple polymer in Figure 27.3 and imagine substituting different configurations of atoms in place of some of the hydrogen atoms or added into the backbone. There are several of these configurations called functional groups that appear in string materials, and these typically dictate the string's properties. Consequently, their group name usually appears in the material's name. For instance, if an amide group is in the repeating unit, it is a polyamide. Some of these groups appear in Figure 27.8.

Molecules that have simple linear backbones are called linear molecules and those that have rings are called cyclic or

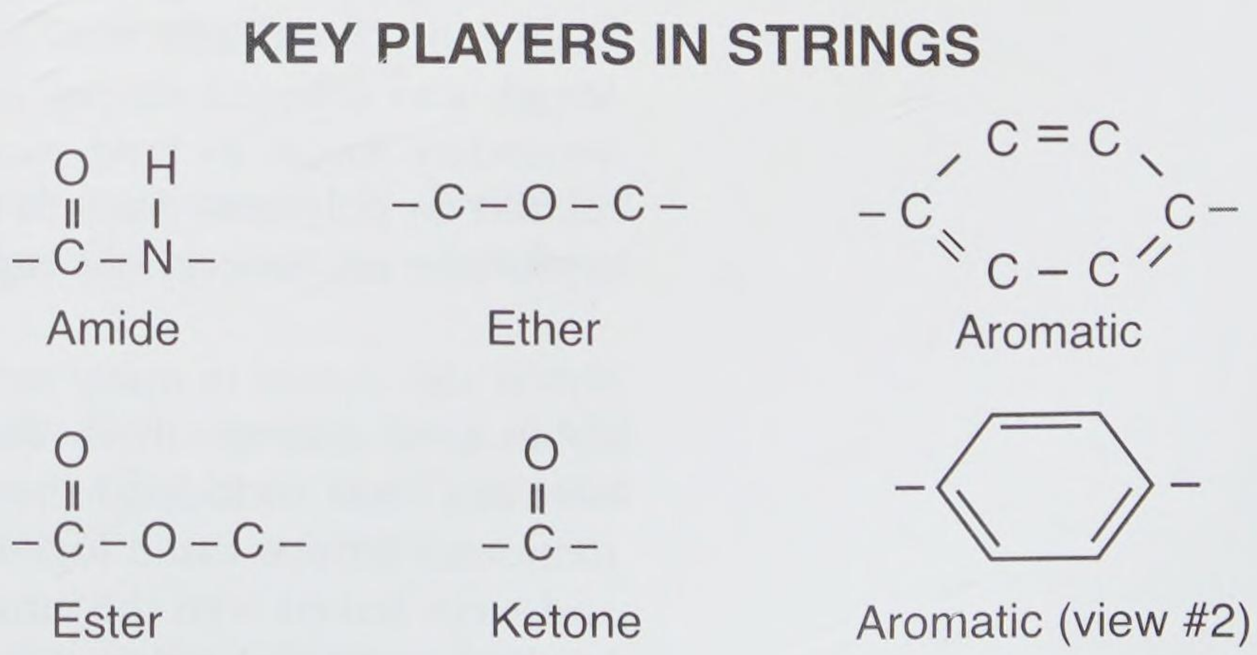
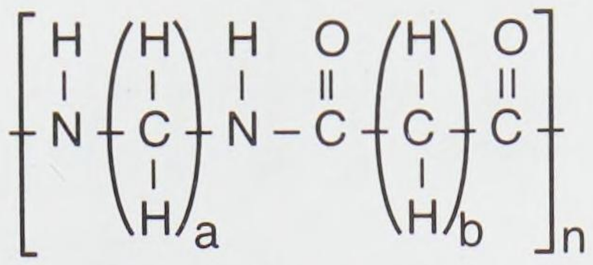


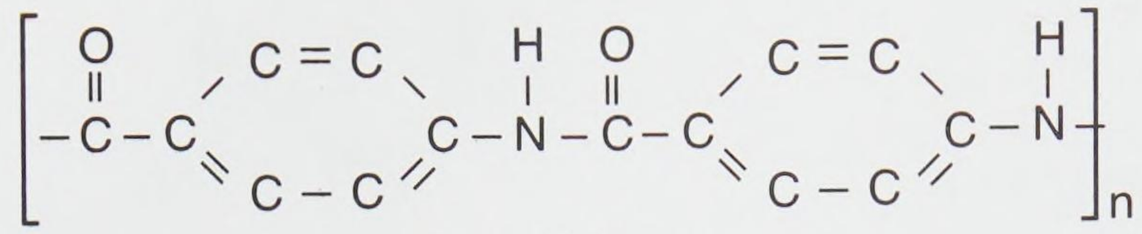
Figure 27.8

Entire classes of materials are defined by these functional groups. They may appear in formulas in different orders than above, but they are all equivalent. Remember, carbon must have four bonds, so where you don't see four lines, the hydrogen atoms are left out for simplicity.

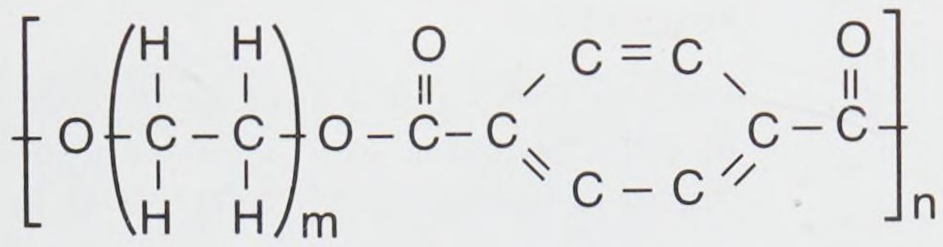
STRING FORMULAS



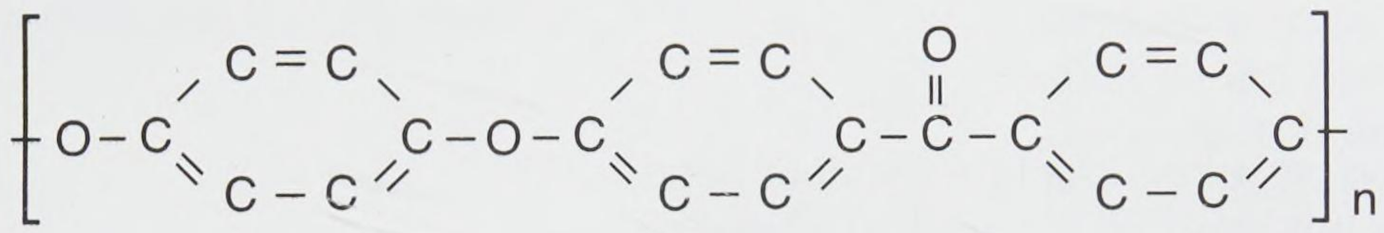
Polyamide: Nylon



Aramid: Kevlar



Polyester: Polyethylene terephthalate (PET)



PEEK: Polyetheretherketone

Figure 27.9

The subscripts a and b in nylon are changed to 6 and 12 accordingly for nylon 6/6 or 6/12, etc. Likewise with polyester. Changing the m subscript results in a different polyester. The n subscript indicates the number of times everything in the brackets will be repeated.

aromatic. Generally speaking, a ring in the backbone will make a molecule stiffer and stronger than a linear molecule. PEEK and all the aramids like Kevlar® have these rings. The complete formulas for string materials are presented in Figure 27.9. These formulas represent the repeating units that are strung together to make the molecule chains.

If you look at these formulas as if they were physical structures made of 2x4's, and if you squint, you can visualize how stiff or flexible that structure will be just by its layout. A few highlights are worth pointing out.

Nylon: Nylons as a group are referred to as polyamides. The C—O and N—H bonds in nylon are polar with the O having a negative end and the H having a positive end. These polar side atoms attract their opposites in neighboring molecules, tending to increase stiffness and tensile strength. These secondary bonding sites combined with the symmetry of the polyamide chain allows these molecules to align themselves side by side or neatly fold back upon themselves in a very close-packed fashion.

These regions of regular alignment are known as crystalline regions. The degree of crystallinity also increases strength and stiffness. Because of the length and different shapes of polymer molecules combined with only weak secondary forces to hold molecules in position, you'll never get 100% crystallinity in polymers. Sixty to seventy percent is high. So most materials have crystalline and amorphous regions mixed together.

Nylon also comes in many varieties. There is nylon 6, nylon 6/6, nylon 12, and nylon 6/12, among others. This differentiation is the number of carbon atoms between (and including) the amide groups. Two numbers indicate that two monomer groups came together with a different number of backbone atoms and were linked with the amide group. This is indicated in the nylon formula by the a and b subscripts.

Kevlar: Kevlar is a strange animal. You'll notice that it is both a polyamide and an aromatic. Hence the name "aramid" which is a shortening of "aromatic polyamide." It is not considered a nylon, however. Its strength and stiffness come from the aromatic rings. These rings are stiff not only because of the shape but by the nature of the bond. All six of the carbon atoms share the elec-

trons of the double bonds. These electrons travel around the circle holding the structure together in a very strong, stiff bond.

Copolymers

A copolymer refers to the structure of the chain itself. The chain of a copolymer has more than one repeating unit within the polymer chain and is created during the polymerization process. Depending on the design of the copolymer chain, you will achieve an average of the properties of chains formed from the individual repeating units or even a synergistic effect that can boost one or more of the properties.

Sometimes, different polymers are blended together in the polymerization process. These materials are known as polymer blends or alloys. Each chain of the different polymers retains its unique configuration during the blending process, and secondary bonds form between the different chains to create the polymer blend material. Depending on the materials and how much of each is blended, you will achieve an average of the properties of the different materials.

Polymers Are Silly

How does the behavior of these polymer chains show up in the whole tennis string? String is schizophrenic. Sometimes it acts like an elastic solid, like when it is being hit by a tennis ball. And sometimes it acts like a slow moving bottle of catsup, as it does if you hang a weight from one end and it continually elongates, or when it loses tension sitting in a tennis racquet. Hitting a tennis ball is instantaneous, and sitting around in a tennis racquet is time dependent. This behavior is known as viscoelasticity — behaving both as a viscous liquid and an elastic solid.

This difference is important in string, but not in metal. If you hang a weight from a metal string, it will stretch just enough to exactly counter the amount of the force. It will maintain that length (with virtually no creep) until the force is removed, whereupon it will instantly return to its starting length. A nylon string, on the other hand, will continue to get longer indefinitely. When the force is removed, it will recover all or most of the elongation, but it too will take time.

But if you impact both with a tennis ball, they both instantaneously absorb and return the energy of the impact force. They respond the same to a fast force but differently to a slow force. Why?

Consider a ball of Silly Putty. If you throw it on the floor, it will bounce. It will act elastically to the force of impact. But if you just lay in on the floor for a couple of hours, it will flatten out under the force of its own weight.

The reason for these different behaviors goes back to the difference between the uniform pattern of atoms in a metal crystal compared to the entangled mass of polymer chains. Upon impact, polymer chains straighten in the amorphous regions and stretch in the crystalline regions. These are instantaneous and recoverable reactions, just like the behavior of atoms in a completely crystalline metal structure. But in a string that is under constant tension over time, chain slippage and disentanglement occur. That's why the string constantly elongates

with a steady force or the tension constantly relaxes in a strung racquet. The chains have time to rearrange themselves and move relative to each other. The secondary forces between chains are not strong enough to prevent this, whereas in a metal, everything is linked by primary forces.

Conclusion

As you can see, there are a lot of variables that can be affected to change the properties of string. String manufacturers must calculate how much work and money is worth tinkering to get 3, 4 or 5% improvements. And typically the big quantum leap improvements in materials occur when an application in an industry much larger than tennis justifies a huge development expense. Then that material can trickle down to other applications.

It's a mind-boggling task when you consider that you can take a formula and organize the atoms in it into different structures called isomers (same formula, different structure). A formula like $C_{40}H_{82}$ can be arranged in millions of different structures. What does it cost to tinker and fine tune in this manner? What will it cost the consumer, and how much is each percent improvement worth to the consumer and what percent is necessary for him to notice?

Jennifer Capriati, United States

28

String Materials

Performance Properties of Synthetic Materials

By Crawford Lindsey

Choosing a string can be a bewildering affair. There are bazillions of constructions and materials, but when you get right down to it, to the untrained eye, most strings look alike. You can't point to features on a string and say, "See this thingamajingie…it increases control (or dampens vibration or whatever)." String features are invisible. That's why you can say almost anything about a string and sell it — once. But when an irate customer comes back to hang you with that string, a fancy jitterbug is no substitute for basic knowledge as to why certain materials actually behave as they do (see Note 1).

What Materials?

The glamour area of leading-edge materials technology is in the stiffer, lighter, and stronger materials. Development for applications requiring these properties make headlines. This is great for making racquets, but what about string? Most of the new post-nylon materials are only used for extreme circumstances like excessive string breakage. The bulk of synthetic string material is still nylon. What are these other materials and how do they compare to nylon?

The materials most commonly used in string are detailed in Table 28.1. Materials in the same family tend to behave alike. The different names within a family are merely different licensed brand names for the same mate-

Table 28.1 **Material Families**

Material	Company	Family
Nylon	Dupont/All	Polyamide
Polyester	Dupont/All	Polyester
Kevlar	Dupont	Aramid
Twaron	Akzo	Aramid
Tecnora	Teijin	Aramid
Vectran	Hoechst Celanese	LCP*
Spectra	Allied Signal	UHMWP*
Dyneema	DSM	UHMWP
Zyex	ICI	PEEK*

Note:
1. *LCP = liquid crystal polymer*
   UHMWP = ultrahigh molecular weight polyethelene
   PEEK = polyetheretherketone
2. *Each family is a different way of aligning and bonding long strings of molecules in straighter, closer packed, and longer parallel chains. The more this is done, the stiffer the material.*

Table 28.2 **Strength**

Material	Strength lb/in^2 (000s)	Times Nylon
Carbon	518	3.6
Tecnora	498	3.5
Dyneema	497	3.5
Kevlar	424	3.0
Spectra	422	3.0
Twaron	421	2.9
Vectran	412	2.9
Steel	399	2.8
Nylon	143	1.0
Polyester	140	1.0
Zyex	115	0.8

Table 28.3 **Stiffness**

Material	Stiffness lb/in^2 (000,000s)	Times Nylon
Carbon	33.3	48
Steel	28.9	41
Spectra	16.4	23
Dyneema	16.2	23
Twaron	11.9	17
Technora	10.5	15
Kevlar	10.3	15
Vectran	9.4	13
Polyester	2.1	3
Nylon	0.7	1
Zyex	0.6	0.9

rial or patent work-arounds for similar but proprietary branded material. Each product may also have several application-specific versions with different strength, stiffness, ultraviolet sensitivity, abrasion resistance, temperature behavior, creep resistance, etc. So, referring to a material just by its name can do it an injustice. It depends on what variety you are working with in what situation. For example, there might be better or worse formulations of polyester for a specific player type.

How are all these materials different from each other in strength, stiffness, and play?

What Is Strength?

Strength is the force necessary to break a square-inch of a material while pulling along its length. The chart at right ranks the materials in terms of tensile strength.

The strength values are big numbers. What do they mean relative to a benchmark that we can relate to? Table 28.2 indicates each material's strength compared to nylon. Carbon and steel are also listed for an additional frame of reference.

What Is Stiffness?

We think of elasticity in terms of how much a material stretches. The force resisting stretch is the elastic force. It competes against the stretching force to return the material to its original shape. The measurement of this elastic restoring force is called modulus. This is the same "modulus" that we encounter in racquets. Modulus is defined as the tension (force per square inch) divided by the strain (elongation divided by original lenght). "Modulus" and "stiffness" are used interchangeably, so the higher the modulus, the stiffer the material. The stiffness chart (Table 28.3) ranks the common string materials and compares each to nylon. The stretch chart (Table 28.4) indicates how much each material stretches at the breaking tension.

Tension and Stretch

Breaking strength tells us how a string material dies; stiffness tells how it lives. Stiffness describes how a material behaves under tension. The tool most often used to show the relationship of strength and stiffness is the "tension/stretch curve" (Figure 28.1). This curve shows the "lifeline" of a material by plotting tension against the percentage

of stretch at each tension when being pulled at a constant rate. Sounds boring, but check it out...it has very interesting implications.

The slope of the curve is the stiffness. For materials like steel, the curve is virtually a straight line at normal stresses, so the slope is the same everywhere (curve A in Figure 28.1). In that case you can say the material has definite stiffness, and you know how it will behave in all circumstances. But for most string materials, the curve changes along its length (like curve B or C), and so, therefore, does its stiffness. Consequently, stiffness is not an absolute term when it comes to string materials.

Instead, string materials behave differently at different tensions. When you say a string is stiff or elastic, it may depend on what tension you are talking about and what other string you are comparing it to. Look at the difference in the two fictional string materials displayed in Figure 28.2. String materials very often have "S-shaped" curves. The slope, or stiffness, at any point along these two curves would indicate something quite different vis-a-vis each other. Each would be considered stiff or soft compared to the other depending on the tension.

Strength and stiffness combine to create some very interesting behaviors (Figure 28.3). Some materials are strong and stretchy and some strong and stiff. The first are tough, the latter brittle. "Brittle" means that it breaks easily on impact. But how can it break if it is so strong? After all, we use Kevlar, which is strong and stiff (and brittle), to prevent breaking. What's the deal?

An example of a boat tied to the dock by a long rope helps to explain this paradox. The rope's job is to bring the boat's velocity to zero as it floats away. The boat

Table 28.4

Stretch at Break

Material	% Stretch
Zyex	34.0
Nylon	18.7
Polyester	14.9
Technora	4.6
Kevlar	3.6
Twaron	3.6
Dyneema	3.5
Vectran	3.3
Spectra	2.9
Carbon	1.5
Steel	1.4

Figure 28.1

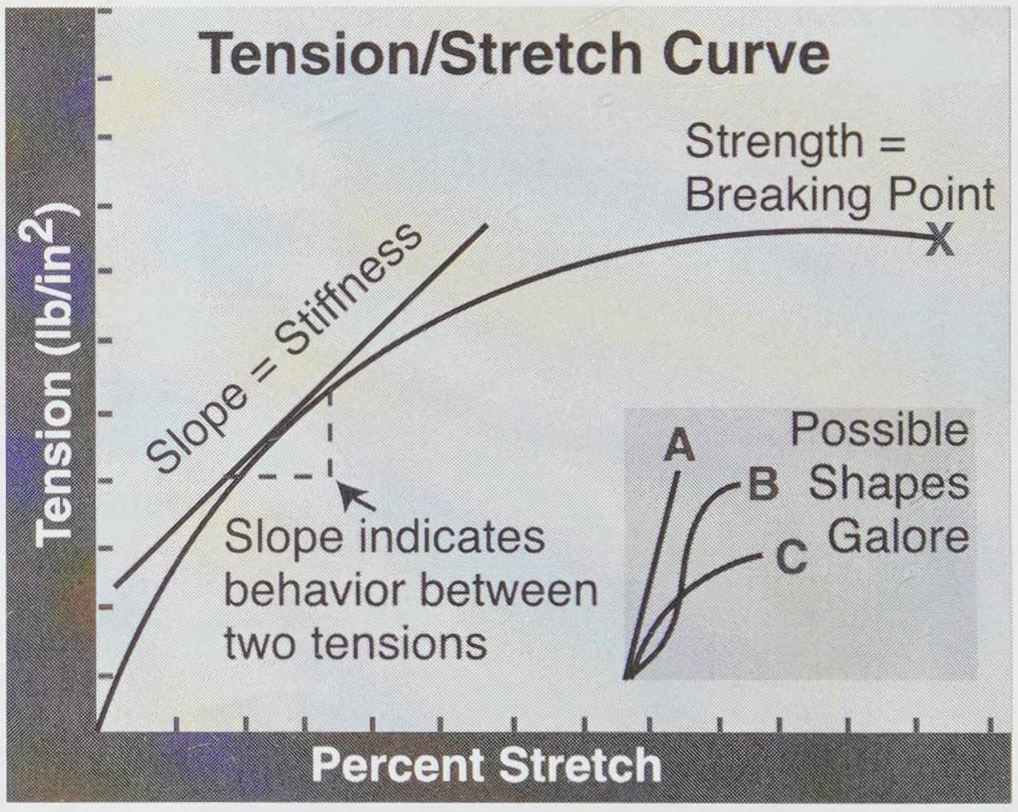


Figure 28.2

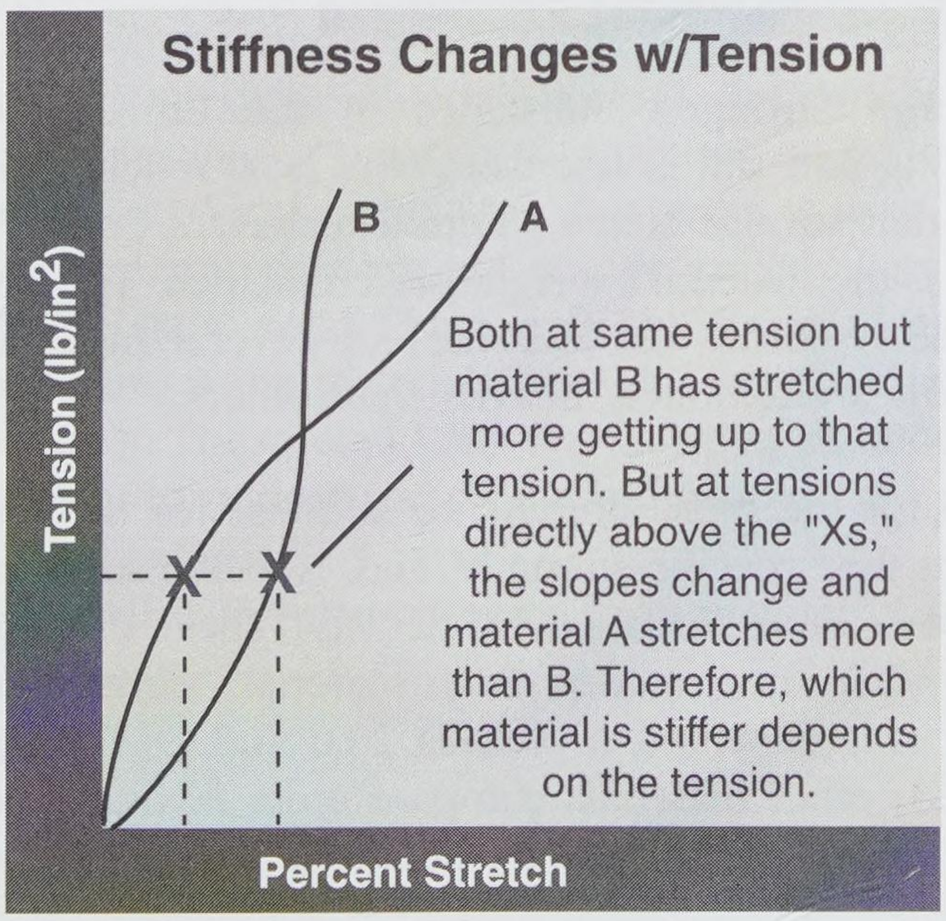
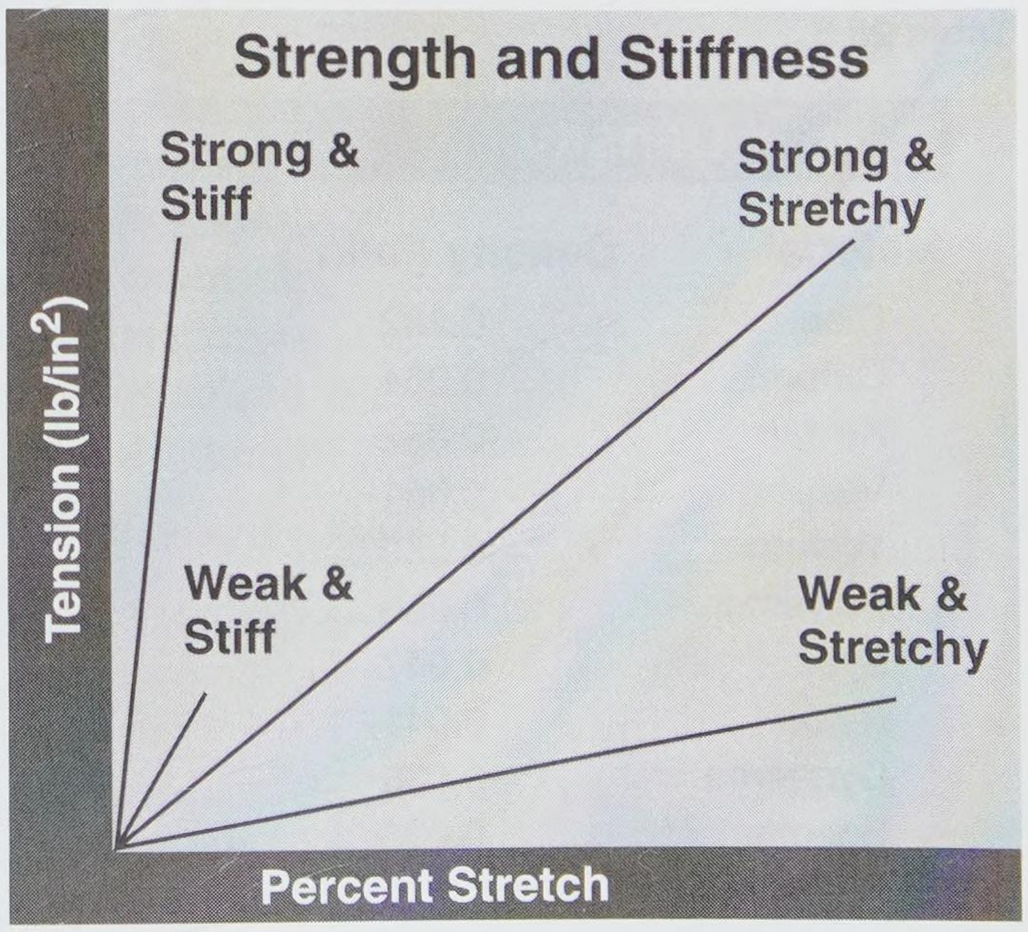


Figure 28.3

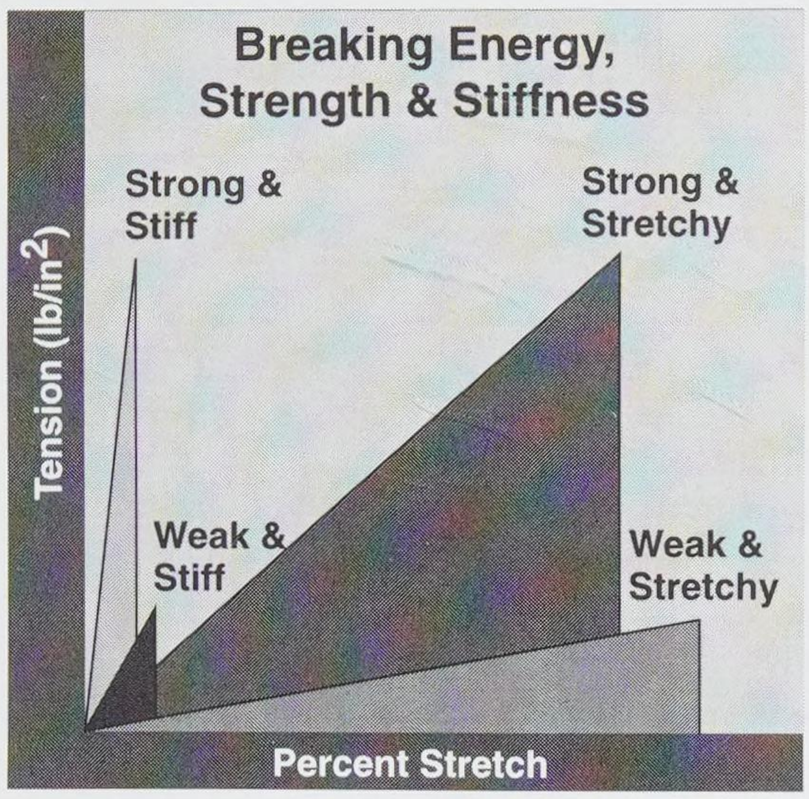


drifts until the line becomes taut. If the rope is very stiff, it won't stretch and the boat comes to a screeching, instantaneous halt. If it is stretchier, it will ease to a leisurely stop. The force exerted on the rope by the boat (and vice versa) is equal to the boat's mass times its acceleration (deceleration). Since the boat weighs the same in each case, the force will be greater when the deceleration is greatest — with the stiff rope. The stiffer rope has to be stronger because it creates higher forces. If it isn't, it will break while the "weaker" one does just fine.

Sometimes stronger is stronger, and at other times, weaker is "stronger." It depends on the situation. Kevlar is used to prevent string breakage because its strength is so high compared to nylon's that it makes up for its lack of stretch and the higher forces it incurs. This is a classic case of property tradeoffs when comparing materials.

If the boats above used a long stiff rope and a short stiff rope of the same material, the longer one would stretch more (because there's more of it, but it stretches the same percentage amount) and thus create less force. The shorter one might break while the longer one holds firm. This applies to racquet strings too. There is less impact force felt on longer strings than shorter ones. Thus, you might be able to get away with a stiffer material on a center main, but not on a shorter side main.

Figure 28.4



The principle is the same as catching a baseball. If you move your hand backwards while catching, you lessen the force because the time of the impact becomes longer as it is spread over a greater distance. A string stretches to do the same thing. The energy is absorbed, the force lessened, and dwell time increases. Figure 28.4 demonstrates how much energy it takes to break the four fictional materials illustrated in the figure. The area under each curve is the energy needed to break each material. It is apparent why the strong brittle material might break on impact. If both the strong materials have the same breaking strength, the stretchy one would require more energy to achieve breaking force because more energy goes into stretching as opposed to increasing the force, as it does in the stiffer material. Depending on the impact energy, a material may or may not build up enough force to break.

Strength and Stiffness by Weight

Ever hear the brag, "Strongest substance in the universe by weight"? Being strong and stiff is one thing, but how much "stuff" does it take to be that strong? That's where a material's density is important. Density is the material's weight per cubic inch. The density table (Table 28.5) shows the densities of several materials. Some string materials are definitely heavier than others.

If you are looking for the most strength or stiffness for the least weight, you look at the "specific strength" and "specific stiffness," which is strength or stiffness divided by density. These are your ultimate strength and stiffness values. Each material's density and specific properties are presented in Tables 28.6 and 28.7. Using steel as a value of 1, other materials compare quite favorably. All materials listed are stronger than steel by weight, although they are not all stiffer.

Table 28.5

Density

Material	Density (lb/in^3)
Steel	0.282
Carbon	0.064
Kevlar	0.052
Vectran	0.051
Technora	0.050
Polyester	0.050
Zyex	0.047
Nylon	0.041
Dyneema	0.035
Spectra	0.035

Strength by Weight

Material	Strength (in.) (000,000s)	Times Steel
Dyneema	14.2	7.5
Spectra	12.0	6.3
Technora	9.9	5.2
Kevlar	8.2	4.3
Vectran	8.2	4.3
Carbon	8.2	4.3
Nylon	3.5	1.8
Polyester	2.8	1.5
Zyex	2.4	1.3
Steel	1.9	1.0

Table 28.6

Stiffness by Weight

Material	Strength (in.) (000,000s)	Times Steel
Carbon	708.6	6.9
Spectra	467.7	4.6
Dyneema	460.6	4.5
Technora	209.0	2.0
Kevlar	196.6	1.9
Vectran	186.0	1.8
Steel	102.7	1.0
Polyester	41.8	0.4
Nylon	16.1	0.2
Zyex	11.9	0.1

Table 28.7

So, just throwing out a wild, random example, a string made with just a little Dyneema (first on the strength by weight list) and a lot of Zyex (last on the list, not counting steel) might be as strong as one made of all Kevlar (middle of the list). Looking at specific properties (properties by weight) like this gives a good indication of why certain combinations of materials have been used in a string to achieve a specific playing characteristic.

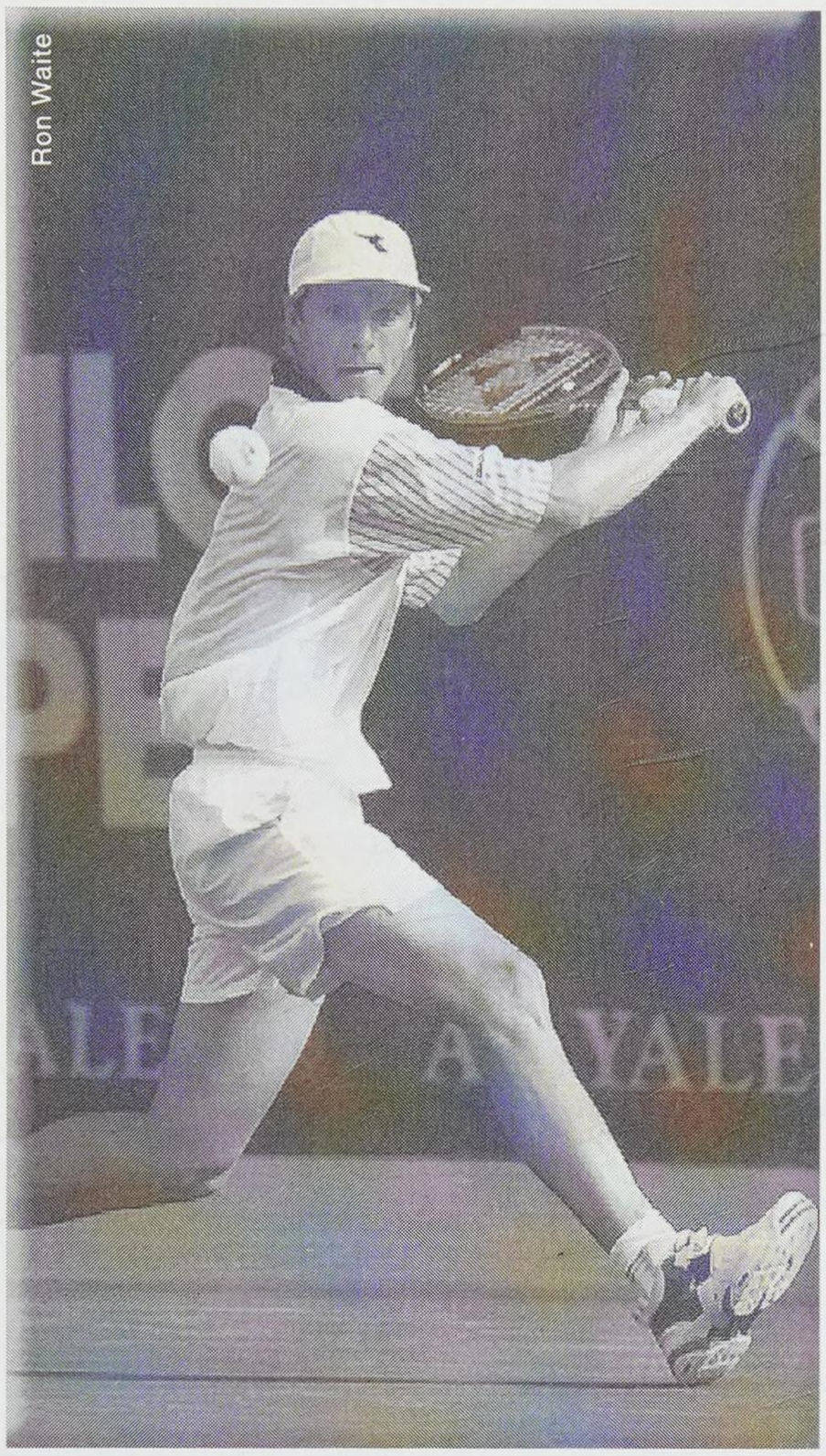
Conclusion

That brings us back to our original question — why is nylon still used despite all the high tech materials? For racquet sports, it has the best combination of stretch and strength which leads to the best combinations of power, control, comfort, and durability. The higher tech materials are usually much stiffer at all tensions. Stiffness works for racquets but not for most strings. That's why so many constructions are designed to provide for a softer feel, even if new materials aren't coming along to do the same thing.

Notes

Note 1. Some caveats: There is a difference between material behavior and the behavior of a structure made of a material. The structural properties of a racquet or string depend both on the material used and how it is used. This chapter deals only with material. Second, the data accumulated here are obtained by static testing — constant rate pulling on material — and are the types supplied by chemical companies. But string is used in dynamic impact situations. These dynamic data are not readily available, so predictions have to be made from the static data. Third, only the standard modulus of materials are compared here. Some materials have dozens of varieties with different customized properties. These could flip flop relative positions on a chart, but they aren't important to understanding the big picture.

Note 2. All data were obtained from material supplier specifications.



Yevgeny Kafelnikov, Russia

29

String Construction

How a Synthetic or Gut String Is Made

By Crawford Lindsey

There's good news and bad news. The good news is, of the hundreds of strings on the market, there really may be some difference between them all. And the bad news … well, it's the same as the good news. How can that be? Aren't there just a few basic constructions, like solid core with one, two, or three wraps, or multifilament, or monofilament like a polyester? That's true, but within each construction the combination of materials, diameters, wraps, twist angles, resins, coatings, and sundry other considerations give every string different properties. How the string is constructed makes a big difference. A string is the sum of its parts. Whenever you add one fiber to another, you always get more than you started with. More strength and more stiffness. Whether you get "less more" or "more more" depends on the composition and orientation of the added material. For example, adding an elastic wrap does not make up for the stiff core and make the string more elastic. It actually adds more stiffness overall. It does, however, make the total string more elastic than if you added a stiffer wrap. Either way, the sum will be stiffer and stronger than either part by itself. And whatever you end up with, you are getting it with the qualification "All other things being equal." It's important to always qualify "more" and "less" with "compared to what."

Core Principles

The Core Dictates. First, consider the core. In a string of a given material, the core will dictate performance because the core is generally the fiber with the greatest diameter. The thicker a fiber, the stiffer, stronger, and

Natural Gut Construction and Play

By Rod Cross

Why Players Like Natural Gut

Natural gut is still the preferred string of many professional players since they like the way it feels and plays. Exactly what that means is hard to pin down, but one outstanding feature from laboratory tests is that natural gut has a smaller dynamic stiffness than any other type of string. All natural gut strings share this property, regardless of which manufacturer makes it. Most nylon strings stretch further than natural gut when they are installed in a racquet, but the initial part of the elastic curve is irrelevant. The relevant thing is how far the string stretches and how much the tension increases when it is used to hit a ball. For example, suppose that a 12 inch length of nylon stretches by 1 inch when it is installed in a racquet and tensioned to 60 lb. When used to hit a ball at 120 mph, the string will stretch further, by about 0.2 inch, and the tension will increase by about 40 lb during the impact. The string returns back to its normal stretched length and normal 60 lb tension when the ball leaves the strings. The extra 0.2 inch stretch lasts only for the time the ball is on the strings, but that's the important part.

A 12 inch length of natural gut will stretch by about 3/4 of an inch when it is tensioned to 60 lb. Gut is less elastic than nylon at tensions up to about 50 lb. But it is more elastic than nylon at tensions above 50 lb. When used to hit a ball at 120 mph, a 12 inch natural gut string will stretch further, by about 0.22 inch, and the tension will increase by about 20 lb during the impact. The extra stretch is only slightly more than with nylon, but the increase in tension is a lot less. This has a significant effect on the softness of the impact. The string tension increases during an impact from 60 lb to a maximum of about 100 lb in a nylon string (when a ball is hit at about 120 mph). With a natural gut string, the tension increases from 60 lb to a maximum of only 80 lb. Since the average tension during the impact is smaller with gut, the impact is softer and the average stiffness of the string plane is lower, as if the racquet was strung at a lower tension.

Another property of natural gut is that it holds its tension better than most other strings. If one racquet is strung with natural gut and another is strung at the same tension with a nylon or polyester string, then after several days or after several sets, the natural gut string will still be close

less elastic it is, and the better it will maintain tension (not all the time — see Chapter 35). It's the work horse of the string. And by being so dominant, it also limits the potential contribution of any other element's properties. As the core goes, so goes the string.The least elastic fiber in a string will also dictate performance. The analogy of a stringbed is a perfect example. No matter how long the main strings might be, if the crosses are very short, stiff, tight, and close together, they will not allow the mains to do their thing. It's the same with string construction — a stiff fiber won't allow the elastic one to stretch as far as it would by itself.

So, in a solid core single wrap string made of only one material like nylon 6/6, the least elastic part would be the thicker core. But if it is a composite string (made of two or more materials), the stiffest part could be either the core or the wraps, regardless of diameter. The performance of a string with a stiff core and elastic wraps will be determined by the core. But a string with an elastic core and stiff wraps is more complicated. It depends on the material, diameter, and orientation of the wrap. A very stiff material can be made thin enough to be as elastic (or nearly so) as a more elastic core material. Or it can be wrapped at an angle to increase its elasticity (more on that later), or both. Thus, there are more options starting with an elastic core.

Material vs. Diameter. Even so, the fiber's material composition is usually more dominant than its diameter. The elastic properties of two fibers must be pretty close for changing diameters to cause their properties to add together instead of one limiting the other.

Material vs. Construction. Materials are also more important than construction. What a string is made of is more important than what it looks like. No matter how you combine nylon elements, you're not going to get gut. Single wrap, double wrap, braided, multifilament — nylon is still nylon..

Materials contribute more to playability, whereas construction has a greater impact on durability. That's why wrap "construction" was invented in the first place — to protect the material in the performance core. And that's where the magic formula comes in — to make all the elements additive and complementary while still getting all the desired characteristics.

Strength vs. Elasticity. Strength and stiffness are always additive. Every element that contributes to the cross section is adding strength, regardless of its other properties. For example, if the core has a tensile strength of 100 pounds and the wraps all add up to 100 pounds, then the string's total tensile strength will be 200 pounds. But if string is as strong as the sum of its parts, it's only as elastic as its least elastic part. And the least elastic is usually the strongest. So what limits one characteristic augments another.

Wraps: To Serve and Protect

Wraps were invented to protect the core — especially from notching (the biggest culprit of breakage). A thick monofilament mate-

rial with a notch can be weaker than a thinner fiber without one. If an outer fiber of a wrapped string notches and breaks, the failure doesn't propagate through the rest of the string like it would if it were a monofilament. The outer layer localizes or isolates the crack.

Single wraps predominate, but double and triple also exist. For a given gauge, the more wraps you have, the smaller the core they are protecting (unless you make the diameters of the wraps smaller instead). But then you have a weaker core surrounded by individually weaker protectors. Now the wrap strategy becomes strength in numbers. "You can get some of us but not all of us."

But with more elements you also get more space between fibers, so you have less fiber material. And with less fiber in the cross-section (the rest is filled with resin), you have less strength. So, the strong core that we originally set out to protect has become weaker, the protectors are weaker, and all of them added up are weaker than we started with. But a funny thing happened on the way to the string factory. As more and more layers and fibers are added in an attempt to protect the core, strength goes down but playability goes up. With more and more parts, the string becomes softer, more flexible, and more playable. Taken to the extreme, we end up with a multifilament. Multifilaments thus have less tensile strength, but perhaps are more durable in terms of longevity. A single fiber will break all at once, a multifiber a little at a time. Which one goes first depends on what kind of forces each endures.

Twists, Angles and Fiber Counts

Twists and Durability. This is where string construction gets fun, and where a string can really be fine tuned. Wraps are twisted around the core, and multifilaments are all twisted around each other. What is the purpose of the twist? If the wrap fibers are all oriented in a straight line along the axis of the core (and sometimes they are), they will contribute the most to tensile strength (the breaking strength when pulling along the string's length) and stiffness. But if one of the fibers breaks, that fiber loses its contribution to the string's properties. If the fiber is twisted around the core instead, its tensile strength will be less (because it's no longer oriented in a straight line), but if it breaks, the fibers on either side of the break are held in place by the friction and braiding effect of neighboring fibers in the twist. A good percentage of the fiber's strength will be lost, but some is retained due to the interaction with fibers around it.

Twists and Elasticity. Twisting also builds elasticity into the construction. In effect, it makes the string more spring-like. For example, a straight metal wire might have a stretching stiffness of thousands of pounds per square inch. If it is coiled into a spring, the force required to stretch it will be much less. It's the same with string — twisted wraps take less force to stretch than straight line wraps.

to its original tension while nylon strings will have dropped a few lb in tension and polyester strings even more. For a professional player, this is important. For an average social player, this is not so important. In fact, social players may prefer nylon or polyester since a loss in tension results in a softer feel. A racquet strung with natural gut may feel too stiff for a social player, especially if it is strung at the same high tension used by professionals.

Why Cows Hate Natural Gut

Natural gut strings are more expensive since they are not as easy to make as synthetic strings. As the name implies, natural gut is made from a natural product, namely the intestines of cows. The small intestine of an animal is essentially a long flexible tube designed to help the animal digest its food. It is naturally quite elastic since it needs to expand and contract to function properly. It takes about three cows to make a single tennis string. Not because the intestine is too short but because the part used to make a string is very thin.

Many people think that natural gut is made from cats. The small intestine of a cat is only 4 feet long and is therefore too short to make a tennis string. The word "catgut" seems to have evolved from the use of natural gut in a musical instrument called a "kit" or perhaps from the name of the town in Germany where the strings were made. The intestine of a cow or sheep is about 120 feet long, enough to string three racquets. The part of the intestine used for making tennis strings (or for making sausage skins from sheep intestines) is the outermost layer called the serosa.

The serosa is removed and cut into long ribbons which must then be cleaned in a series of salt and other chemical baths over several days. A sufficient number of ribbons (about 18) is assembled and twisted as a long string and dried under tension over several days in a temperature and humidity controlled room. The string must be thoroughly dried and not just dried on the outside. After drying, the string has the appearance of ordinary string or rope. The string is then polished using a fine grit sanding machine to form a smooth, round and clear string. Polishing may involve rotating a long stretched string while the polishing machine travels along the length of the string or it may involve passing the string through a small polishing head. The final process is to treat the string with a protective coating such as polyurethane to improve abrasion resistance and to prevent moisture entering the string.

Twist Angle and Elasticity. The angle of the twist is also important. The tighter the wrap (the greater the angle compared to the core), the more elastic it will be. But it will also have less tensile strength — the greater the angle, the more of the load the core will have to bear. But as the string loses tensile strength at increased angles, it gains in hoop (or bending) strength which comes into play on knots and turns between grommet holes.

Twist Angle and Energy Return. A tighter wrap may also diminish the string's energy return. This is because the very spring-like action that makes the string more elastic can rob it of resiliency. The force of ball impact is absorbed spirally which makes it soft, but is given back the same way, which limits the unified directionality available for the energy return. In a straight line of fibers, all the fibers are moving in the same direction. With twisted fibers, each fiber is moving in somewhat different directions relative to each other and to the core. Not only does this disperse the energy but it also causes more friction, which uses up even more energy.

Twist Angle and Fiber Count. The angle of the twist also determines the number of fibers you can fit on any wrap layer. The higher the angle, the fewer fibers of a given diameter that can fit. But whether it is twenty fibers or one fiber, it is the same amount of material in either case. For example, imagine a core with a six-fiber-wrap arranged in a straight line. As the angle of that wrap is increased, the orientation of each fiber to the cross section becomes greater and the cross section of those fibers become elliptical. The elliptical cross section becomes wider with every increase in angle. As it gets wider, fibers must be eliminated to fit the circumference of the string. Ultimately, as the angle approaches 90 degrees, you are left with just one fiber wrapped perpendicularly around the core. But though you lost fibers along the way, you had to increase the length of the remaining fibers to account for the tighter wrap. So you end up with the same total amount of material but fewer fibers and much different properties. Basically, the volume is being filled in a different configuration, but it is still the same volume and thus the same amount of material.

Resin: The Glue that Bonds

Bonding. Fibers get all the glory, but resin is the unsung hero. Without resin, any fiber is too limp to string. The resin bonds all the fibers together and gives the string its body and shape. Without the resin, a string would be just like a rope or twine. Resin is typically applied to every fiber at the intersection where they all come together from the bobbins to be twisted onto the core. If a fiber is not "wetted," premature fray, breakage, tension loss, and poor resiliency can result. In general, the resin is more elastic than the fibers to keep it from breaking down when the string is stretched and to allow the fibers to do their thing.

Resin Types. Resins come in different flavors. There are different starting compounds and additives that contribute to making strings different from each other. Resin can be stiff or elastic, depending on how the pieces are intended to work together. But not every resin bonds with every fiber. For that reason, nylon fibers will usually have a nylon resin, Zyex a urethane, and Kevlar might have either. So not all combinations are possible.

How Much Resin? The amount depends on the number of fibers. The more fibers there are, the more resin is necessary to fill the gaps between them and to bond every fiber to each other. For example, the "void volume" of a 16

gauge multifilament may be as high as 22–24%. That is the amount of space that will be occupied by resin and coating.

It's ironic that the more fibers there are, the less performance material there is. Or more to the point, more fibers means less fiber. This also means less strength and tension maintenance (remember, all other things being equal). The properties of the wraps, core, and resin in a string don't add up to what they would be if the string had been a monofilament of the same total diameter. (Which brings us to another irony, the monofilament will be stronger, but because of the catastrophic consequences of notching, it might not last as long. Strength and durability are not the same thing.)

Resin and Performance. Resin itself isn't a performance material, but instead enables other materials to perform. First, it protects each component fiber and the string in total. It acts somewhat like a windshield safety glass. As in the glass, there are dissimilar layers that prevent crack propagation. Each crack is isolated by the "glue line." If one fiber gets nicked, the crack is not propagated throughout the string. This is why you can notch a multifilament and not get a catastrophic break like you might in a monofilament.

Secondly, resin transfers the load from fiber to fiber by shear forces. Without resin, you'd get more breakage because fibers couldn't as easily transfer the load to each other. Some would end up bearing a disproportionate amount and break. Resin by itself is not a strength component because it has no directional fiber alignment, but it helps strength by judiciously transferring the load.

And last, resin allows a multi-part string to be one and many at the same time. In other words, though it bonds fibers together as a whole, each fiber can still behave somewhat individually. The human muscle is an example. Depending on how you flex a certain muscle, different fibers will do different work, as evidenced by the muscle's different shapes. It's the same with strings. Depending on the speed, angle, and location of ball impact, the individual fibers will elongate in different ways. Theoretically, individually responding fibers will distribute the load more efficiently than a solid core which will deform uniformly. So, even though the resin holds the fibers together, if one wants to stretch a little more than another, the resin gives and takes to balance it all out.

Coatings: Slip Sliding Away

The coating is for durability and stringability. Usually the materials are the same as the resins because the same primary consideration is true — finding a coating that will bond with the fibers. But a coating can have many different additives and can have multiple layers, each with a different mixture and thickness.

Durability. A coating can be hard or soft. If it is hard, it will resist abrasion, friction burn, and string wear. If it is soft, it will resist string movement and wear because the strings stick together more. Each is a different approach to the same thing — durability.

Lubrication. Lubricants are also added to the coating or as a last layer itself. A lubricant serves the same purpose as waxing the string to facilitate weaving. Lubricants are usually a silicone or light oil, though materials like teflon are also used. The balancing act is to find a lubricant that will ease stringing, not gum up the clamps, and will evaporate quickly to prevent excessive string movement.

Coatings also contain other additives such as coloring and plasticizers which keep the outer glue line between fibers from becoming too stiff and brittle when the string is bent during stringing.

How Are Strings Different?

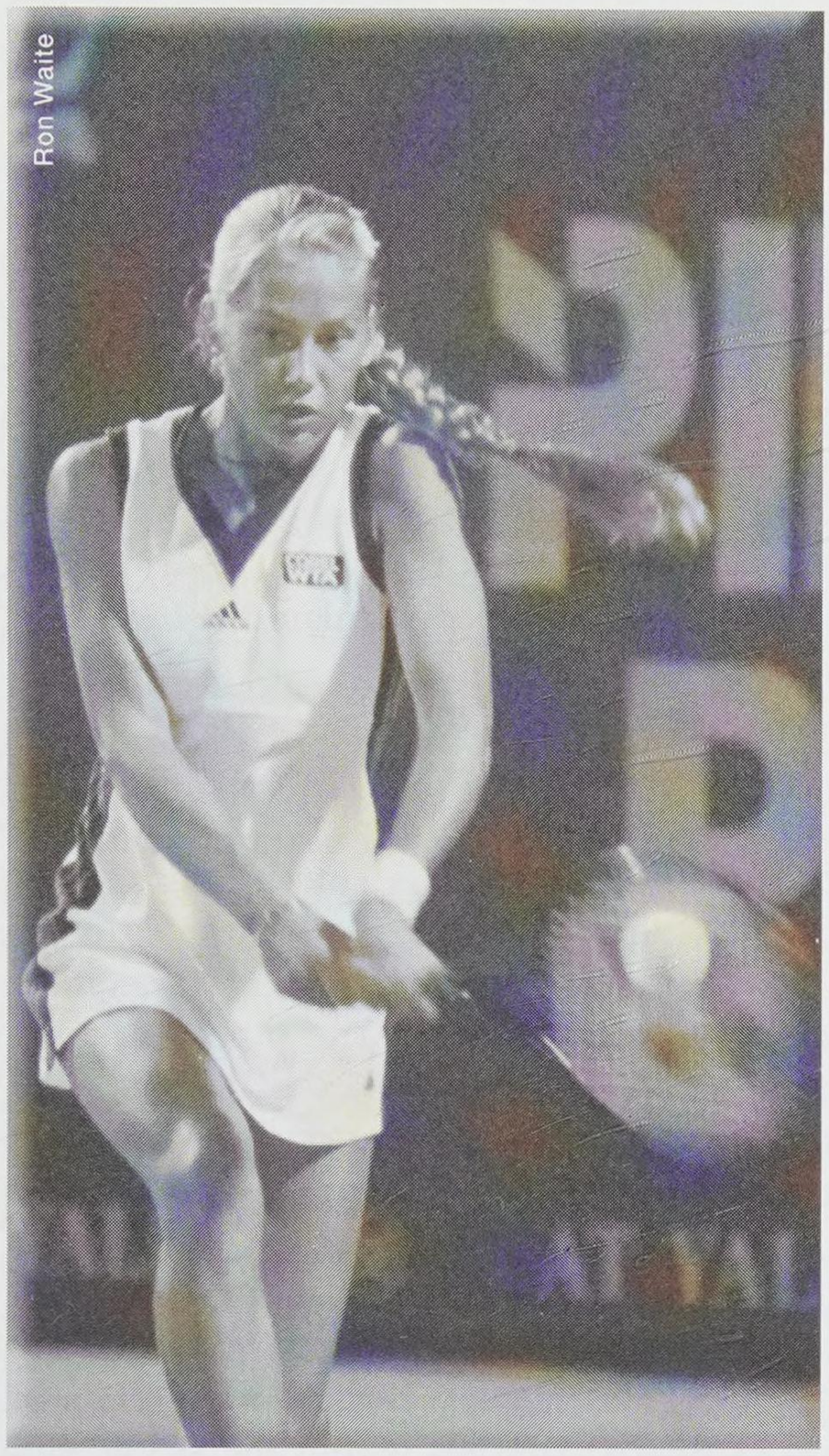
Need you ask? Everything so far has answered that. But it's so confusing because everything seems to be a tradeoff. To gain in one property, you lose somewhere else. Each time you change one factor, you change both durability and playability. So, if there is just one difference between strings, then there must be at least two … but probably many more you may never know about.

Draw Ratio. For example, consider two strings with a nylon 6 core and outer wrap. Though made of the same material, the individual fibers can be prepared differently before putting them together in a string. One such difference is the "draw ratio." Nylon fibers originally get extruded at a larger diameter than the final string. While they are still hot, they are pulled or "drawn" down in diameter. The draw ratio is the starting diameter compared to the final diameter. The draw ratio determines how stiff and strong a filament is. When the hot plastic first comes out of the nozzle, it is very weak. Pulling it orients and aligns the molecular chains. As it is pulled, stiffness and strength are added. But there is an optimum point. If it's pulled too much, it becomes stiff and brittle. So, the same nylon 6 core can be less stiff by not drawing it as much, while the outer wrap could be stiffer by drawing it more. The same nylon 6 construction can have different cores and wraps, and you won't know until you play.

And, if that's not enough, some manufacturers heat and draw the completed string again for a final "orientation." So, saying a string is a certain material like nylon 6 and a certain construction like a single wrap does not tell the whole story. Different draw ratios, wind angles, resins, coatings, and final orientation treatments can all make a difference to what looks like the same material and construction.

Why the Cost Differences?

It looks like the same string! Why does one cost $10 at retail and the other $23? Well, as we have seen, they probably aren't the same string. Each is finely tuned, and it takes knowledge, R&D and machinery to optimize the performance gain of each tune-up. That's what you're paying for. And eeking out little differences in the lab can result in big differences on the court. You are paying for performance, not materials, labor, or packaging. You are paying for the intangibles that make one string play better than another. Sometimes that's hard to swallow in a black and white material world, especially with regard to something that seems as simple as a piece of nylon. You're not paying for features, you're paying for benefits. Sure, a company can charge anything it wants with a rationale like that, but it can only get it if the product delivers. And if it does, it's worth it.



Anna Kournikova, Russia

30

String Properties

String Properties and What They Mean

By Rod Cross and Crawford Lindsey

As simple and innocent as string appears, it is very difficult to test. It has many complicated properties, and the results are sometimes contrary to expectation. Testing it is like opening a Pandora's Box of new concepts and ways of thinking about string.

We tested 89 strings. Strings were between 15 and 18 gauge and represented a cross-section of materials. Prior to tension testing, each new piece of string was weighed, gauge measured, and friction tested. Each single piece of string, measuring 320 mm (12.6 inches) long, was mounted into the tensioning apparatus and clamped. The string was tensioned to 20 kg (44.1 lbs) and the elongation was measured. Tension was raised to 28 kg (61.7 lbs) and held there for 10 seconds. Elongation was measured again.

To test tension loss, the string was allowed to sit for 1000 seconds (16.7 minutes) while data signals from a strain gauge were processed every one or two seconds to record tension. Many previous tests had shown that the rate of tension loss becomes linear (on a logarithmic scale) before that time, and tension loss can be accurately extrapolated for any time beyond 1000 seconds.

The string was then impacted by a hammer swinging like a pendulum. The hammer weight (one pound including lead head and wooden shaft) and speed at impact (8.63 ft/sec) combined to simulate the force necessary to hit a serve 120 mph. Ten separate impacts were recorded. For each impact, change in tension (before, during,

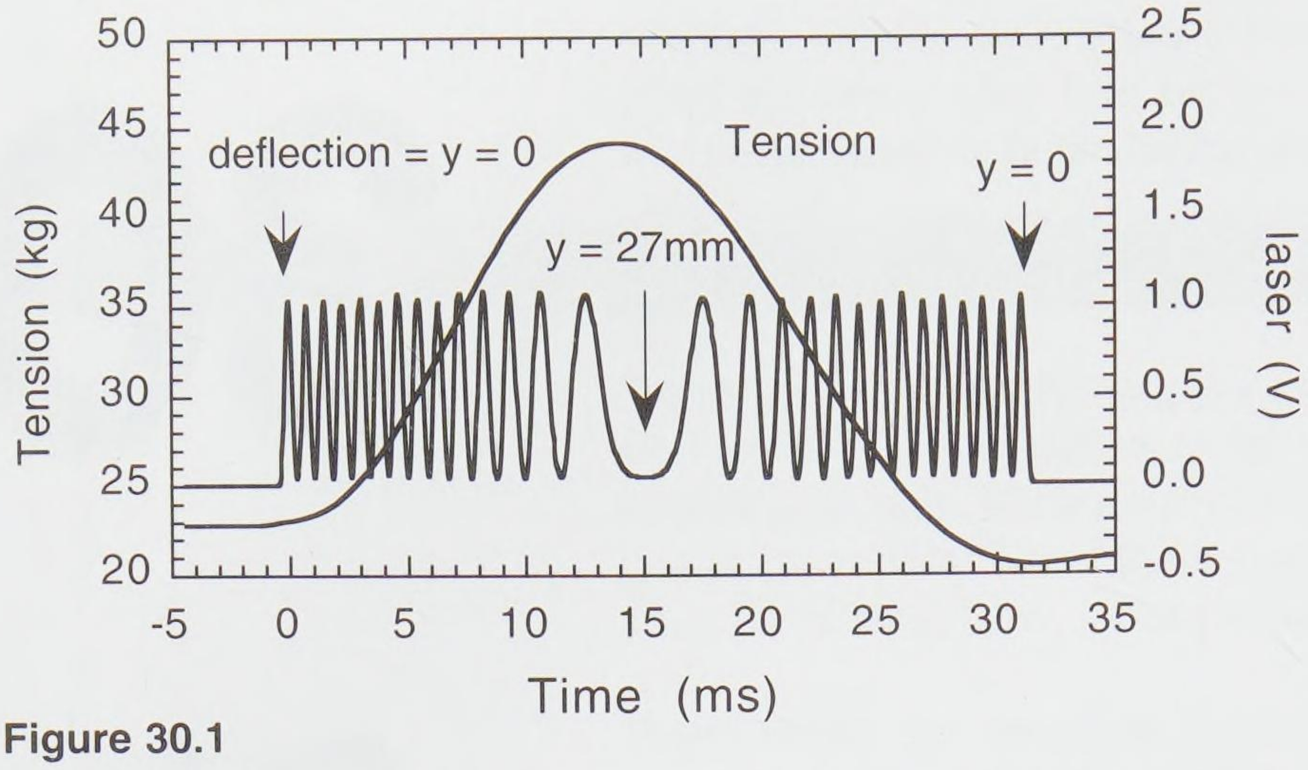


Figure 30.1

Tension vs time during an impact with a polyester string and the corresponding laser signal used to measure string displacement. Each new maximum in the laser signal corresponds to an extra 2 mm displacement. The tension starts to rise only after the string has moved a few mm.

after impact), string deflection, duration of hammer on strings, force on strings and hammer, and in-going and out-going hammer velocity were recorded.

Deflection and hammer speed data were collected by digitizing the signals from a laser that passed through an optical grid on the hammer as it impacted the string. Figure 30.1 shows the oscilloscope presentation of this information. Stiffness was calculated using all of this data.

There are two astounding facts that stand out from all the data collected. First, gut is simply amazing. It ranks "highest" (i.e., at the extreme end – qualitative interpretation being left to the player) in every category except friction. The other astonishment is that out-going hammer speed ("power") is about 95% of the in-going speed for every string – no matter the tension, type of string or the number of impacts. This later result is presented in greater detail in Chapter 32.

Other observations also jump out. Strings of similar material all group together. (There were five main groups of material in our tests — gut, zyex, nylon, polyester, and kevlar.) Material composition is the biggest differentiator between strings, not the construction of those materials within the string. But for a given material, construction can make a difference.

Dynamic Stiffness — It Determines Almost Everything

What Is Dynamic Stiffness? The most important physical property of a string is its dynamic stiffness. The stiffness of a string tells us how far it will stretch when the string tension is increased. The dynamic stiffness was measured after the string was tensioned to 28 kg (61.7 lbs). The string was left for 1000 seconds (16.7 minutes) to measure the loss in tension with time. It was then stretched rapidly by hitting it sideways with a hammer so that it stretched an extra 6 mm (1/4 inch) or so lengthwise. The string tension rose and fell within a period of about 30 ms (milliseconds, or thousandths of a second), at which time the hammer bounced off the string. We measured the increase in tension, DT, and the increase in length, DL. The dynamic stiffness, k, is given by k = DT/DL and it is measured in lb/in. For example, if the tension increases by 50 lb when an impact stretches the string lengthwise by 0.25 in., then k = 50/0.25 = 200 lb/in. This is a typical result for a nylon string. With a very stiff string like kevlar, the tension increased by about 120 lb when it was stretched by 0.2 inch, so k = 120/0.2= 600 lb/in. The softest string was natural gut, with k = 108 lb/in.

Measuring Dynamic Stiffness. We tested string samples of free length 320 mm (12.6 inch), held firmly in metal jaws. "Free length" is the distance between the jaws at the entry point of the string. A one pound hammer was swung into each string at a speed of 8.63 ft/sec to impact the middle of the string and to

simulate the effect of serving a ball at about 120 mph. (The hammer weight and speed reproduced the impact energy calculated for one string's share of the impact of a 120 mph serve.) The hammer deflected the string sideways by about 1 inch, resulting in a lengthwise stretch of about 1/4 inch (note: sideways deflection and lengthwise stretch are not the same thing). The resulting measurement of k therefore refers to a string of length 12.6 inch. If we had tested a string twice as long, it would have stretched twice as far and k would have been half as stiff.

Quick Stretches vs Slow Stretches. Impacting a string with a hammer simulates the effect of hitting a tennis ball. It is not exactly the same, but the important thing is the rapid stretch rather than a slow stretch. During a slow stretch (Figure 30.2), such as when you string a racquet or when it is pulled slowly in a pull test machine, the string will keep stretching even if the tension is held constant. This phenomenon is known as creep. If the string has settled down at say 60 lb for a few days, then as soon as you increase the tension above 60 lb, the same thing happens. For example, if you increase the tension from 60 lb to 80 lb by pulling on the string, it will stretch immediately by a certain amount, but it keeps stretching forever at the higher tension. Measuring DT/DL under those circumstances obviously does not give a consistent measure of k. In order to get a reliable measurement of k without the effects of creep, it is necessary to stretch the string quickly by impacting it instead of pulling it and allow it to return quickly to its pre-stretch length before it has a chance to creep further (Figure 30.1).

On the first impact, a few molecular bonds get broken and the string stretches a bit further than usual. After that, it settles down and it is then possible to measure k reliably. That is why we impacted the string 10 times with the hammer. The dynamic stiffness varied a tiny bit from the first impact to the last, but usually by no more than 3%, despite the fact that each impact caused the string tension to drop immediately after each impact. To smooth out this variation in k, we took the average of all 10 impacts for every string tested.

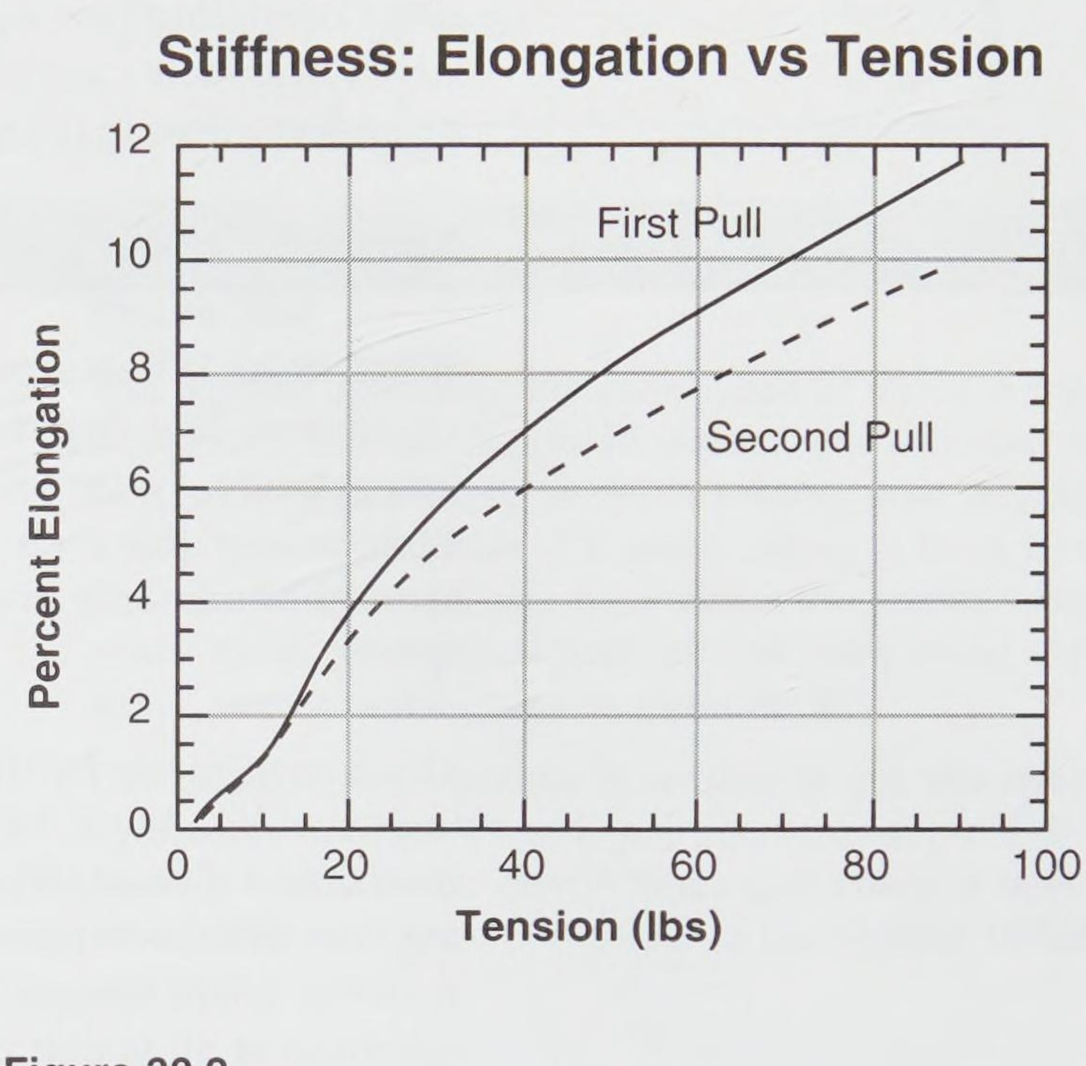


Figure 30.2

The graph shows percent elongation measured in an Instron pull machine, stretching at 4 in./min. This is a lengthwise static pull similar to a stringing machine. The first pull acts to pretension the string. The second pull shows that the string is stiffer after a prestretch. The stiffness (the slope of the curve) varies with tension, but is roughly constant above 40 lb.

In the impact tests, the string was stretched at about 1000 in./min by hitting it with a hammer. Stiffness is defined as the impact tension rise divided by the impact elongation (note: elongation is not the same as deflection).

Stiffness and Materials. The dynamic stiffness of a string, for a lengthwise stretch, depends mainly on the type of material and not the tension. However, all strings were tested at the same starting tension of 28 kg. We did not test the strings at other starting tensions, so we did not specifically look at a large range of different string tensions. However, the tension dropped after each impact, and that allowed us to measure k over a small range of string tensions. The 3% variation in k over this small range of tensions for a given string was negligible compared with the 800% difference in k between gut and kevlar. We tested two gut strings and both had k = 108 lb/in. Kevlar strings ranged from 500

to 886 lb/in. Polyester strings ranged from k = 251 to 310 lb/in. Most of the strings we tested were nylon, and all had values of k between 161 and 225 lb/in. Only one zyex string was available, with k = 145 lb/in.

In terms of stiffness, this makes comparing one type of string with another relatively easy. Despite the fact that we tested hundreds of different strings, there were really only five catagories of string. In order of increasing stiffness they are gut, zyex, nylon, polyester and kevlar (aramid). For a given material, different construction methods can make a small difference to the stiffness, but the biggest difference in stiffness comes when you switch to a different material. Comparing one nylon with another is not so easy. The differences are not large, in which case other effects such as tension loss with time are then just as important in making the comparison.

Impact Force and Duration — It's What You Feel

What Is It? When a player hits the ball, each string in the racquet stretches by an amount that depends on its length and its dynamic stiffness. The player is not actually aware of this and couldn't care less whether the center main stretches by 0.2 inch or 0.3 inch. The player is only aware of what is felt through the handle and what happens to the ball in terms of its flight path and spin.

The sound made by the strings and the ball is probably an important psychological effect, but it has no effect on the flight of the ball and only a tiny effect on the feel. Some players like both the sound and the feel of the strings vibrating, but the force transmitted to the handle by this vibration is tiny compared with the main impact force. The only reason players are aware of the string vibration at all is that the strings vibrate for about one second after the impact, while the frame vibrations die out much faster, in about 30 ms.

So, as far as the player is concerned, the most important effects of the strings are felt in terms of the impact force and the duration of the force. Both these effects depend on string tension and dynamic stiffness, which together, help determine string plane stiffness. But that's getting ahead of ourselves because the actual string plane stiffness depends on the number of strings in the racquet, the tension and the dynamic stiffness. But to begin to sort this out, we first need to look at the behavior of a single string.

Measurements. The force on a single string and the impact duration both depend on string tension and dynamic stiffness in a way that is entirely predictable but very surprising. The force varies with time during an impact. It starts off at zero when the hammer (or a ball) first contacts the string, it increases to a maximum value when the string has stretched as far as it will go, then the force drops back to zero as the hammer bounces off the string. We measured only the maximum or the peak force (Figure 30.3) as well as the duration of the force (Figure 30.7). (Actually, we measured the increase in tension in the string (Figure 30.4 and 30.5) and the sideways displacement of the string (Figure 30.6), but it is easy to calculate the sideways force from that data.)

The increase in tension and the peak force during an impact both increase as stiffness (k) increases. For example, on a gut string, the tension increased by 22 lb and the peak force was 28 lb. On a kevlar string, the tension increased typically by about 100 lb and the peak force was typically about 44 lb. The

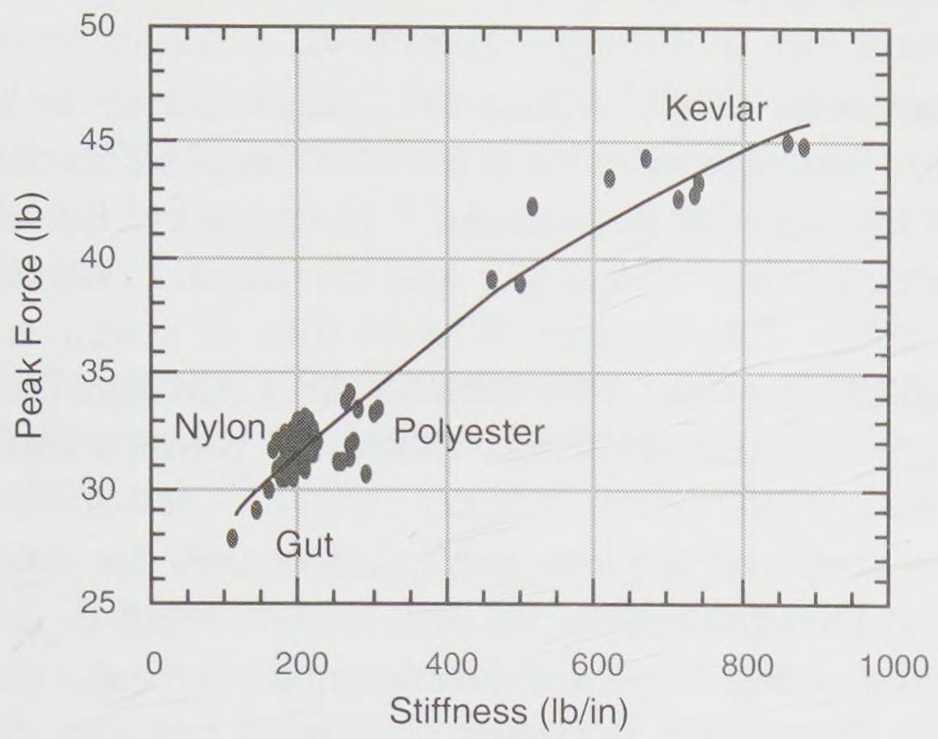


Figure 30.3

Peak force is the peak sideways (in the direction of the deflection) force on the string, calculated from the deflection and the increase in tension. In a fully strung, hand-held racquet, the peak force on the stringbed (i.e., not a single string) is about 10 times larger and it is the same as the peak force on the ball. Ball speed depends on both the force and its duration. When force goes up, duration usually goes down. Peak force is largest on kevlars and smallest on gut. That's why gut feels soft and kevlar feels stiff.

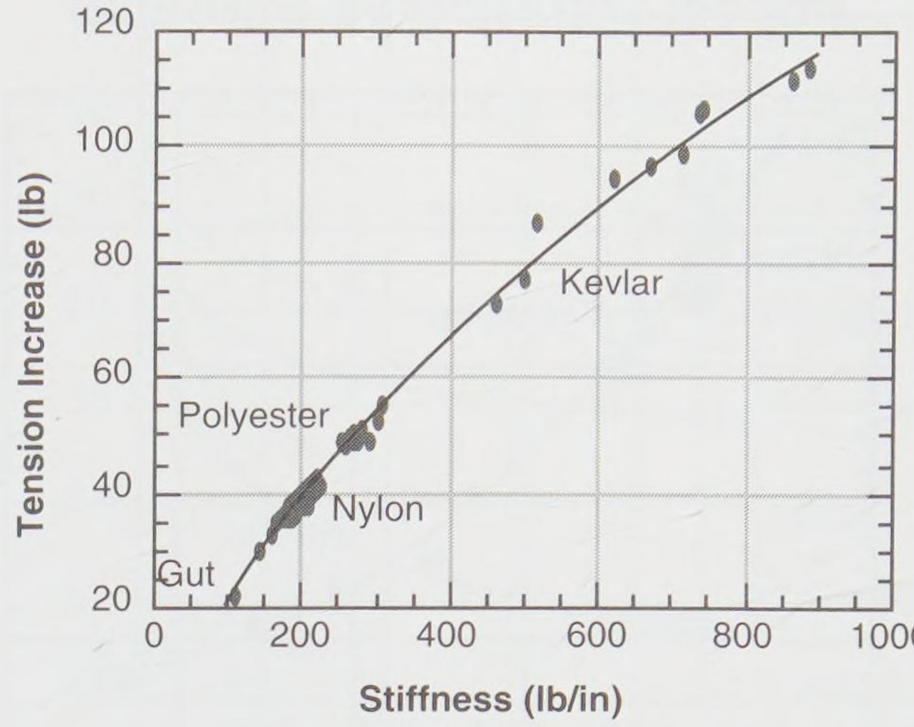


Figure 30.4

The biggest difference between types of string is the increase in tension during an impact. The increase in tension for a gut (20 lb) is much smaller than any other string. The tension rise in a kevlar string is huge by comparison. All nylon strings were about the same, rising about 40 lb. Similarly, all poly strings were about the same, with a tension rise of about 50 lb.

Compared to the increase in tension of gut, the average nylon rose about 1.5 times gut, polyester about 2.5 times gut, and kevlar over 4 times gut. There is over a 5 times difference from the lowest to the highest tension rise in the graph.

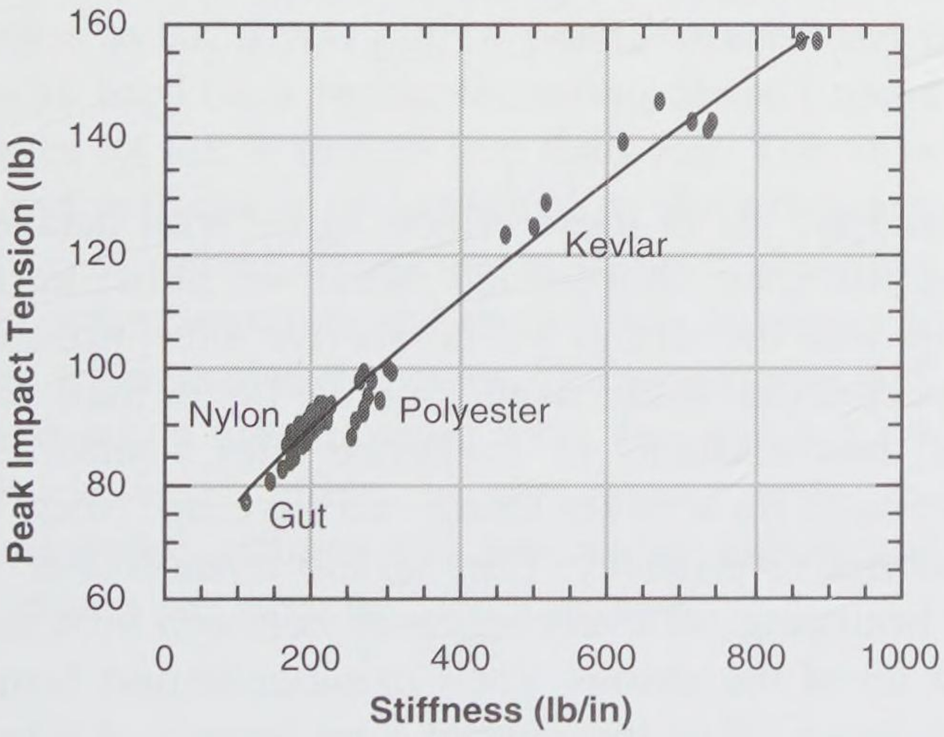


Figure 30.5

The graph shows all 89 strings. Peak Impact Tension is the original tension plus the increase in tension during the hammer impact, averaged over 10 impacts. For example, gut was at 55 lb on average before each impact. The tension increased by 22 lb from 55 lb to 77 lb during each impact.

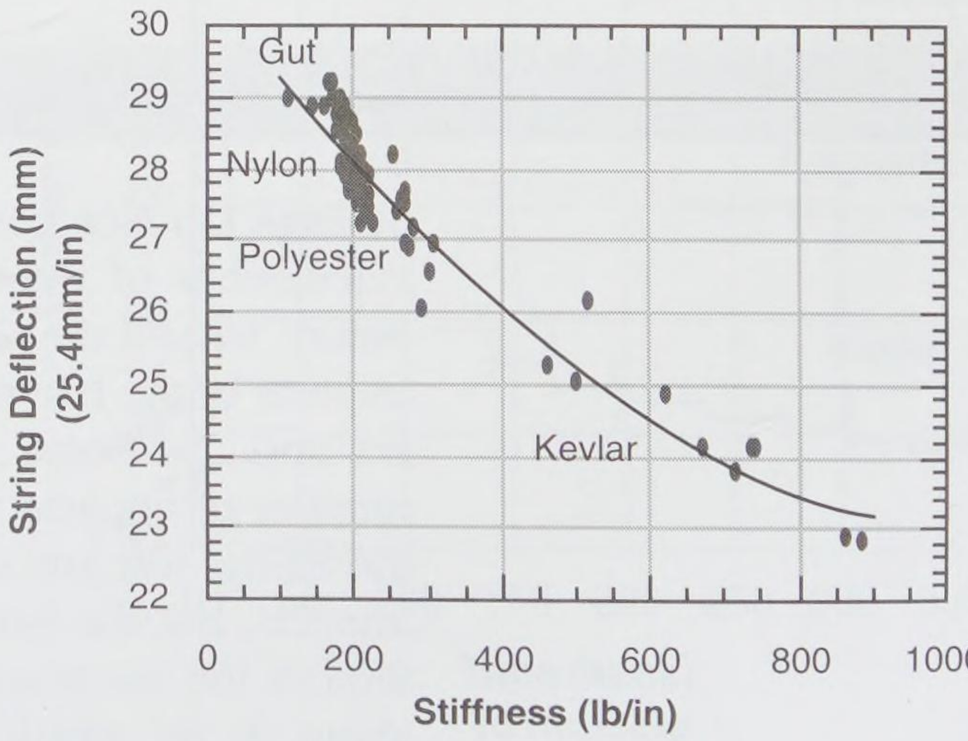


Figure 30.6

During each impact, the hammer pushes the string sideways by the amount (on average) shown in the graph. The same thing will happen on each string in a fully strung racquet, depending on how hard you hit the ball.

In general, the stiffer the string, the less deflection. However, some stiffer strings may have the same or higher deflection than softer strings. This can happen when a stiffer string loses so much tension during impact that it will stretch more than expected during the impact.

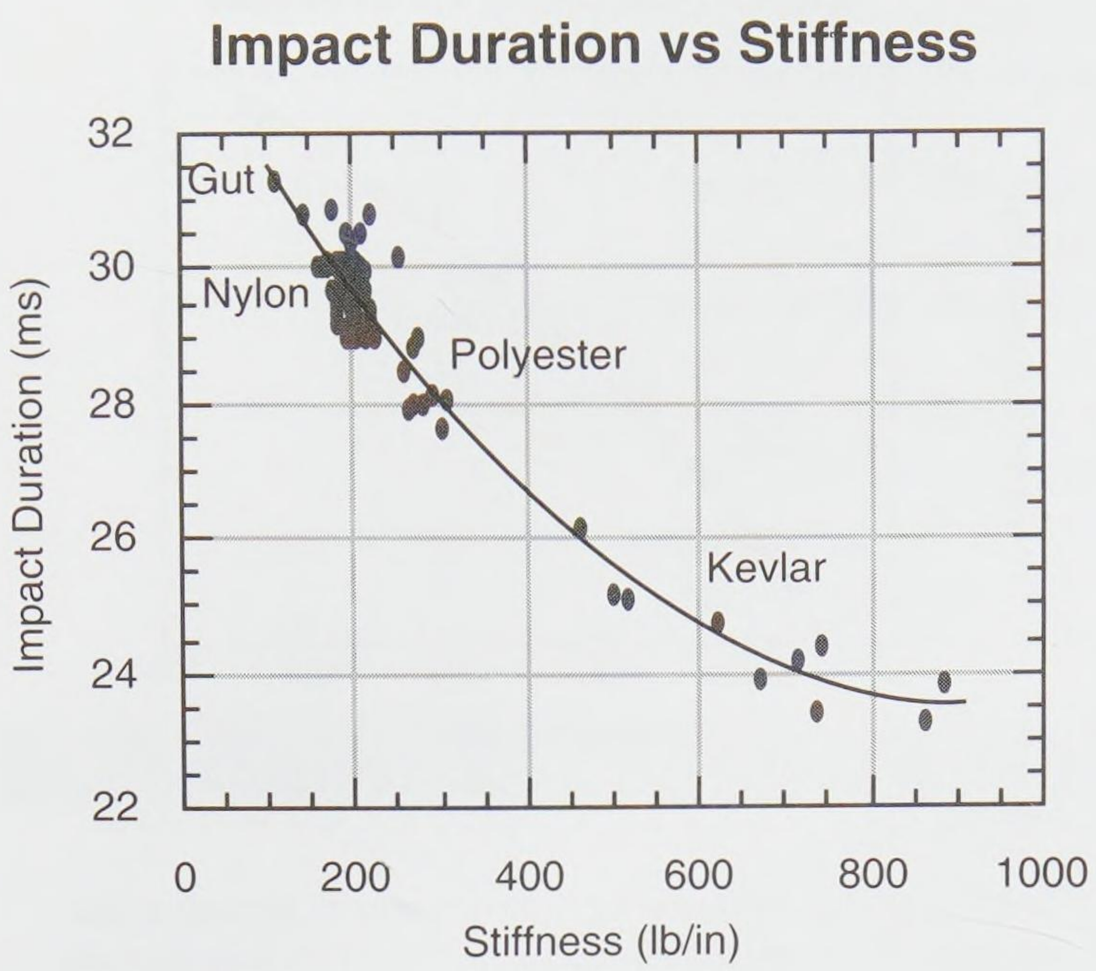


Figure 30.7

Impact duration is the time that the hammer spent in contact with the string before it bounced off, measured in millisec (ms). 30 ms = 0.030 sec.

The impact duration of a ball on a strung racquet is about 5 ms. The hammer was heavier than a ball and a single string is not as stiff as the whole stringbed, hence the impact duration was about 6 times longer. Dividing these results by 6 will give a good estimate of the time spent by a ball on a strung racquet.

impact duration was about 31 ms on gut and about 24 ms on kevlar.

Explanations. These numbers might look a bit strange at first sight. Kevlar is about 6 times stiffer than gut and the tension rises about 5 times more, but the peak force is only 50% larger and the impact duration is only about 20% shorter. Why is this? If kevlar is 6 times stiffer than gut, why isn't tension rise and peak force 6 times more and impact duration 6 times less?

Kevlar is very stiff, so it doesn't deflect very far when hit by the hammer. Being so stiff, only a small deflection of the string causes the tension to increase dramatically, more than doubling the initial tension. But the tension is directed along the string, not perpendicular in the direction of impact. Consequently, a very large increase in tension along the length of the kevlar string, combined with a small sideways deflection, results in a relatively small force acting in the direction of the impact. The force on the hammer is only about 50% larger with a stiffer string than the force on a gut string. The net result is that though the peak force on the kevlar string is larger (50% larger), that force acts for a shorter time (20% shorter), and the hammer is ejected at exactly the same speed for both strings.

Impact force and duration will cause different strings to feel differently with respect to shock and feel, but they both conspire to eject the hammer at the same speed from each of the different strings.

Calculations

Strings Do Not Lose Much Energy. All of these effects agree with detailed calculations of what should happen (See Chapter 13). What we didn't know before we did the tests was how well the strings would survive after repeated impacts of the hammer. We expected that some would survive better than others and that some would lose their elasticity or resilience after a sufficient number of impacts. After 200 impacts on a nylon string, we gave up. None of the strings lost any of their resilience or elasticity. They all lost tension after 10 impacts, but the hammer kept bouncing off every string at between 95% and 98% of the incoming speed for all of the strings. Each hammer impact lasted about 30 ms, which is six times longer than the normal 5 ms impact of a tennis ball on the strings. Two hundred hammer impacts is, therefore, equivalent in effect to 1200 normal impacts, each equivalent to a 120 mph serve.

If a string loses no energy during an impact, then the hammer will bounce off the string with exactly the same speed as the incoming speed. In fact, we found that the energy loss was not zero, but it was close enough to zero to make no significant difference in the calculations.

We did not test strings at tensions other than the starting tension of 62 pounds, but we did do some calculations to predict the results. (Actually, our impact

tests tested strings at tensions ranging from 34 to 62 pounds because each string was at a different tension at impact depending on how much static, Figure 30.8, and impact tension, Figure 30.9, it had lost.) The calculations are shown in Figures 30.10-30.14 and are explained below.

Deflection: Perpendicular vs Linear Stiffness (or Deflection Stiffness vs Pulling Stiffness). As the graphs demonstrate, the sideways deflection of the string is smallest for stiff strings and largest for soft strings, but it does not vary by as much as one might expect for different strings, nor does it vary much by changing tensions. For example, if tension is 55 lb, then the string deflects by 29 mm for gut and about 22 mm for a kevlar string. The sideways deflection increases at lower string tensions, but only by 1 or 2 mm (Figure 30.10). Intuitively, one might expect the deflection to double when the tension is halved, but this is not the case. The reason is that the string gets stiffer, in a direction perpendicular to the string, the more it deflects.

Strings act a bit like ordinary springs in that the harder you pull, the more they will stretch. But there is a difference. The stiffness of an ordinary spring doesn't change when you stretch it. If you pull a spring twice as hard it will stretch twice as far. If you pull or push sideways twice as as hard on a string, the string doesn't move twice as far. It moves less than this. The sideways stiffness is proportional to the string tension, so when the string is pushed sideways, the string stretches and the tension increases so the stiffness also increases. The lengthwise stiffness might also increase a bit, but this is a relatively small effect. In our impact tests, the lengthwise stiffness did not change significantly when we stretched each string.

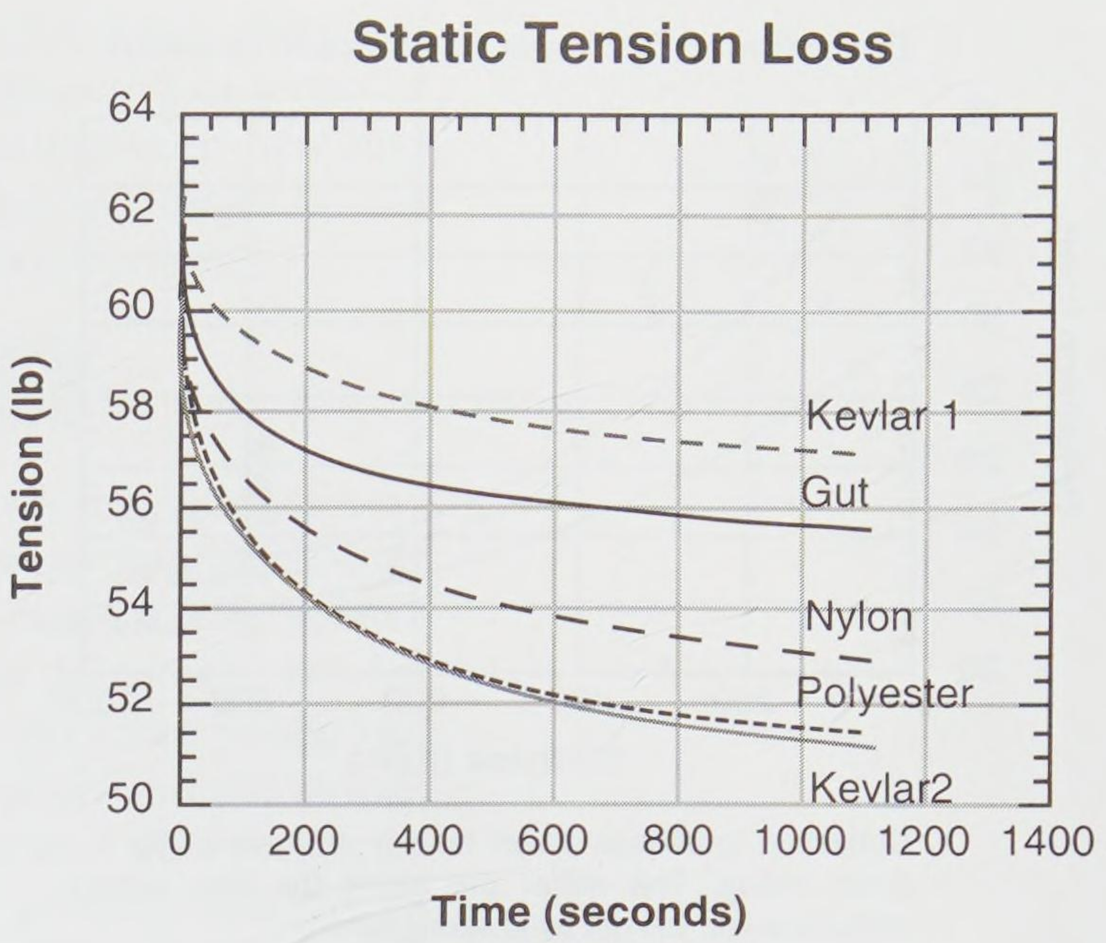


Figure 30.8

Tension vs time after clamping at 28.0 kg (61.7 lb). All strings lose tension rapidly during the first 100 sec, and then more slowly after that. All strings were held at 61.7 lb for 10 seconds before clamping. Kevlars had a wide range of tension loss with time, but most lost tension severely in the impact tests.

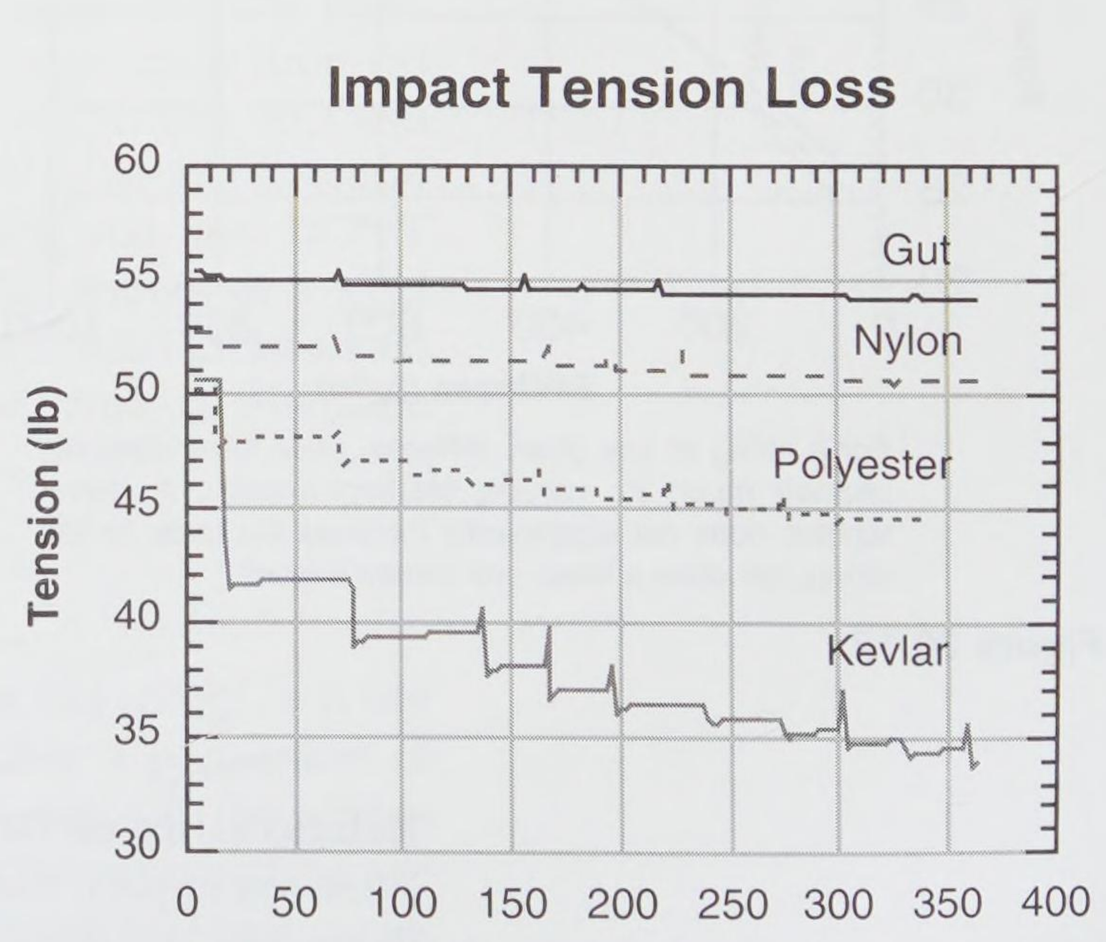
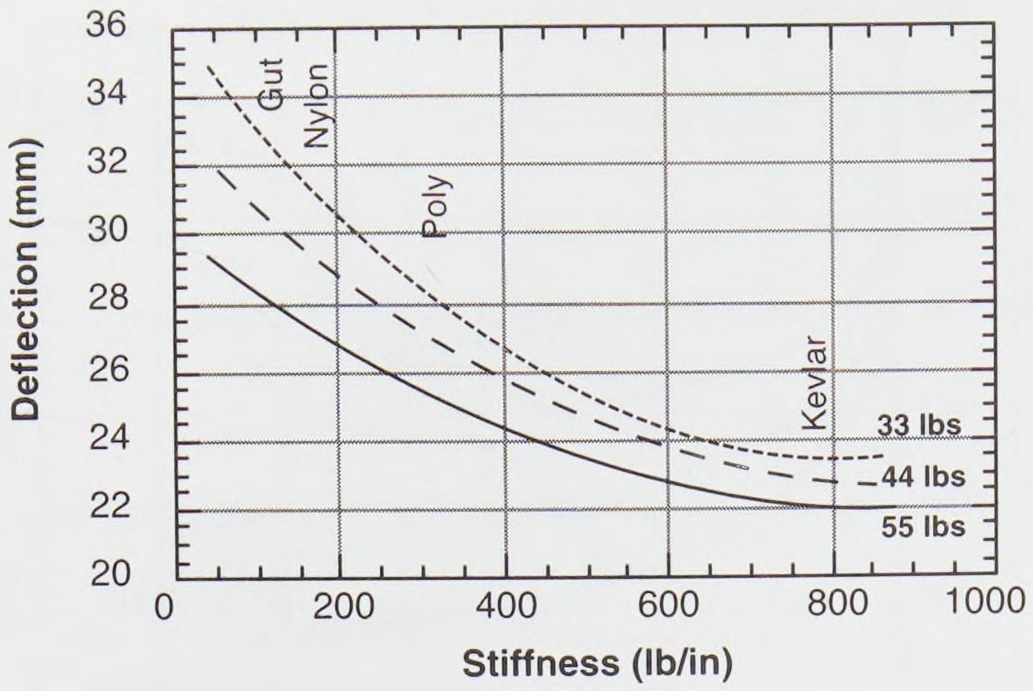


Figure 30.9

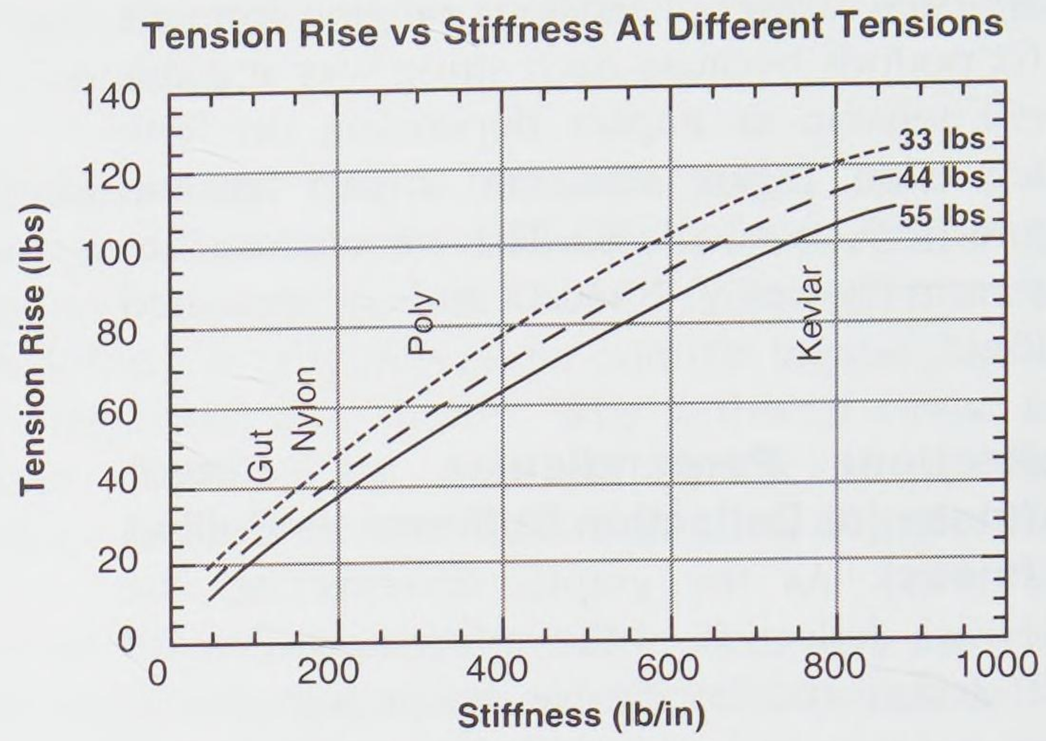
The graph shows tension vs time while each string was subject to 10 impacts. The tension rise during each impact was not properly recorded.For this test, data was only recorded every 2 seconds and whether or not the impact itself (the spikes in the graph) was captured was just a matter of chance. It was not relevant to the test at hand and should be ignored.The loss in tension after each impact is clear, especially after the first impact.

One impact with the hammer is roughly equivalent in effect to 6 very fast serves since one hammer impact lasted about 6 times longer than a normal impact of the ball on the strings.



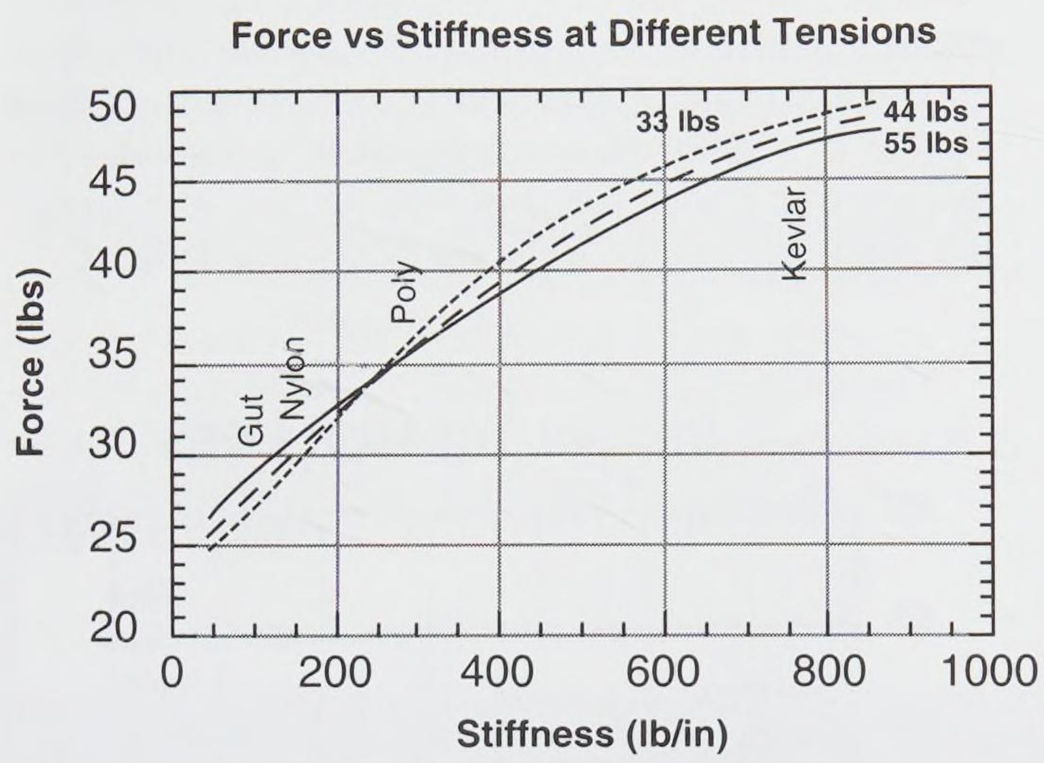
Deflection increases when tension is lowered for a given string. The stiffer the string the less extra deflection you will get by lowering tension.

Figure 30.10



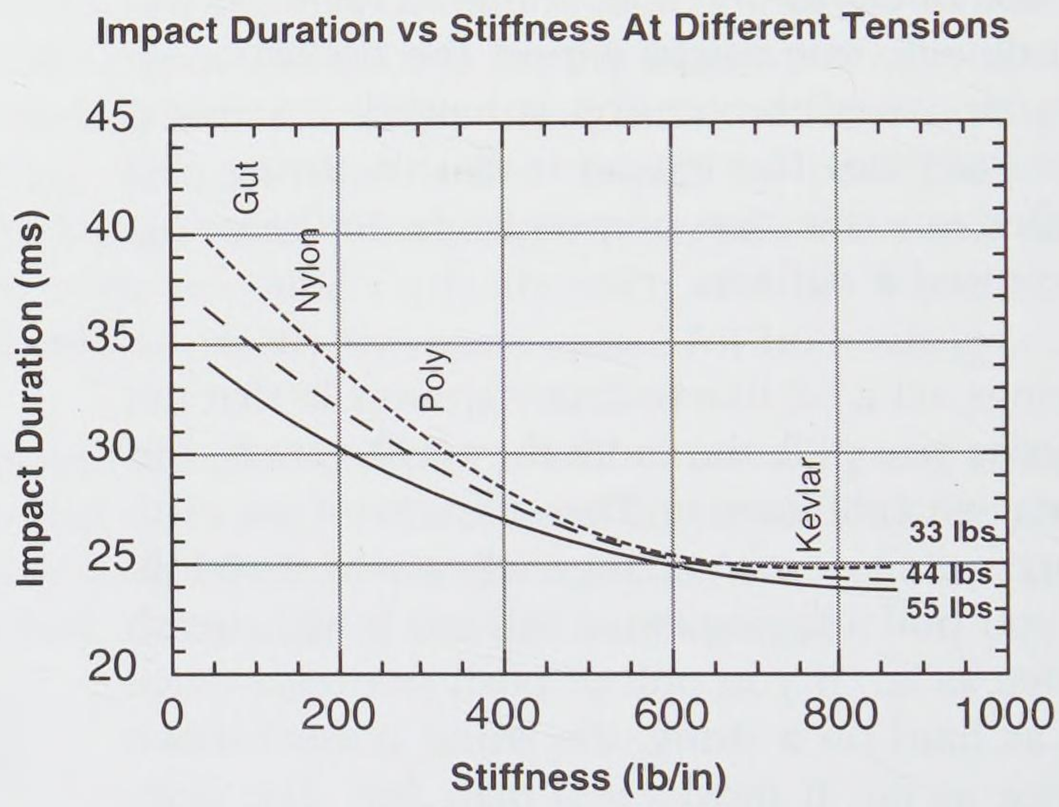
Impact tension rises more for a string at lower tension compared to higher tension. The stiffer the string, the greater that difference.

Figure 30.11



For a string of any given stiffness, peak force does not change much by varying starting tension. A higher tension does not significantly increase the force on the string, nor does a lower one lessen it much.

Figure 30.12



Impact duration is more significantly increased by lowering tension for soft strings than for stiffer strings.

Figure 30.13

Relationship of Deflection, Force, and Tension. If a string is subject to a sideways impact, the increase in tension, when the tension is low, is larger than that when the string is initially at a high tension (Figure 30.11). The reason is that when a string starts off at high tension, it is already fairly stiff in a sideways direction. The hammer comes to rest fairly quickly with only a relatively small stretch and a small increase in tension. If a string starts off at a low tension, the hammer pushes the string a relatively long way before it comes to rest. As a result, the peak tension is only slightly smaller if you start with a string at low tension.

Furthermore, the peak force on any given string hardly varies at all if you change the initial tension. The peak force increases if you use a stiff string such as kevlar, but it doesn't really matter if you string kevlar at 33 lbs or 55 lbs. The peak force on the strings will be about the same. The same is true of nylons, polys and gut. Regardless of the initial tension, or the subsequent loss in ten-

sion over time, the peak force on the string will be about the same. New and old strings should therefore feel much the same when measured in terms of the peak force on the strings. The main difference in feel between new and old strings will probably be due to the impact duration, which is longer on old strings at low tension. Is that the only difference? What about power?

Do Strings Ever Lose Power or Resilience?

Difference Between New and Old Strings. Most players report that new strings perform better than old strings. New strings are crisper and give better ball control, while old strings are less lively and are often described as "dead." By dead, most players seem to mean that old strings lack the power of new strings. However, some players say the opposite, that they get more power out of the strings as they get older. New strings are at high tension and old strings are at a lower tension. Most players will tell you that strings lose their resilience with age or use. But our tests show that they don't. It is possible that strings will lose resilience if they are stretched almost to the breaking point, but we didn't stretch the strings that far. We stretched them only as far as they stretch in a 120 mph serve. So we are left with some really tough questions here. Do old strings actually lose power or don't they? If they do, then how come the hammer keeps bouncing off at the same speed after lots of impacts? If they don't, then how come most players think they do?

Steel Ball Drop Test. Further tests were done to look at this. A two pound steel boule was dropped from a height of 100 inches onto the strings of several racquets, with old and new strings, with the racquet clamped to the floor. The ball was held by an electromagnet so that it could be more easily dropped in the middle of the strings. The ball bounced to a height of between 89.7 and 90.3 inches off all the strings. This variation is too small to make any practical difference. So, that settles it. Old strings DON'T lose power and they DON'T lose resilience. They absorb all of the impact energy of a hammer or a steel ball and give most of it back, regardless of how old they are. The ball bounced off the strings at 95% of the incoming speed, exactly the same as when the hammer bounced off a single string.

Player Perceptions of Power

When a player says that new strings give more power than old strings, it is not a comment based on any accurate measurement, but rather a perception of power. The player might genuinely think the ball is coming off the strings faster, when in fact, it is not. How can this be possible? That is something we can't answer and we are not even certain it is true, but there is clearly a difference in what players experience and what the above measurements and calculations are telling us. Some suggestions follow.

Old Strings Are Looser and Have MORE Power. We know that strings lose tension with time and with use. We also know that if you want more power you should string at a lower tension. That's because the strings will absorb more of the impact energy and the ball will absorb less, so less energy will be wasted in the ball. At least, that's the case with a tennis ball. A steel ball doesn't compress as much and loses no energy. That's why the steel ball was used to test the strings. But this tells us that old strings will have MORE power than new strings when they are used to hit a tennis ball. That's a bit embarrassing. We can't use that argument to explain why old strings seem to have LESS power.

Effect of Impact Duration on Power and Control. Perhaps there is a clue in the difference in impact duration between old and new strings. A ball or a hammer will spend more time on a soft string than a stiff string. So, as a string ages and the tension drops, the ball will spend more time on the strings. Though it will still come off at the same speed (maybe even a slightly higher speed if we believe that low tension = more power) it will come off at a different angle. Why?

On strings at high tension, the ball will come off the strings after about 4.5 ms. With strings at low tension, the ball comes off after about 5 ms. For a serve or a smash, the racquet head swings through an angle of about 8 degrees in 4.5 ms or about 9 degrees in 5 ms, depending on how fast the racquet is swung. For a groundstroke, where the racquet is swung at a slower speed, the racquet will swing through an angle of only 3 or 4 degrees during the time the ball is on the strings. A one degree change in the rebound angle means that the ball will land in a different spot, but a good player should be able to compensate by hitting the ball a bit earlier, one degree before the usual impact point. Nevertheless, if a player finds that the ball lands in a different spot, the question is whether the player will perceive this as a change in ball speed or a change in rebound angle. However, we don't think this is the effect we're looking for.

Psychological Effects: Is Power All In Your Head? Perhaps there are psychological effects involved here as well. If a ball stays on the strings for only a short time before it bounces off, then players might assume that it also comes off at a higher speed because it came off faster. The word faster is ambiguous itself. Faster can mean sooner or it can mean higher speed. Players sometimes complain that pressureless balls are like bullets since they come off the strings so fast. They might spend a shorter time on the strings, but that doesn't mean they come off at a higher speed. Suppose a ball stays on string A for 4.5 ms and bounces off at 60 mph. On string B, it might stay on the string for 5.0 ms and still bounce off at 60 mph. But the player might think it came off at a higher speed from string A because it came off sooner and felt crisper. We are only guessing here. No one has done an experiment with a radar gun to check this out, but it might actually be true. There is definitely nothing wrong with the physics of this. The speed off the strings depends on the force acting on the ball as well as the duration of the force. A small force acting for a long time can generate the same speed as a big force acting for a short time. That's exactly what we measured with the hammer bouncing off different types of string.

Sound and Power. The sound of the impact might also play psychological tricks. If you hit a drum at high tension, you get a crisper sound than at low tension. If you use a string dampener, the shot sounds different (better for some and worse for others), but it can't make any difference to the speed of the ball since only a microscopic amount of energy is involved in the string vibrations.

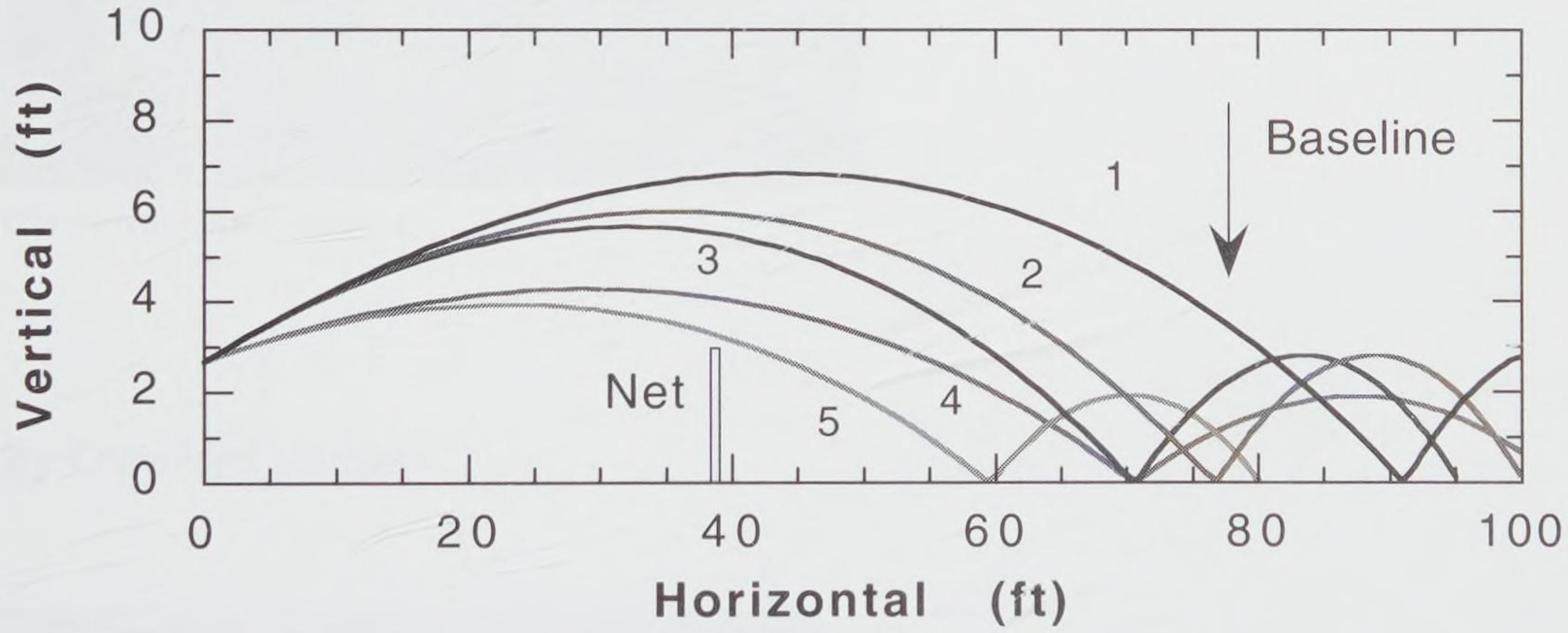
Strings Don't Lose Power, They Lose Trajectory. There is another possible reason why old strings might seem to lose power when in fact they don't. Suppose you pick up a racquet with new strings and hit the ball consistently over the net to land on the baseline. Then you pick up the same model racquet but with older or well-used strings. If you play the same shot and the ball keeps landing short or in the net, what will you conclude? Will you say the old strings have lost power? What if you have to hit the ball a lot harder to get it

over the net or onto the baseline? Will you still say that the old strings have lost power? You will probably answer yes to both questions. But suppose the ball comes off both sets of strings with exactly the same speed when you swing the racquet with the same speed. Then the strings have the same power, but the ball could come off the strings at a different vertical angle. The ball might bounce off new strings at a higher angle than off old strings, possibly because new strings give a better grip on the ball than old strings. It might have something to do with the way the strings slide or don't slide across each other sideways in the string plane. That would explain why the ball clears the net with the new strings and lands short or in the net with old strings.

The question now is, can a player judge the speed better than the vertical angle, or is it the other way around? Again, no one we know of has tested this with a radar gun, so we can't say for sure. Until that is done, we cannot give a definite answer to the question why old strings seem to lose power even though there is no loss of resilience.

In the mean time, have a look at the ball trajectories in Figure 30.14. These were calculated on a computer. The ball slows down through the air due to air resistance and lands on the court at a slower speed than when it was first hit. Can you guess which ball started off with the highest speed? Which one started off with the lowest speed?

Trajectories, Spin and Shot Placement



1 = 70 mph, no spin, 10°
2 = 70 mph, 16 revolutions/second, 10°
3 = 70 mph, 28 revolutions/second, 10°
4 = 70 mph, no spin, 6°
5 = 70 mph, 16 revolutions/second, 6°
Note: the angles of 6 and 10 degrees are the launch angles above the horizontal

Figure 30.14

The answers are that they all started at the same speed, but the spin or the angle was different in each case. The trajectories are shown side-on. The player's view is from above and behind, so the angles won't be as easy to pick. A player might say, "I'm not getting any power out of these strings," but a spectator might say, "The ball is coming off the strings at a low angle." Power describes ball speed, and control describes a consistent or expected ball angle. If the trajectories (4) and (5) are consistently low with old strings, and trajectories (1) - (3) are consistently high with new strings, then control is good, but

it might seem that power is different. If the trajectories are not consistent, then the problem is with control, and that is probably a sign that the string tension is too low. At low tension, the ball stays on the strings a bit longer, giving the racquet more time to rotate about the long axis through the handle. If the ball strikes the strings off-axis then the racquet will rotate about the long axis and the ball will come off the strings at an angle that is either too low or too high depending on whether the ball strikes below or above the axis.

Goran Ivanisevic, Croatia

31

Power and Strings Part 1

An Introduction to Elastic Energy

By Crawford Lindsey

String Tension and Power

Tennis players live by the cliche "string loose for power and tight for control." But though many live by it, many don't understand it. In fact, many courtside arguments on this topic have been more vituperative than on a disputed match point line call. So, is this dictum a law of the universe or a convenient myth? The crux of the matter is this — somewhere inside of us is the little kid looking at the world for the first time saying, "Daddy, if I pull the racquet string very, very tight, then it will have more power in it, and it will send the ball away faster and farther." The reasoning, of course, is that there is more energy in the string, so there must be more power. A little thought experiment will help to expose the fallacy of this statement.

Steel Balls and Bounce Height. Where does this energy that's all couped up in the tighter string go? Imagine two racquets infinitely secured in all directions so only the string face will deflect upon impact. One is strung at 50 and one at 70 pounds. Since we want to know what is happening in the strings only, we will use nondeformable steel balls for our test. This eliminates the dynamics of a deformable rubber ball interacting with the strings. We will thus be able to see which strings have the most power.

Drop a steel ball weighing the same as a tennis ball on each racquet (Figure 31.1). Which ball will rebound higher? What will happen? The possibilities for ball rebound are shown in Figures 31.1-31.3.

Follow The Bouncing Balls

Tension & Rebound

Does Lower Tension Rebound More?

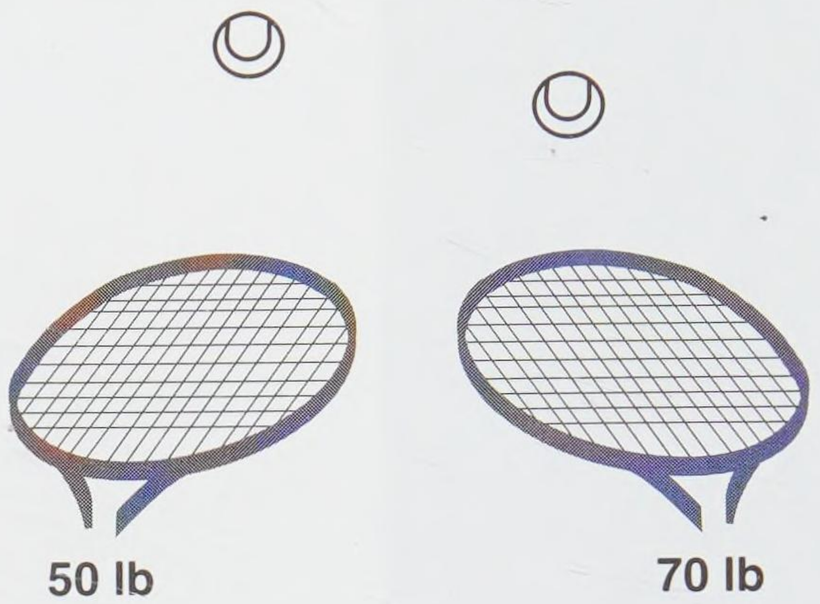


Figure 31.1

Guess #1:

The 50 lb stringbed will deflect more and rebound the ball higher because of a trampoline effect.

Does Each Tension Rebound The Same?

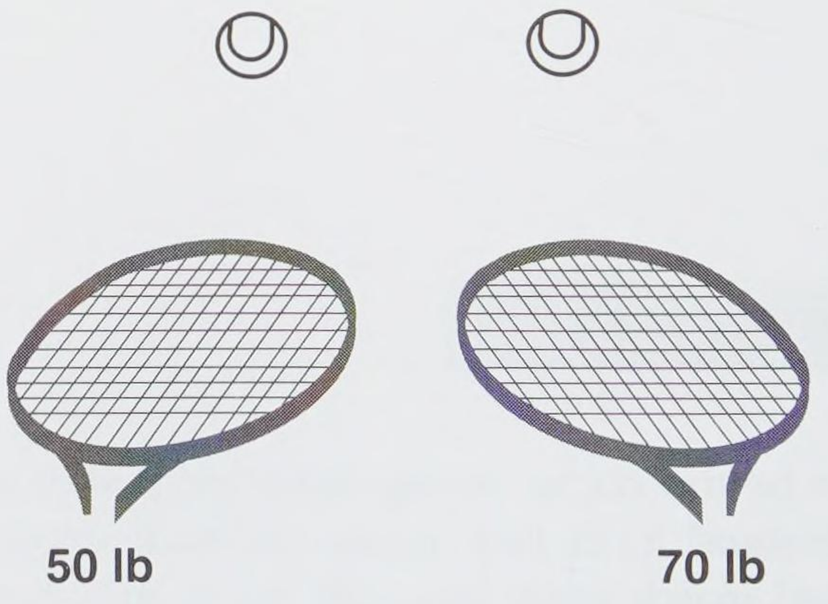


Figure 31.2

Guess #2: It will bounce the same height on both racquets.

You might say that since no energy is getting lost in ball deformation because they are made of steel, all the kinetic energy of the balls is going into the strings. Since each ball has the same energy at impact and each stringbed will receive the same energy, then each ball might bounce the same height with each tension. But then you rethink. The 50 pound stringbed will deflect more than the one at 70 pounds. Does energy get used up in stretching the string? If so, then the 70 lb stringbed will lose less energy stretching.

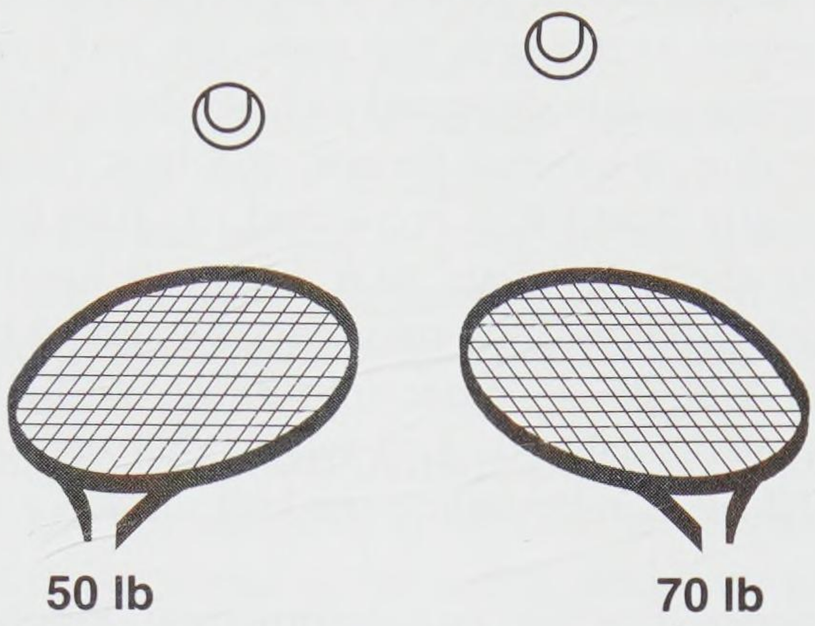


Figure 31.3

Guess #3: So now you're convinced that the bounce will be higher on the 70 pound racquet because more energy is "available" since it took more energy to stretch the 70 lb strings tighter. And round and round you go.

Now you are more confused than ever because all three scenarios seem reasonable. What happens is this. Strings don't actually "have" power. The energy in the strung racquet, whether at 50 or 70 pounds, is not available to the steel ball. That energy is all distributed in the equilibrium balancing of all the stresses and strains as the racquet and strings pull on each other. The only energy available to return to the ball is the energy added by the impact. The preexisting tension in the string is irrelevant. That's not extra energy, it's equilibrium energy, or by definition, zero energy. The racquet starts in equilibrium and ends in equilibrium. In between, energy in equals energy out. There is nothing extra in either racquet to give. The energy is just passed around a little differently in each racquet and with the steel ball.

Since there is no extra "power" or energy obtained in tensioning the strings, and since there is no energy going into deforming the ball, the only place energy can be lost is in stretching the strings. As will be explained in more detail in Chapter 32, all strings lose only about 5% of the incoming energy in stretching. And in the typical ranges of playing tensions, there is little difference in energy loss at different tensions. The answer to our conundrum is therefore clear. The steel balls will bounce to the same height.

The reason this is an important conclusion is that we have determined that the "intrinsic power" of all strings at all tensions is about the same. We did this by eliminating energy loss in the racquets and balls by clamping the racquets so they could not recoil and using steel balls so the ball could not deform. But we know that tennis balls don't bounce to the same height when dropped on a clamped, stationary racquet (the looser stringbed bounces the ball higher). And since different strings have the same intrinsic power, the answer must lie in the ball or the dynamics of the interaction between the ball and strings at different tensions. Let's take a look.

Tennis Balls and Bounce Height. Now we are back in the real world of rubber tennis balls. But our little trip to Steel Ball Land has taught us one very important thing. Power is all about energy conservation in colliding objects. A tennis ball's impact energy is the same (because the steel balls weighed the

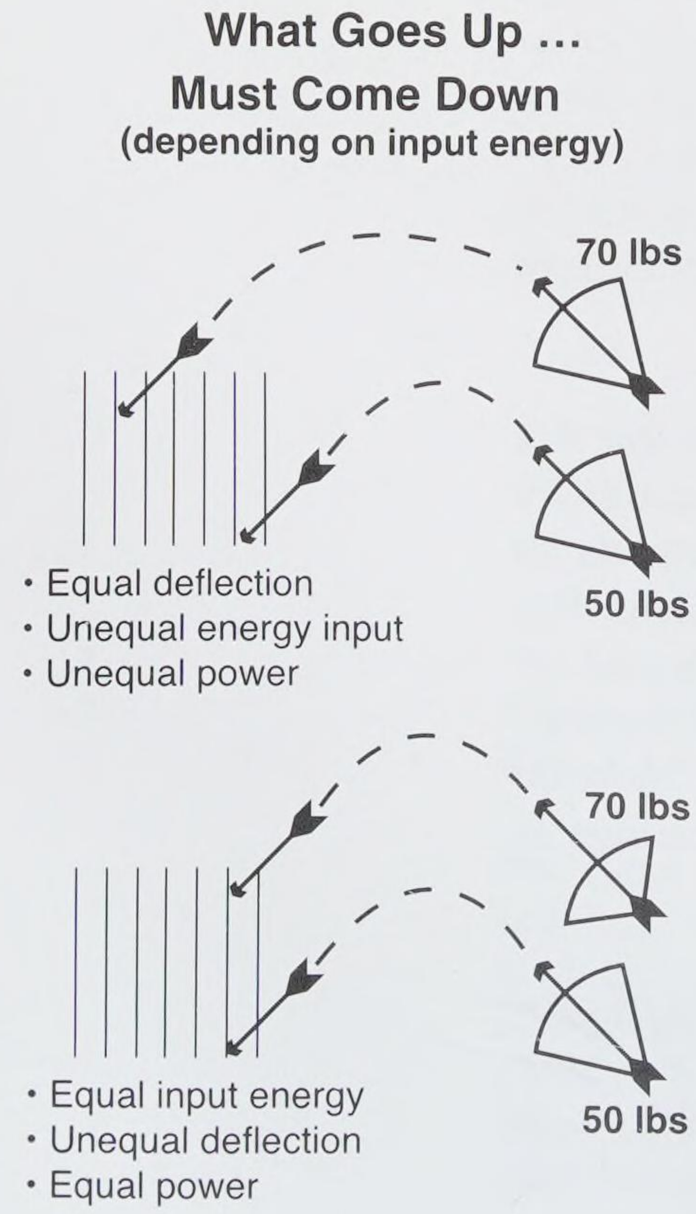
same as a tennis ball), and thus, the amount available to give back to the balls is the same. But the tennis ball is much more compressible, so the stiffer the strings are in comparison to the ball, the more the ball compresses. The key point here is that a tennis ball is designed to lose about 45% of its share of the stored impact energy due to internal friction and heat during its deformation. (The energy isn't actually "lost." It is converted to other forms that are not of any use to propelling the ball.) Thus, that energy is lost for accelerating the ball. The relative deformability of the two objects is going to determine where the incoming energy will go — more into string or ball deformation (see Chapter 32 for a detailed analysis). And where that energy goes determines how much help it will be in rebounding the ball.

Because so much energy goes into ball deformation, kinetic energy conservation is of the utmost importance in creating tennis power. As we have seen, one way to do this is to make the ball harder. That would be illegal. So for tennis, the equation becomes very simple. For the range of typical playing tensions, the looser the strings, the more energy will be stored in the strings and returned to the ball. The tighter the strings, the more the ball is compressed, and the more energy is lost.

There is a point, however, where more energy is lost in stretching and moving the string than the energy saved by not compressing the ball. The tension where that occurs is somewhere in the 40-pound range, depending on swing and incoming velocity. At that point, lowering tension will produce less power.

So, lower tensions are not by their nature more powerful than higher ones. It depends on the nature of the colliding objects, the dynamics of the collision, and how low you go. But if one of the objects is a tennis ball, then for the range of common playing tensions, lower tensions mean more power (that's the good news, but the bad news is that it is not a "significant," nor, to most players, a perceptible amount more, as we will see in Chapter 32). But it's not that lower tensions are more powerful, but that they are more capable of storing and returning a ball's impact kinetic energy without turning some of it on the ball itself and crushing it.

Figure 31.4



The Bow and Arrow Analogy. But you still can't get rid of that feeling that tighter strings somehow have more power, can you? Still, you're secretly thinking, "But what about all that extra energy I put into the string by pulling it to 70 pounds? Isn't it just like a bow and arrow? Won't the tighter bow send the arrow farther?"

The bow and arrow is a little different because it is not a collision and the input energy is provided solely by the operator. As shown in Figure 31.4, the archer fully extends his arm holding the bow and pulls the string back to his cheek with the other. The tighter bow (70 lbs) is not more powerful because of the starting tension, but because it takes more input energy to pull the string back an equal distance to the cheek than with the 50 lb bow. Unequal energy went into each pull, so unequal energy comes out. What is equal here is the string deflection back to the cheek, not the energy needed to do the deflecting.

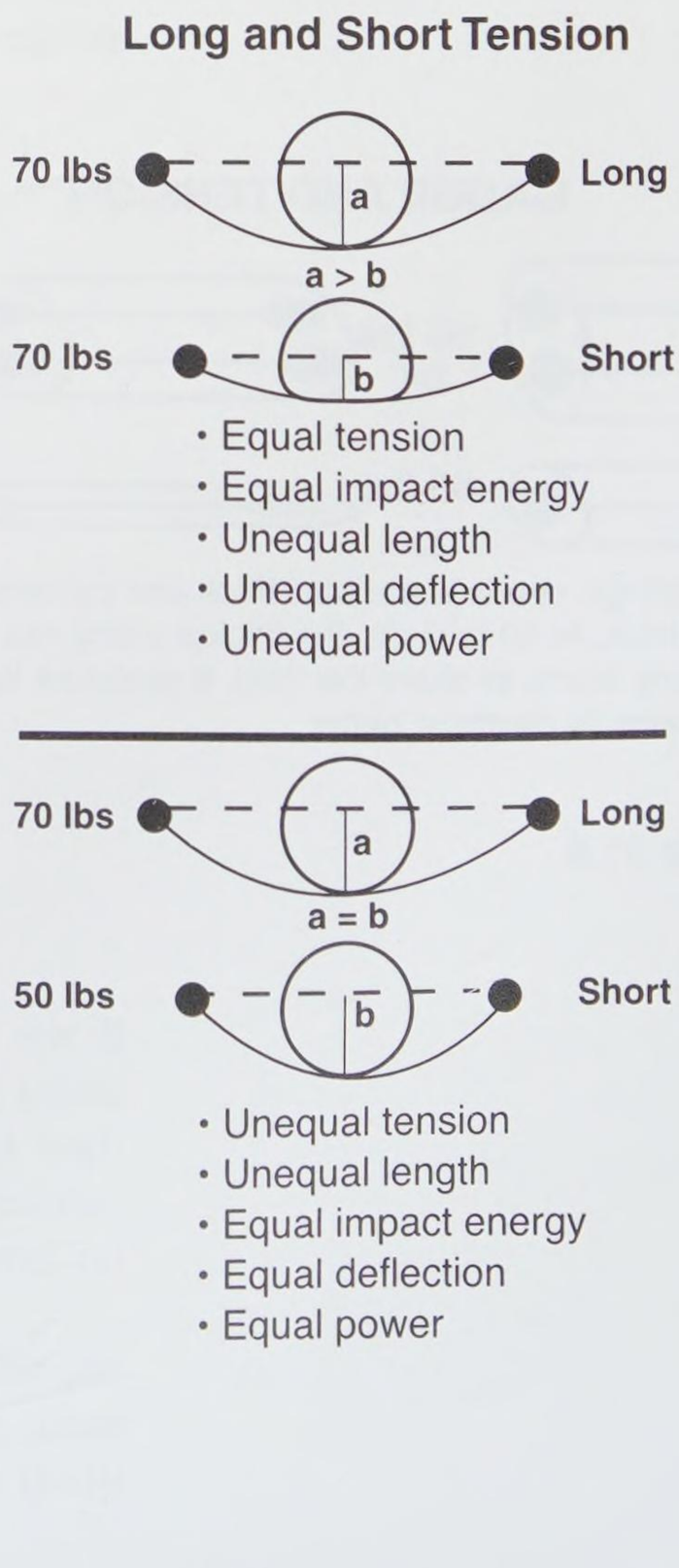
And if you still won't give up and are thinking that pulling tension on the stringing machine is like pulling on a bow and arrow, you're right. There is more energy locked up in that 70 pound racquet, but it is just that — locked. To release it, you have to cut out the strings. Imagine an ant crawling along your center main (imagine you're stringing and haven't installed the crosses yet). If you cut the string just behind its path, the ant will go flying further with the tighter racquet. But ant hurling is another sport and a different application of the tensioned string that doesn't help one's forehand much.

We conclude that looser does result in more power, but not because the strings are more powerful, but because they crush the ball less and more of the energy is thus available to be stored in the strings and then given back. It's about energy conservation and transfer during impact, not inherent power or production. It's not that loose strings have more energy, it's that up to a point, they absorb more and give back more when interacting with a tennis ball. They are better transformers; they are not the energy plant.

String Length and Power

The same sort of reasoning applies to long vs short strings, all other things being equal (Figure 31.5). In other words, with steel balls, both a 13-inch and a 15-inch string of the same gauge and material and at ther same tension are potentially equally as powerful depending on the nature of the collision. The longer string will deflect more, however. The reason is simple. When a string is pulled from the end or impacted in the center, it stretches the same everywhere. So, if you stretch a string 10%, every inch along the string is stretched 10%. If one string is 15 inches and the other 13, then the 15-inch string will stretch 2" x 10%, or 0.2 inches more than the 13-inch string. A long string isn't more stretchy; it just has more inches to stretch and thus deflects more on impact.

Figure 31.5



Of course, for a tennis ball, the results are the same as in the loose vs tight situation. More deflection means more cushioning, less ball deformation, and more energy return. Again, it's about energy conservation and location of conservation, not energy creation. All things being equal, longer strings convert more incoming energy into outgoing energy. Longer strings have a better exchange rate than shorter strings at the same tension.

Different deflections for different lengths is, of course, why proportional tensioning exists. For the above reasons, different length strings at the same tension will be saving different amounts of incoming energy. If you're looking for power, the shorter strings will give energy away. If it's control you want, longer strings at the same tension might introduce too much power or variability into your hit.

The basic stringing formula for proportional tensioning is to reduce tension in the same proportion as the difference in length of two adjacent strings. This is a good starting point, though it is essentially arbitrary for the following three reasons.

(1) First, it assumes that for every x percentage increase or decrease in tension, there will be a constant x percentage increase

or decrease in elongation. Instead, elongation changes proportionately more at lower tensions than higher tensions. So, as you lower the tension on the shorter string, you may gain more than a proportional amount of deflection.

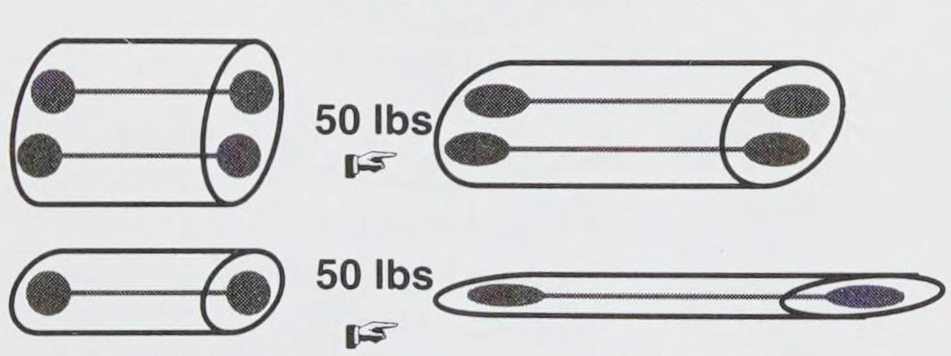
(2) So, using the basic formula may actually over-compensate a little. But that's okay because each string going outward from the impact is exposed to less impact. That's because the ball is round and the impact is absorbed outward from the first impact point. So each outward string needs to be even looser to stretch the same amount as the one next to it because it is enduring slightly less impact energy.

(3) And last, longer strings at higher tensions will also lose tension at a faster rate. Even as you're lowering the tension on the shorter string, the longer, tighter string is losing creep tension faster than the shorter one. Your tension differential will therefore end up less than the difference in your reference tensions for the two strings.

So, if anything, you might want to err on the side of lowering tension on the shorter, looser string by more than the same proportion as the difference in string lengths. Obviously, unless you're a mathematician, trial and error will be your guide.

String Gauge and Power

If you tension a 15 gauge monofilament string to 50 pounds, it is being tensioned equally from the outside to the inside (at least after the tension has had time to equilibrate inside the string). The entire cross-section is being pulled to 50 pounds and is elongating the same amount in the process. By this reasoning, you can think of a 17 gauge string as embedded in the middle of the 15 gauge string. Now imagine that you are pulling this string on a continuous tension machine. What if we could get to this buried 17 gauge string by melting away the outside layers? What would happen? As the layers melt away, the string will get longer and longer with each disappearing layer. Why? It is exactly the same piece of nylon that was there to begin with. It's not that the nylon suddenly became more elastic because it is thinner. It's that the 50 pounds seems like more to the string now than it did before. It's still the same 50 pounds, but now the number of pounds per square inch of cross-section is much greater. Each polymer chain is not sharing the 50 pound weight with as many other chains as before, so it must stretch further to build up enough tension to resist the load. So it's not that thinner is stretchier, but that thinner carries more load per square inch than thicker, so it stretches more (Figure 31.6).



Two strings one is two atoms thick and the other one atom thick. At 50 pounds, the thicker string has twice as many atoms to share the load. It stretches less and maintains its diameter better.

Figure 31.6

If we now put all the layers back on and then increase the tension until the stress in pounds per square inch equals its 17 gauge counterpart, we'll find that they have both elongated the same amount. The nylon elongates the same amount when subjected to the same stress in pounds per square inch, no matter how thin or thick it is.

So, different gauge strings may be at the same tension, but they are not at the same stress. It is the stress that determines how tight or loose the string feels upon impact. Also, if you pull a 16 and a 17 gauge to equal tensions, not only

will the 17 gauge elongate more to get to that tension, but the higher the tension, the greater the difference in their diameters will become compared to the difference in their starting diameters. This in turn means the stress increases faster as tensions increase for a thinner string compared to a thicker one.

And then the same reasoning as above applies: the tennis ball will bounce higher off the 17 gauge because it stretches more, absorbs more energy of impact, and then gives it back. The pounds per square inch are greater in the thin string but the amount of energy being added to it is the same. (Naturally, all is not always so simple. Chapter 35 describes some exceptions to this principle.)

So, a long, thin, loose string has as much power in it to start with as a short, fat, tight one. If you play tennis with steel balls, it won't make any difference. But if you use tennis balls, the first one will (usually) give you more energy return than the latter.

Elena Dementieva, Russia

32

Power and Strings Part 2

How Much Do Strings Affect Power?

By Rod Cross

The most surprising results to come from the string tests described in Chapter 30 are that

- strings don't lose power with use
- tension makes almost no difference to the energy loss in a string
- there is no significant difference in energy loss between different string types.

This is so contrary to common belief that it seems to be wrong. Most players at an elite level replace their strings many times each year since they say that the strings feel dead after five or six sets, as if the strings have lost power or resiliance (see Note 1). Most players also prefer the "crisp" or "lively" feel of a new set of strings. Consequently we will look more closely in this chapter at the effect of different string tensions and string types on the energy loss in the ball. Impacting a string with a metal hammer or a steel ball is the best way to measure the energy lost in the strings, but it does not tell us how much energy is lost in the ball when the string type or tension is varied.

The conventional wisdom is to string a racquet at low tension for more power. The reasoning is that the ball will squash less in a soft impact, so it will lose less energy and come off the strings at a higher speed.

Furthermore, the racquet will vibrate less in a soft impact, since the force on the strings is smaller, so the racquet bends by a smaller amount and less energy is wasted in racquet vibrations. If less energy is wasted in both the ball and the racquet, then the laws of physics guarantee that the ball will come off the strings at a higher speed. However, the question that needs to be asked is, by how much?

Suppose the string tension is reduced from 60 to 50 pounds. If you can serve a ball at 120 mph with strings at 60 pounds, then you can serve it even faster if you drop the tension to 50 pounds. But as you will see in this chapter, measurements and calculations indicate that you will serve the ball at about 120.8 mph if you drop the tension to 50 pounds. The increase is ridiculously small and probably not noticeable.

Old strings do NOT lose power. They will feel softer since the string tension drops over time and with use, but if anything, the ball will come off the strings a tiny bit faster as the strings get older. The sensation that old strings feel dead and have reduced power is probably due to the fact that the force of the ball on the strings is reduced at low string tension, with the result that the force acting on the hand is also reduced. If the force on the hand is reduced, players may interpret that as a dead feel, and they may equate that with the feel of hitting the ball at a lower speed. The racquet will vibrate less when the tension is reduced, so players will also notice a reduction in the vibration level, as if the ball was being hit at a lower speed. Conversely, many elite players tend to prefer relatively stiff strings such as polyester or kevlar, possibly because the force on the hand is greater, and that will give them the sensation of greater power.

Serve Speed

When a player serves a ball, the racquet is swung at a speed that varies from one end of the racquet to the other. The tip travels the fastest, and the handle travels the slowest. If the speed at the impact point is V, then the ball will be served at a speed v given by

$$v = (1 + e_A)V \qquad (32.1)$$

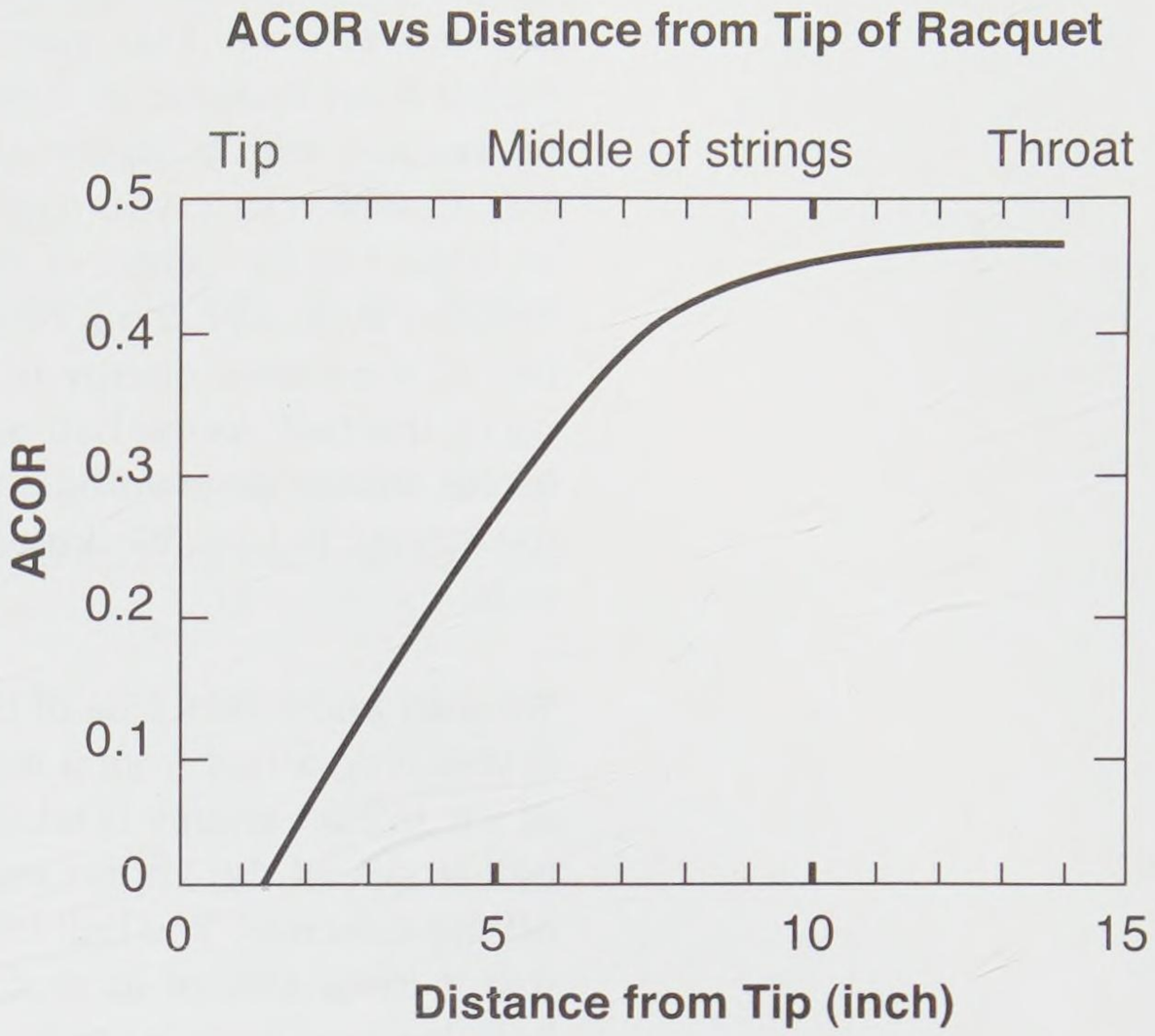
where e_A is the apparent coefficient of restitution (ACOR) at the impact point. Equation (32.1) is derived in Chapter 11. The value of e_A can be measured by firing a ball at right angles to the string plane and measuring the rebound and incident speeds. e_A is the ratio of these two speeds, so it a number that measures how well the ball bounces off the strings. For example, if the ball rebounds at 40% of the incident speed, then $e_A = 0.4$. If it bounces at 10% of the incident speed, then $e_A = 0.1$. If e_A is 0.4, then the serve speed will be 1.4V or 40% greater than the racquet speed. If $e_A = 0.1$, then the serve speed will be 1.1V or 10% greater than the racquet speed.

Changing the string type or tension has no effect on the racquet speed. Consequently, the only effect that we need to consider is the effect on e_A. The value of e_A increases when the string tension is lowered, but only by a small amount. Similarly, changing from nylon to natural gut will reduce the string plane stiffness, but e_A will increase by only a small amount.

Typical ACOR Values

A typical graph of ACOR vs impact position on the strings is shown in Figure 32.1. All racquets and all strings give a similar result to this in that the ACOR is largest near the throat and smallest near the tip. If you think about this for a second, it means that racquet power is largest near the throat and smallest near the tip. Racquet manufacturers don't tell you this. They tell you that racquet power is biggest in the sweetspot region near the middle of the strings. It turns out that we are both correct. If the racquet is at rest when the ball hits it, then the power is a maximum near the throat. However, if you swing the racquet to hit the ball, then the point of maximum power shifts away from the throat to a point near the middle of the strings. That's because the middle of the strings is traveling at a higher speed than the throat, so the ball then comes off faster from the middle of the strings. The ACOR in the middle of the strings is usually about 0.3 for a light racquet, about 0.4 for medium weight racquets, and about 0.5 for a heavy racquet. The ball, therefore, bounces faster or higher off a heavy racquet, but the disadvantage is that you can't swing a heavy racquet as fast as a light racquet.

Figure 32.1



You can measure the increase in ACOR with racquet mass yourself very easily. Place the racquet on the floor and put your foot on the handle. Then the racquet behaves as if it is as heavy as the floor. Then drop a ball onto the middle of the strings. The ball bounces much higher since it can't transfer any of its energy to the racquet. The energy is taken up partly by the strings and partly by the fact that the ball squashes a bit, but almost all of the energy stored in the strings is then given back to the ball. If you drop the ball directly on the floor, then the floor is too stiff to absorb any of the impact energy so it won't have any energy to give back to the ball.

Increasing Power by Increasing Bounce

Equation 32.1 tells us something that is perhaps very obvious. If you swing the racquet twice as fast, so that V is doubled, then the ball is served twice as fast since v is also doubled. It also tells us something that is not quite so obvious. If you can somehow double the ACOR, then the ball speed won't be twice as fast. It will only be about 20% faster. Doubling swing speed doubles ball speed; doubling bounce speed from the strings adds only 1/5 to the total ball speed. For example, suppose that you somehow increase the ACOR from 0.25 to 0.5. One way to do this is to serve from a point near the throat instead of a point near the tip. But then, the racquet speed V is different, since the tip will travel faster than the throat. But suppose you use a super-dooper ball and a super-dooper racquet with super-dooper strings so that the ACOR is doubled from 0.25 to 0.5 for an impact on a particular spot of the strings. Then V is the same, and v will increase from 1.25V to 1.5V. That's an increase in serve speed of 0.25V, or an increase of 20%.

More realistically, you might get a 10% increase in ACOR if you use a heavy racquet with strings at low tension, in which case ACOR could increase from say 0.40 to 0.44. That gives an increase in serve speed from 1.40V to 1.44V, which is an increase of 0.04V, or an increase of 3%. But if you go for a heavier racquet with a bigger ACOR, then you won't be able to swing it quite as fast, in which case you might actually end up serving the ball a bit slower. For an impact in the center of the strings, the ACOR is typically about 0.4. The ball stiffness is usually about equal to the string plane stiffness, in which case, about half of the elastic energy is taken up by the strings and the other half is taken up by the ball. As the ball comes off the strings, about 95% of the energy stored in the strings is given back to the ball, and about 5% of the energy stored in the strings is lost. We know that from the hammer and the steel ball impact tests.

We also know that 45% of the energy stored in a ball is lost during a collision. If you drop a ball from a height of 100 inches onto a concrete surface, then all of the impact energy is taken up by the ball when it squashes. In that case, the ball stores all the impact energy, but it loses a lot of this energy as it bounces off the concrete. The ball bounces to a height of about 55 inches, which means that it loses 45% of its stored energy. This is specified by Rule 3 of tennis. A ball dropped from a height of 100 inches onto a concrete slab must bounce to a height of between 53 and 58 inches.

Increasing Bounce Speed by Lowering Tension

What happens if we change string tension, keeping racquet mass and speed fixed, while hitting in the same stringbed location? If you decrease the string tension, then the ball will absorb less of the impact energy by squashing a bit less, and the strings will absorb more since they will stretch a bit more. The ball still loses 45% of whatever energy it stores, and the strings still lose 5% of whatever energy they store. Since the strings store more energy than the ball when they are at low tension, and they give more of it back, then the ball will bounce a bit faster or a bit higher.

A simple calculation (which is done in the next section) indicates that if you drop the tension by half from 60 to 30 lb, then the ACOR will increase by 7% from 0.40 to 0.43. That's a handy increase, but no one would want to drop the tension that far. A drop from 60 to 50 lb is more reasonable, in which case the ACOR will increase by 2.3% from 0.40 to 0.41 and the serve speed will increase from 1.40V to 1.41V. That's an increase of only 0.7%, not worth bothering about. Similarly, if the strings lose 10% of their stored energy instead of 5%, then the effect on the serve speed is also insignificant.

Stored Energy vs Lost Energy in Strings and Balls

So why does reducing tension have so little affect on bounce speed and, ultimately, serve speed? The ball and the strings both act like springs, as described in Chapter 13. If you apply a force F to a spring, it will compress by an amount x given by $F = kx$, where k is called the spring constant or the stiffness of the spring. The spring will then store an amount of elastic energy given by $E = kx^2/2$.

The ball and the strings both have a spring constant about 200 lb/in., but of course this can vary from one string or ball to another. Now suppose the ball

has a spring constant k_B and the strings have a spring constant k_S. If they push against each other, then the force on the ball is the same as the force on the strings, so

$$F = k_B x_B = k_S x_S$$

where x_B = compression of the ball and x_S = distance moved by the strings with respect to the frame.

The energy stored in the ball is $E_B = k_B x_B^2/2$ and the energy stored in the strings is $E_S = k_S x_S^2/2$. The total elastic energy stored in both the ball and the strings is

$$E_T = k_B x_B^2 /2 + k_S x_S^2/2,$$

so the fraction of E_T stored in the strings is

$$E_S /E_T = (k_S x_S^2)/(k_B x_B^2 + k_S x_S^2).$$

But $x_B = k_S x_S/k_B$ so (with a little algebra that we will skip here):

$$\frac{E_S}{E_T} = \frac{k_B}{(k_B + k_S)} \qquad (32.3)$$

What If the Ball and Strings Are Equally Stiff?

We will now apply the last formula to a case where the ball and the strings are equally stiff and where the ball is incident with 200 units of kinetic energy (the actual units are not important here). We first consider a case where the racquet head is clamped so that all the kinetic energy of the incident ball is converted to elastic energy in the ball and the strings. None of this energy will be wasted in pushing the racquet head backwards, and none is wasted in causing the racquet frame to vibrate. Similarly, no extra energy is given to the ball by swinging the racquet towards the ball.

For example, if $k_S = k_B$ = 200 lb/in., then E_S/E_T = 200/400 = 0.5, so half the total elastic energy is stored in the ball and half is stored in the strings (see Note 2). Since the total energy available is 200 units, then 100 units are stored in the ball and 100 units are stored in the strings. Since 5% of the energy in the strings is lost and 45% of the energy in the ball is lost, the total energy lost is 5 + 45 = 50 units or 25% of the total elastic energy. The ball will therefore be ejected out of the strings with 200 - 50 = 150 units of kinetic energy. Since it was incident with 200 units of kinetic energy, the ratio of the rebound speed of the ball to its incident speed will be given by e = COR = (square root of 150/200) = 0.866. Note that the COR and the ACOR are equal when the racquet head is clamped, as explained in Chapter 11. If the racquet is hand held, and if the ball impacts in the middle of the strings so that the frame does not vibrate, then the COR is unaffected, but the ACOR is less than the COR.

Effect of Lowering Tension on Stored Energy and Power

Now suppose we reduce the string tension by half, so k_S = 100 lb/in. Then E_S/E_T = 200/300 = 2/3 so 2/3 of the total elastic energy is stored in the strings. Of the initial 200 units of energy, 133 units are stored in the strings and 67 units are stored in the ball. The string loses 0.05 x 133 = 6.6 units and the ball loses 0.45 x 67 = 30.1 units. The total energy lost is 6.6 + 30.1 = 36.7 units = 18.4% of the total. The energy lost is less than before, so the ball will come off the strings a bit faster. In this case, the ball will be ejected with 200 - 36.7 = 163.3 units of energy, so e = COR = (square root of 163.3/200) = 0.904.

How Does This Affect the Serve Speed?

We have just shown that when the string plane stiffness is halved, the COR increases by only 4.3%, from 0.866 to 0.904. This is when the racquet head is clamped. What we are really interested in is the increase in ball speed when the racquet is swung towards the ball. To calculate that, we can use the results found in Chapter 12 to calculate the ACOR, e_A.

For a hand-held racquet, the ACOR is given by

$$e_A = \frac{(eM_e - m)}{(M_e + m)} \qquad (32.4)$$

where M_e is the effective mass of the racquet, m is the mass of the ball and e is the COR. For an impact in the middle of the strings, M_e is about half the actual mass of the racquet. A typical value for M_e is about 170 gm, m = 57 gm, so e_A = 0.40 when e = 0.866, and e_A = 0.43 when e = 0.904. This represents a 7% increase in e_A, even though there is only a 4.3% increase in e. Halving the string tension from say 60 lb to 30 lb therefore increases the serve speed from 1.40V to 1.43V or by 2.1%. Consequently, decreasing the string tension from 60 lb to 50 lb will increase the serve speed by 1/3 of this, or by a miserable 0.7%.

We are assuming here that string plane stiffness is proportional to string tension. This is valid for small deflections of the string plane, but in a very fast serve, the string plane will deflect by an inch or more, and then the string plane stiffness increases considerably during the impact. The increase in stiffness is typically about 30% for natural gut strings, but it can be a factor of three for kevlar strings. The string tension can increase from 60 lb to 180 lb during a fast serve with a kevlar string. The tension drops back to 60 lb when the ball leaves the strings, but during the time the ball is on the strings, the string plane stiffness is up to 3 times larger than the 60 lb stiffness. Furthermore, strings at low tension will deflect more than strings at high tension, with the result that the string plane stiffness at low tension will be almost the same as string plane stiffness at high tension. As a result, the actual increase in serve speed will be less than 0.7% when the tension is reduced from 60 to 50 lb.

The best one could possibly do to increase the serve speed is to change the rules of tennis so that no energy is lost in the ball or the strings at all, and then get the manufacturers to come up with the appropriate lossless balls and strings. Then e = 1, and for the the above racquet, e_A = 0.50. One could serve this magic ball at a speed v = $(1+ e_A)V$ = 1.5V rather than at speed v = 1.4V

using a conventional ball. A 140 mph serve would then become a 150 mph serve. At the opposite extreme, suppose that we use a totally bounceless plasticene ball with e = 0. Then e_A = -0.25 for the above racquet, and the ball would be served at speed v = 0.75V. The ball would stick to the strings when it was served (or perhaps get sliced up into several pieces), and the racquet would slow down from speed V to speed 0.75 when it hit the ball. Even if the ball was ejected out of the strings, it would be impossible to return the ball since it would roll along the court instead of bouncing.

Energy Calculations

We have estimated the effect on serve speed of changing string tension by considering the effect on the ACOR. The same answer is obtained if we examine the way in which energy is shared between the racquet and the ball. When you serve a tennis ball, only about 1/4 of the initial kinetic energy in the racquet is converted to elastic energy in the ball and the strings. This is explained in detail in Chapter 12. If the strings are as stiff as the ball, then half of the elastic energy is stored in the strings and the other half is stored in the ball. Essentially none of the energy in the strings is lost and almost half of the energy in the ball is lost. As a result, about 25% of the stored elastic energy is lost, and hence, 1/4 of that, or 6.25%, of the initial racquet energy is lost.

If the strings are only half as stiff as the ball, then 2/3 of the elastic energy is stored in the strings and 1/3 is stored in the ball. If the ball loses half its stored energy and the strings lose none, then 1/6, or 16.7% ,of the elastic energy is lost, and hence, 1/4 of that, or 4.2%, of the initial racquet energy is lost. Dropping the string tension by half therefore increases the total energy available by only 2%. Some of this is taken up by the ball and some is retained in the racquet.

Suppose that the ball is served at speed v = 1.41V, where V is the speed of the racquet at the impact point (1.41 looks like a strange number to pick, but it is as good as any other number, and it happens to be the square root of 2, which simplifies the following calculation). The ball speed will be 1.41V at a point near the middle of the strings where the effective mass is about 1/2 the actual racquet mass, M. Consequently, the kinetic energy of the racquet before the collision is about $MV^2/4$, and the kinetic energy of the ball after the collision is $\frac{1}{2}\,mv^2 = \frac{1}{2}\,m(1.41V)^2 = mV^2$. The energy given to the ball divided by the initial racquet energy is therefore about 4m/M. This ratio is about 4 x 57/340 = 2/3 for most racquets. We should really consider rotation and translation of the racquet to get an accurate value of the initial energy in the racquet (see Note 3), but a rough estimate is good enough for our purpose here. For a serve, the ball ends up with roughly 2/3 of the initial racquet energy, meaning that about 1/3 of the energy is retained in the racquet or lost. If the racquet came to a complete stop, then all of the racquet energy that is not lost would be given to the ball, but that doesn't happen when a ball is served.

Suppose that at a string tension of 60 lb, the ball ends up with 67% of the initial racquet energy. If we decrease the string tension from 60 lb to 30 lb and all of the 2% extra energy available goes into the ball, then the ball will end up with 69% of the initial racquet energy. That represents a 3% increase in the energy in the ball or a 1.5% increase in ball speed (see Note 4). Consequently, a decrease in string tension from 60 lb to 50 lb will result in an increase in ball speed of less than 1%.

Notes

1. If a 12-inch length of string is stretched until the tension is say 60 lb, it will stretch by about 1 inch, and then it will be about 13 inches long. If the pull force is removed, then the string will shrink back towards its original length, but it will remain slightly stretched, about 1/4 inch longer than its original length. In this respect, strings are not perfectly resiliant, meaning that they do not return to their original shape or length when they are stretched and when the stretching force is removed. However, this has no relevance to the behaviour of a string in a racquet. When a string is installed in a racquet then each 12-inch length will stretch by about one inch, but if the string is tied to the frame, then it will remain at that stretched length for the rest of its life. Every time a ball is hit, the strings stretch a bit further, but they always return to their strung length when the ball flies off the strings. In that respect strings are 100% resiliant. However, the tension drops a tiny amount every time a ball is hit. Strings never lose resiliance in a racquet, but they do lose tension.

2. The stiffness of a tennis ball depends on how fast it is compressed. The rules of tennis (Appendix C) specify that a ball must compress by about 1/4 inch under a load of 18 lb so the ball stiffness is about 18/0.25 = 72 lb/inch when the ball is compressed slowly. In SI units, this corresponds to a compression of 6.25 mm under a load of 8.2 kg (80 N) and a stiffness of 80/0.00625 = 12.8 kN/m. When a ball is tested according to the rules it is compressed much more slowly than during a normal bounce. A ball incident at 13.5 m/s on concrete compresses by about 4 mm when the force on the ball is 240 N and the stiffness is then 240/0.004 = 60 kN/m (337 lb/in). The ball is much stiffer during a fast compression since it takes about 0.2 ms for the wall to buckle. After the wall buckles, the stiffness drops to about 17 kN/m (96 lb/in). Further details are given in Chapter 37.

3. The kinetic energy of a racquet is

$$KE = \tfrac{1}{2}MV_{cm}^{\ 2} + \tfrac{1}{2}I_{cm}\omega^2$$

where M is the actual racquet mass, V_{cm} is the speed of its center of mass, I_{cm} is the moment of inertia about its center of mass, and ω is the angular speed of the racquet. If the racquet is rotating about an axis that is a distance h from its center of mass, then the moment of inertia, I, about that axis is given by the parallel axis theorem

$$I = I_{cm} + Mh^2$$

The velocity of the center of mass is $V_{cm} = h\omega$ so

$$KE = \tfrac{1}{2}Mh^2\omega^2 + \tfrac{1}{2}I_{cm}\omega^2 = \tfrac{1}{2}I\omega^2$$

Alternatively we can write this as

$$KE = \tfrac{1}{2}MV^2 \quad \text{where} \quad V^2 = V_{cm}^{\ 2} + I_{cm}\omega^2/M$$

V is the speed of a point on the racquet a few cm past the balance point of the racquet. For example, suppose that a uniform racquet of length L = 70 cm

is rotating at ω = 80 rad/s about an axis that is 10 cm beyond the butt end of the handle. Since the balance point is in the middle of the racquet (at the center of mass), then h = 45 cm, $V_{cm} = h\omega$ = 36 m/s , $I_{cm} = ML^2/12$ and

$$V^2 = 362 + (0.49/12) \times 802 \quad \text{so} \quad V = 39.5 \text{ m/s}.$$

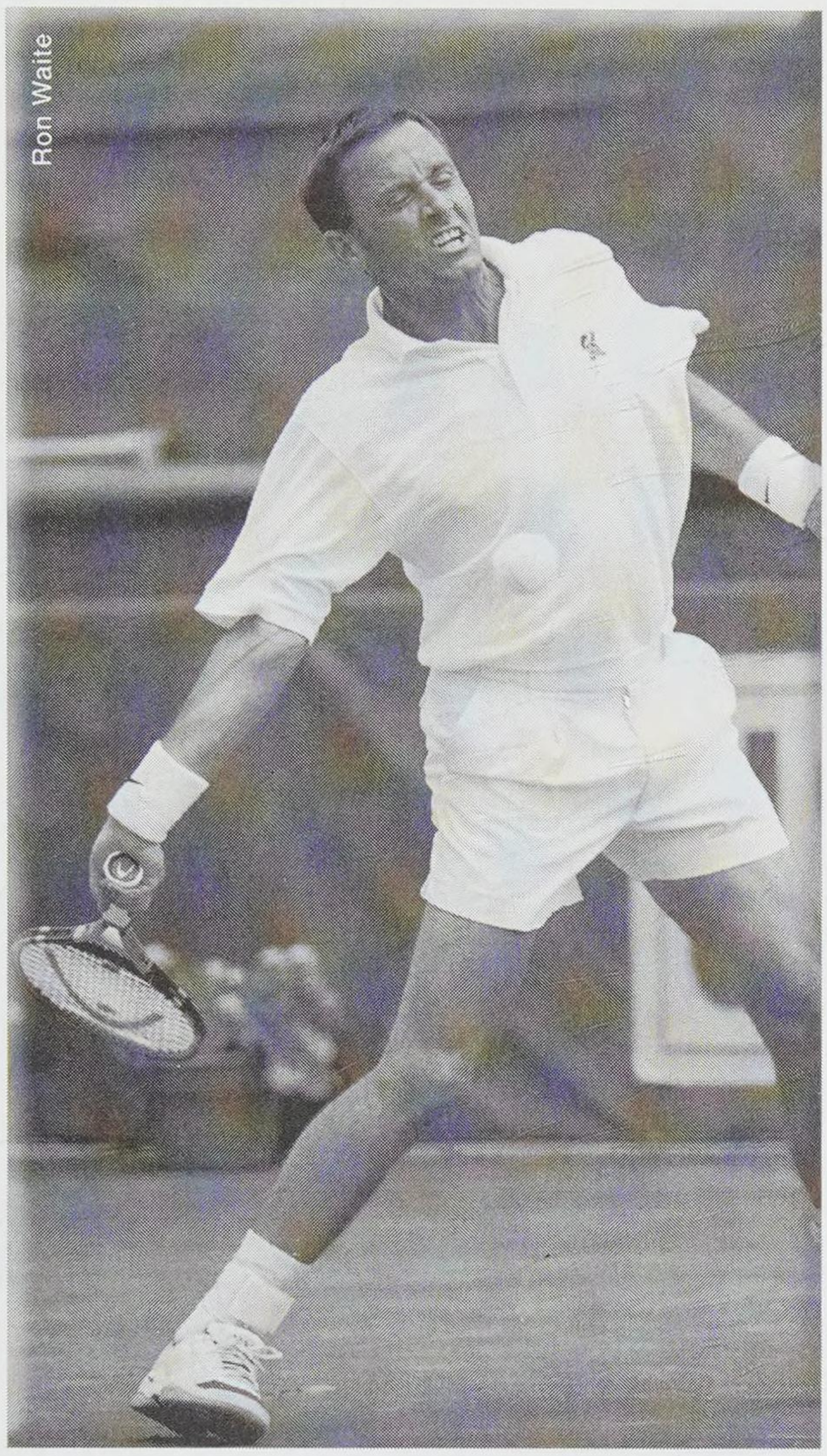
If $V = d\omega$ then d = 0.493 m, meaning that V is the speed of a point 4.3 cm past the balance point (towards the middle of the strings).

The kinetic energy of a racquet can also be expressed as $\frac{1}{2}M_eV^2$, where V is the speed of the impact point, and M_e is the equivalent mass at the impact point. This relation is exact during and immediately after a racquet and ball collision, but not before the collision. Prior to a collision with the ball, the racquet can be swung about any axis that the player chooses. It turns out that in this situation, $\frac{1}{2}M_eV^2$ is typically about 10% less than the actual kinetic energy of the racquet.

4. There are many mathematical tricks that physicists use to simplify calculations so that the calculations can be done in their head rather than on a calculator. Students these days are trained to use calculators rather than having to rely on "mental" arithmetic, so these tricks are not as well known as they should be. Most of the tricks give an answer that is close enough for practical purposes, but they are not exactly correct. For example, if I buy 20 items from the supermarket, then I expect to pay around $80 since the average price of each item is typically about $4. Similarly π^2 is 9.87, but that is close enough to 10 if I want to figure out the answer to a calculation in my head. If I want the square of 6.5, then that's about 6 x 7 = 42, but I know I should add 0.25 to the answer, so the answer is actually 42.25. I can add up all the numbers from 1 to 100 in my head since 1 + 99 = 100, 2 + 98 = 100, 3 + 97 = 100. There are about 50 such pairs of 100 so the answer is about 5000.

If I increase x by 10%, then x^2 will increase by about 20%. For example, if x = 10 then an increase of 10% gives x = 11, and x^2 increases from 100 to 121. That's an increase of 21%, but it's close enough to 20% not to worry about. If the kinetic energy of a tennis ball increases by 3% then the increase in speed is actually 1.489%, so my estimate of 1.5% was pretty close. The general rule is that if $y = x^n$, then an increase in x of p% will give an increase in y of about np%. The answer improves as p gets smaller. Mathematically, $dy/dx = nx^{(n-1)} = ny/x$ so $dy/y = n(dx/x)$, where dy is a small increase in y and dx is a small increase in x.

One of my favorite tricks (admittedly a bit esoteric) is to get the quality factor, Q, of a damped vibration such as the damped sine wave in Figure 7.2. The quality factor of a vibrating system is formally defined as Q = 2π x (energy stored)/(energy lost each cycle). A high Q oscillation is one that damps out very slowly since very little energy is lost during each cycle. A low Q oscillation is one that damps quickly. I can tell by looking at Figure 7.2 that the Q is about 15 since that's the number of cycles I can count. It can be shown mathematically that the amplitude of an oscillation decreases to 1/27 th of its original amplitude after Q cycles.



Mikael Pernfors, Sweden

33

String Tension

Factors in Tension and Tension Loss

By Crawford Lindsey

Once upon a time, string tensioning was done by hand. The string was twisted around a wooden dowel to apply tension. The stringer plucked the string, listening for just the right "reference" tone, and then "clamped off" with an awl.

The crosses were done differently. Since weaving damps tonal pitch, stringers would try to match the humps in the weave instead. This didn't necessarily match the mains, but an experienced stringer could judge what tension (hump) he was pulling compared to the mains and how it would affect the overall stringbed stiffness.

In each case, stringers were trying to achieve the same string stiffness on each string — not the same tension. The mains were tonally set such that as you go out from the center mains, the pitch gets lower. Shorter strings are stiffer at the same tension than longer strings, so stringers would pull each shorter main with less tension, thus giving it a lower pitch but close to the same stiffness as the string on each side of it. This technique was based on the principle that a string's frequency varies depending on the length, gauge, and tension of the string.

Matching humps, though not as scientific, did the same thing. Hump size varied in stiffness depending on the pattern density and the tension at each point. Experience dictated the "reference hump." This was the art of the stringbed. Even though the technique seems prehistoric compared to our age of continuous pull digital readout machines, there was a lot of science there also.

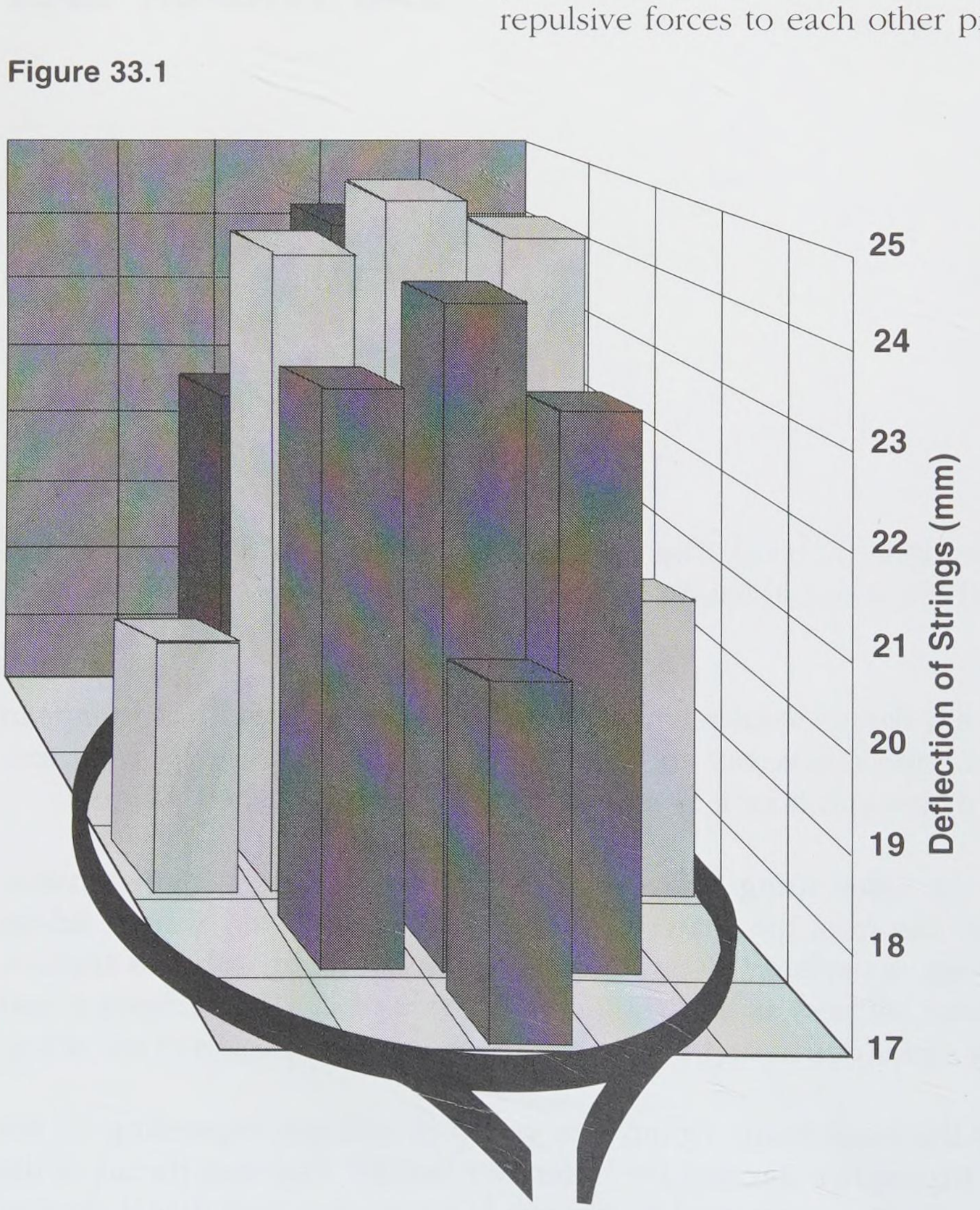
We have come a long way since those days, however, the key both then and now is stringbed stiffness, not string tension. Tension is just a means of achieving different degrees of stiffness at different locations. It is not the end in itself.

What Is Tension?

Newton's Third Law of Motion says that every action must have an equal and opposite reaction. Newton wasn't a stringer, but he knew that if the reference stringing tension were 50 pounds, but the string pulled back at more than 50 pounds, then the stringing machine or its operator would go flying out the window. It's the same with ball impact. The string deflects just enough to exactly match the combined energy of the incoming ball and the swing of the racquet. Your racquet is an oddly shaped atomic computer that instantaneously figures out how much to deflect to bring the ball to a complete stop and then turn it around. This is really kind of magical. What is happening?

It's all about tension. My computer does not fall through my desk because the force of its weight is exactly matched by the desk. The desk does this by becoming tensioned (actually, compressed in this case). It deforms just like strings do. Deformation is necessary to create tension or compression. Deformation is all about pushing and pulling on atoms. All objects are made of atoms in relative equilibrium, and atoms are made of a balance of positively and negatively charged particles. When atoms are pushed closer together, past their equilibrium positions (compression), same charge particles build up repulsive forces to each other preventing them from being pushed any closer by an external force of that magnitude. If atoms are pulled apart, the attractive forces of opposite charges try to pull them back into equilibrium. The further you pull them apart, the greater the force required to do so (up to breaking tensile strength). This is what we feel as tension.

Figure 33.1



But not all tensions are created equal. If you pull every string in a racquet at the same tension, which is typically the case, then every string will deflect a different amount when hit by a tennis ball. Figure 33.1 shows a graphic of a representative string plane deflection at different impact points.

The variances from point to point are a result of pulling different length strings at the same tension. So, when you ask a stringer to string a racquet at 50 pounds, you are really saying, "I want every point on my racquet to play differently from every other point by the amount caused by a 50 pound tension." Well, that's not what you mean at all, though that is what you're getting. Most players probably think that 50 pounds will make their racquets play the same everywhere. In general, and all things

being equal except string length, identical tensions will mean different deflections and stiffnesses. Strings with the same stiffness will have equal deflections but different tensions. The stiffness resulting from any given reference tension is totally relative to the length, gauge, materials, construction, and properties of a string. If you hang a weight from two unequal length strings at the same tension, the longer string deflects more. For this reason, some players have their racquets strung by the "proportional stringing method." With this method, the string tension is lowered on each string in direct proportion to its percentage length of the longest main strings.

Oh, if this were the only complication in determining how tension will affect the feel of the stringbed. The rest of this chapter details the real bugaboo — string tension is always changing.

String Tension Is a Moving Target

As soon as you pull and clamp the string, it is a different tension. Ten seconds later, it's different again. Two weeks later, different still. Every day your racquet is at a different (lower) tension than the day before. The reference tension is the setting on the stringing machine for the brief instant of the pull on a spring tension machine or until it is clamped on a continuous pull machine. So, if the tension is never the same anyway, what's the big deal about tension? Players and stringers obsess about a pound here and a pound there. Does any of this make a difference if it is just going to go away?

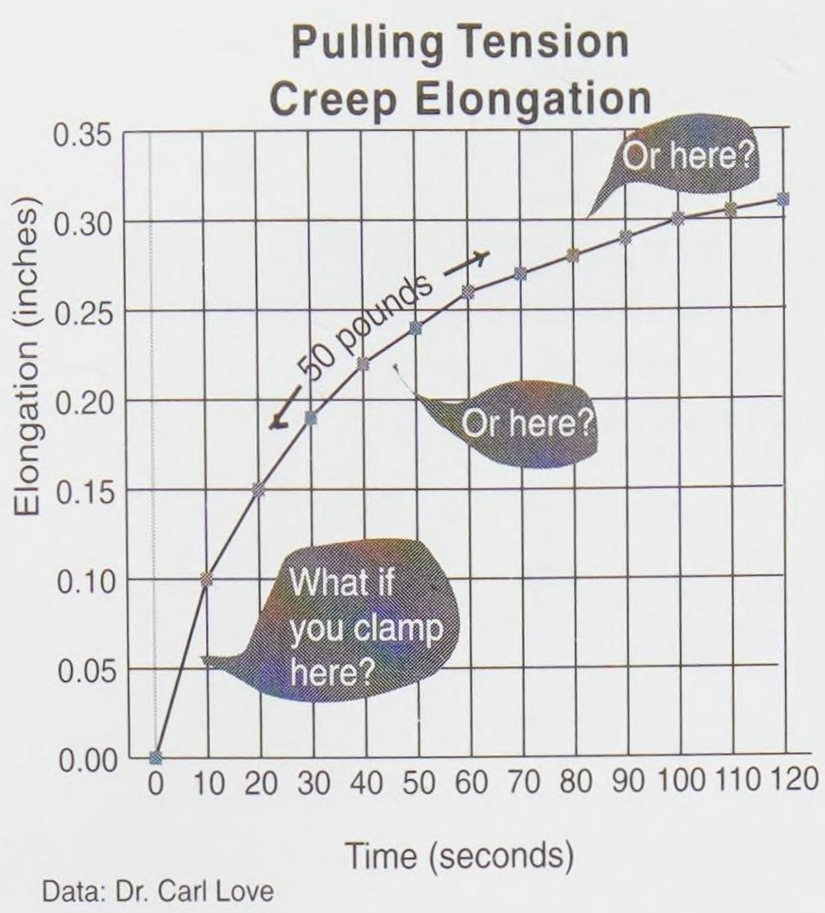
Where Does It Go and by How Much?

String loses tension by a process called "creep." It works in two ways — creep elongation and creep tension relaxation. A string pulled at a constant force will elongate forever. It does so at a decreasing rate as time goes by, however. Tension a string in a continuous pull machine and come back a few years later, and the tension head will be at the end of the track. On the other hand, a string held at a constant length (as in a racquet) can't get longer, so, instead, it loses tension at a continuously decreasing rate. It's easy to picture elongation during pulling, but tension relaxation is more difficult because you haven't actually let go of the string. Where does this tension go and why? After all, we pulled it to 50, tied it off at 50, and now the racquet is holding it there at 50. Right? Wrong. We will see that a string loses tension for the same reason it continues to elongate. It all has to do with the alignment and stretching of molecules. But they don't all stretch the same amount at the same rate. This results in a tension equalization between them as time goes by.

How much tension are we talking about? Figure 33.2 shows the two faces of creep.

Why Does It Go?

As you can see, most of the creep happens in the first 20 to 30 seconds, whether through elongation or tension loss. This happens because string has a split personality. It is viscoelastic which means it has both viscous (which takes time) and elastic (which happens instantly) behavior. If you were to stand on top of a vat of honey, the force of your weight would cause you to slowly sink. If instead you stood on top of a big pogo stick, you would immediately depress the spring to equal your weight. In both cases, the weight is the same, but in



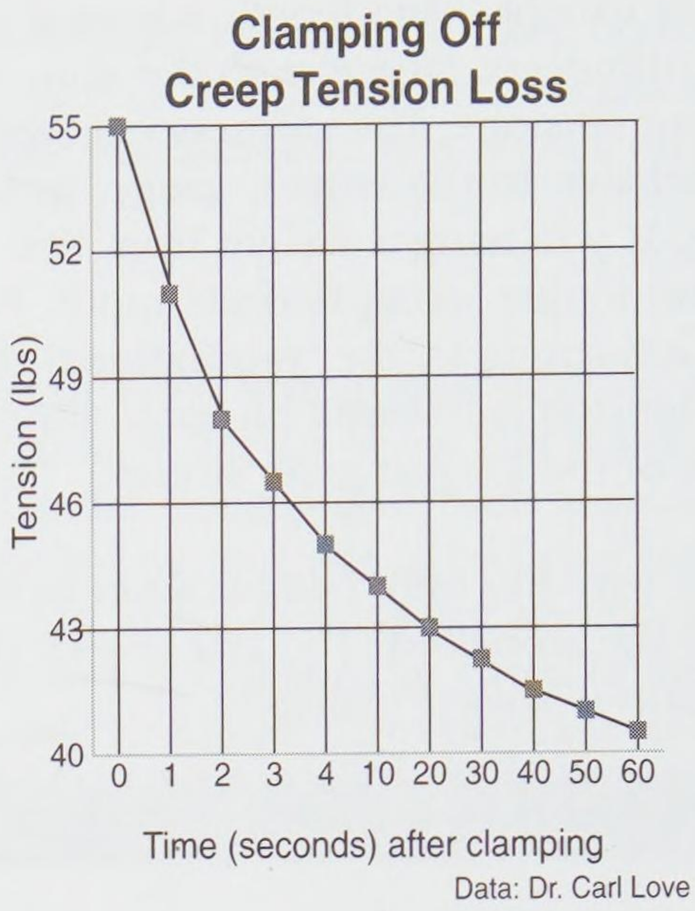
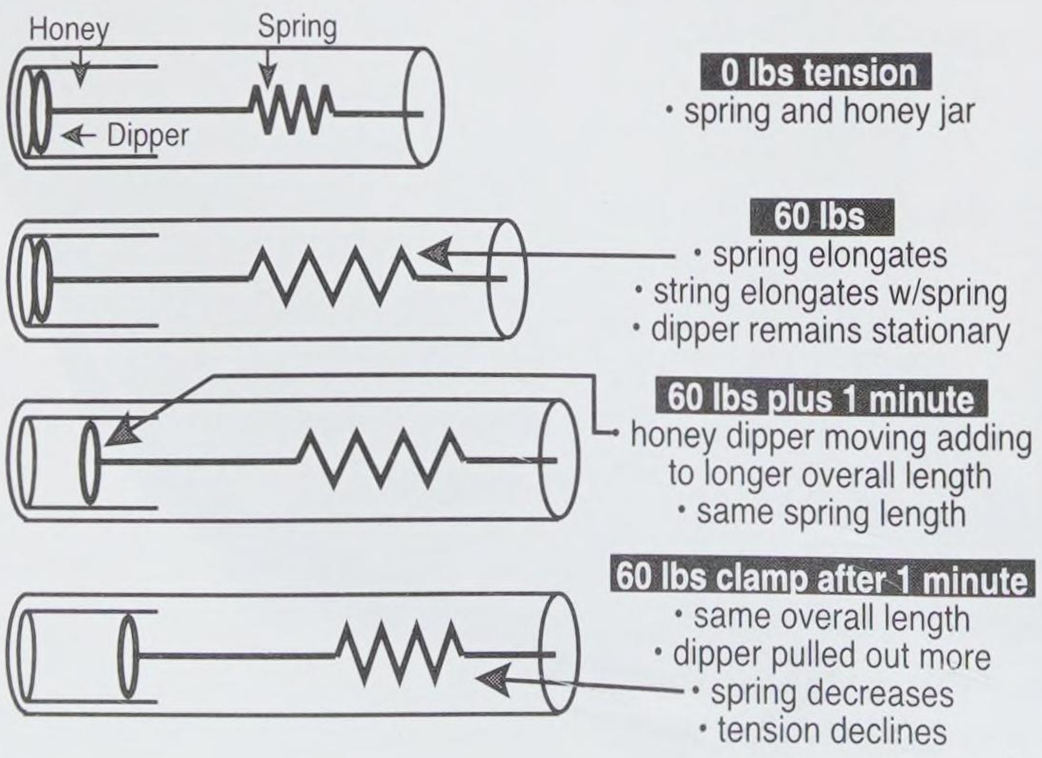


Figure 33.2

Typical behavior of a nylon string when clamping immediately after pulling tension. The majority of the tension loss occurs in the first 10 to 30 seconds (as with elongation in graph at left). String is always moving — it elongates forever as long as you are pulling on it. The second you clamp it off and constrain its length, it begins losing tension. It is never both a constant length and tension at the same time. This is the stringer's uncertainty principle: if you know either the tension or the length, you can't exactly know the other. One or the other is always changing.

If you leave a string at 50 pounds in the tension head of a continuous pull machine, the string will continue to elongate. The string is at 50 pounds no matter when you clamp off, but the internal configuration of the string depends on when you do it. It's as if early on the string is just faking it, never really getting to 50. As you keep pulling, it gives up the fight, stabilizes, and says, "Ok, you win, I will stay here if you just stop pulling on me." On the steep part of the curve, all the polymer molecule chains (the molecular-sized parts) haven't had time to share the load equally, so when you clamp off, there will be a substantial tension equalization between molecule chains. The overall tension will then go down. See right.

Figure 33.3



one, the result of the force is time dependent, and in the other, it is instantaneous. String is a combination of the two.

To visualize how this split personality contributes to elongation and tension loss in a string, let's take our honey and spring analogy further. String elongates as if it were composed of a coiled steel spring attached to a dipper in a jar of honey (Figure 33.3). When the string is tensioned, the spring immediately elongates but the dipper stays put. As the continuous pull machine keeps pulling, the spring remains stretched at the same length, but the dipper starts to move in the honey. As it does so, the string continues to elongate. Now, if we clamp off, the time dependent pulling on the dipper continues because it is still being pulled by the force of the spring. But since the length of the string is now fixed, the spring must decrease its elongation to make up for the movement

("elongation") of the dipper. And the tension necessary to maintain the spring's new length is less, so its tension goes down. This in turn results in less pull on the honey dipper, so it pulls out less and the tension in turn declines, but less than before. This creep circle continues indefinitely. This is a simplified analogy of the actual process, but it allows us to visualize how this creepy behavior is possible.

The real life counterpart is this: the force of tensioning or impact is immediately reacted to by the atomic pogo sticks (the bonds between atoms) in the straight sections of the polymer chains, but it takes time for the other less oriented chain molecules to unfold, straighten, untangle, and move relative to each other. As a result, some molecule chains are bearing more of the load than others when the string is first pulled. The faster and harder it is pulled, the more of the load these chains will bear in proportion to the rest. When the string is clamped, time takes over and the tensions on the various chains begin to equalize. As they do, the overall tension goes down, though the length stays the same, as it must. It's the same as happens in a racquet when strings are pulled at different tensions. As soon as you release the clamp between the two strings, they equalize their tension, yet keep their length.

Tension Loss While Stringing

What does this mean to a stringer, if anything? Figure 33.4 demonstrates what happens to a 70-pound string throughout the stringing process. With a force

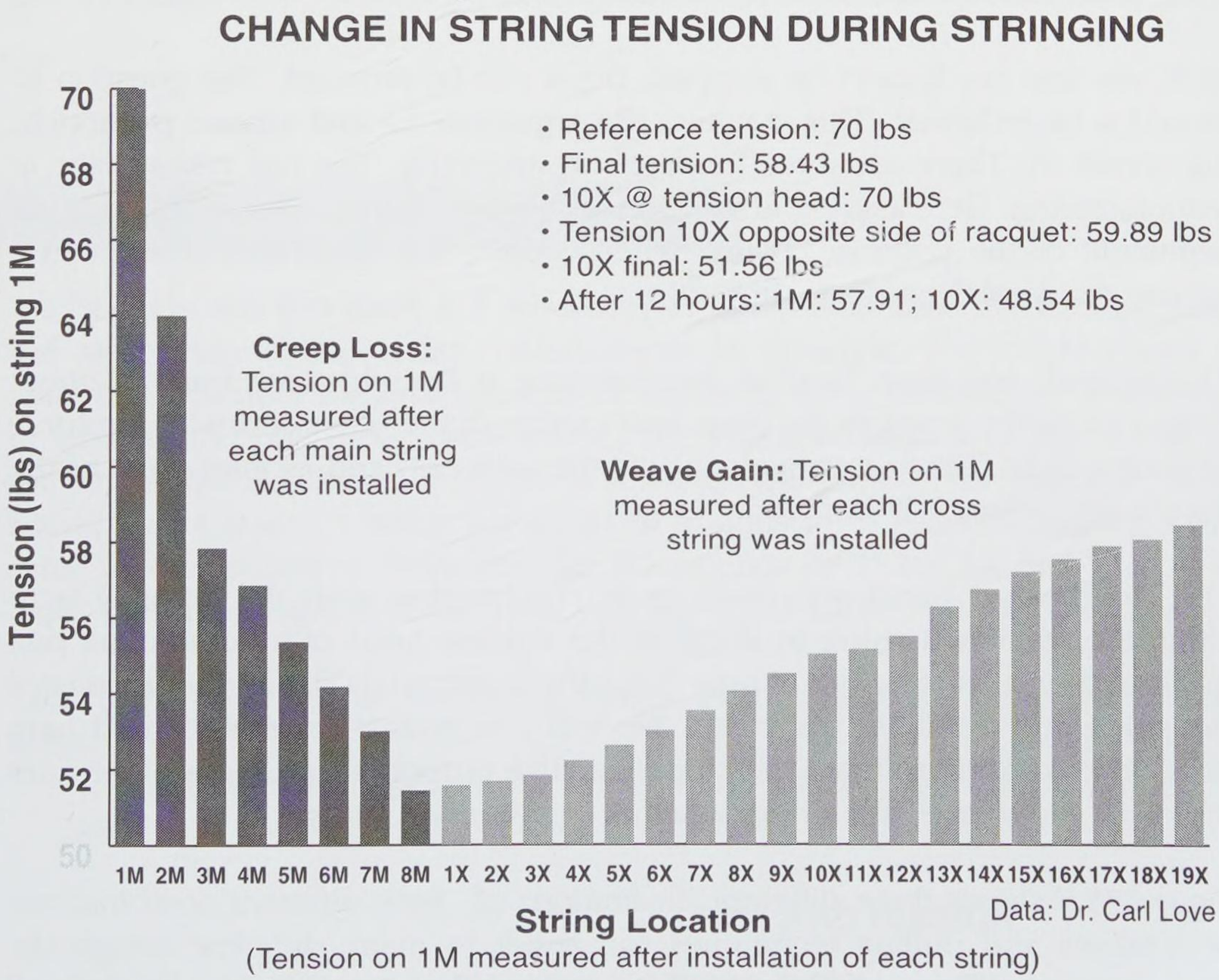


Figure 33.4

String 1RM (first right main) was monitored throughout the stringing process. The tensions above are the tension on string 1RM immediately after each string in the racquet was installed. While the mains are being installed, tension goes way down due to creep. During cross string installation, tension goes back up because each weave elongates the main a little and frame deformation occurs pulling in the sides, thus increasing the tension 0.2–0.5 pounds per string. Creep is still occurring at this point, but it has stabilized at a much slower rate.

gauge placed on the first main and the center cross, tension was measured throughout the stringing procedure.

As you can see, during stringing, the main string tension drops way below reference tension (the same is happening for every other string also). But then the weaving of the cross strings brings it back up. Each hump in the weave elongates the main string a little, thus increasing its tension and the frame distorts, pulling the sides in and lengthening the head from tip to throat.

And, of course, the same applies to the cross strings, but with a catch. Not only does the same tension loss occur after clamping, but it does so by a different amount on different parts of the string. That's because, though the string is 70 pounds at the tension head, it is only a percentage of that as you travel across the racquet. The furthest part of the string from the tension head is not being pulled as tightly because the tension is gobbled up by friction in the weaves. The difference can be as much as 15 pounds depending on the texture of the string and the reference tension. When you clamp off, the end with the higher tension will creep more and faster than the end at lower tension. And the cross string going back in the opposite direction will do the same in the opposite order. So it cancels itself out. But again, 70 pounds isn't always 70 pounds.

Tension Loss and Prestretching

Every string has a different structure, material, and degree of polymer chain alignment, so each string will creep differently than the next. So, is there anything you can do about it? Are you destined to be a victim of creep?

Well, yes and no. It can't be stopped, but it can be reduced. The question is, should it be reduced? This is where the argument for and against prestretching comes in. There are three kinds of prestretching. The first takes place in manufacturing. Here a string is stretched at high temperature to get the desired alignment of the polymer chains. When cooled, this alignment is set, determining many of the string's properties.

The second, and most familiar, prestretching is the old door knob method. Here you tie the string to the door knob and pull the other end with the force of your weight. This is to straighten (get the coils out) and/or take some of the stretch out before it is installed.

The last method (used separately or in combination with the second) is to allow the tensioned string to dwell in the tension head of a continuous pull machine for an extra length of time (e.g., 10-30 seconds). This further elongates the string and stabilizes the creep. We will concentrate on this method here because it is easiest to work with for illustrative purposes, and it actually comes into play every time you string, whether you know it or not.

Figure 33.5 shows three different illustrations of how different combinations of tensions and pulling techniques can result in many different stringbeds. Two racquets strung at 60 pounds with and without extra tension head dwell time or prestretching will be very different stringbeds. The prestretched racquet will be much stiffer. But if you're not looking for a stiffer feel, that's not good. To compensate, you could still prestretch, but string at a lower tension to get a softer stringbed. But then you ask, why bother with the prestretch if I could just end up at the same final tension by using a higher tension to begin with and without all the extra effort?

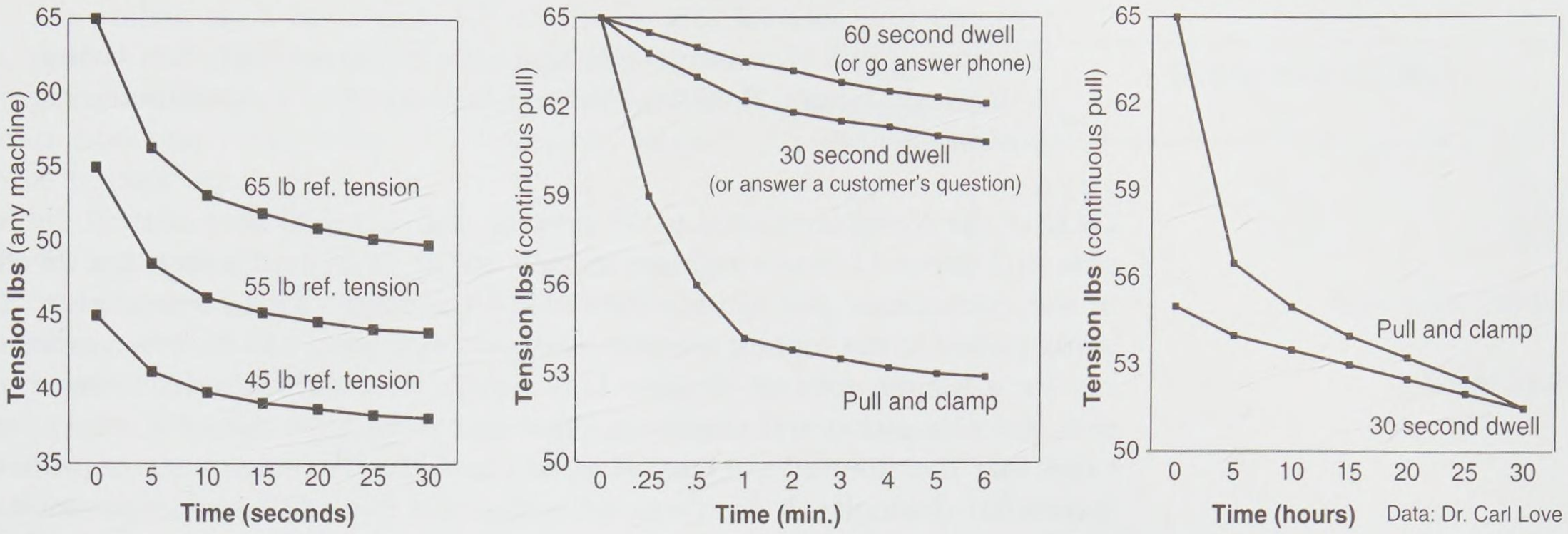


Figure 33.5

Higher reference tensions lose more tension faster during stringing. The 65 lb string lost 23.4%, the 55 lb string lost 20.2%, and the 45 lb string lost 15.3% tension in 30 seconds. The difference between final and reference tension thus depends on reference tension. Each string started 10 pounds apart and ended up only 5 or 6 pounds apart.

Three stringers starting at the same reference tension does not mean that they will all end at the same final tension. Stringing technique, specifically, tension head dwell time or prestretching, can make a big difference. These are completely different stringbeds. The more you elongate, the less tension you lose and the more stable the tension over time, but it's also much stiffer.

Two stringers starting at different reference tensions but using different methods end up at the same final tension. (Actually, the graphs just intersect and are still losing tension at different rates.) The question is, are these similar stringbeds when they intersect? Even though you end up in the same place, does it matter how you got there? See text for explanation.

Time: Every Second Counts

We begin to see that there is a whole series of combinations that end up with the same internal molecular configuration. In principle, you could create a chart that describes the final tensions arrived at by using different combinations of tension and pull time. There should be several combinations that arrive at the same equilibrium tension after a suitable time period. For example, after sitting for 24 hours, a string pulled to 70 pounds for 3 seconds might be the same as one pulled to 50 pounds for 60 seconds or 55 for 30, or 60 for 7. If that's true, then does it matter how you get there? Table 33.1 explores this.

The figure shows some surprising results. First, let's make some generalizations.

1. The longer you pull on the string at any tension, the more it elongates and the smaller the diameter becomes.

Table 33.1

TIME AND TENSION

Tension	Pull Time	Diameter	Stress @ Pull	Elongation %	12 Hr-Tension	12 Hr-Stress
70 lbs	5 s	-4.57%	35,508 lbs/in^2	13.64%	48.10 lbs	24,399 lbs/in^2
65 lbs	60 s	-6.90%	35,039 lbs/in^2	14.87%	58.20 lbs	31,373 lbs/in^2
60 lbs	60 s	-7.12%	32,747 lbs/in^2	13.78%	55.98 lbs	30,553 lbs/in^2
55 lbs	60 s	-5.84%	29,894 lbs/in^2	13.71%	51.57 lbs	28,030 lbs/in^2
50 lbs	60 s	-5.62%	26,842 lbs/in^2	12.09%	46.65 lbs	25,044 lbs/in^2

Data: Dr. Carl Love

2. A string at a lower tension pulled for a longer time may elongate more and become thinner than a string pulled at a higher tension for a shorter time interval.
3. If time of pull is constant (e.g., 60 seconds in this case), then change in diameter, length, final tension, and final stress declines as reference tension declines.

Look at the string tensioned to 70 pounds and clamped immediately. Its tension and stress 12 hours later are closest to the 50 pound string that dwelled in the continuous pull tension head for 60 seconds. That's remarkable. The string pulled to the highest tension equals the one pulled to the lowest tension, but for a longer amount of time. Yet, though close to the same 12-hour tension, the elongation and change in diameters were quite different, which indicates that the internal configuration of the molecular alignment is probably somewhat dissimilar. If so, they still will play a little differently, but not as if they were strung 20 pounds apart.

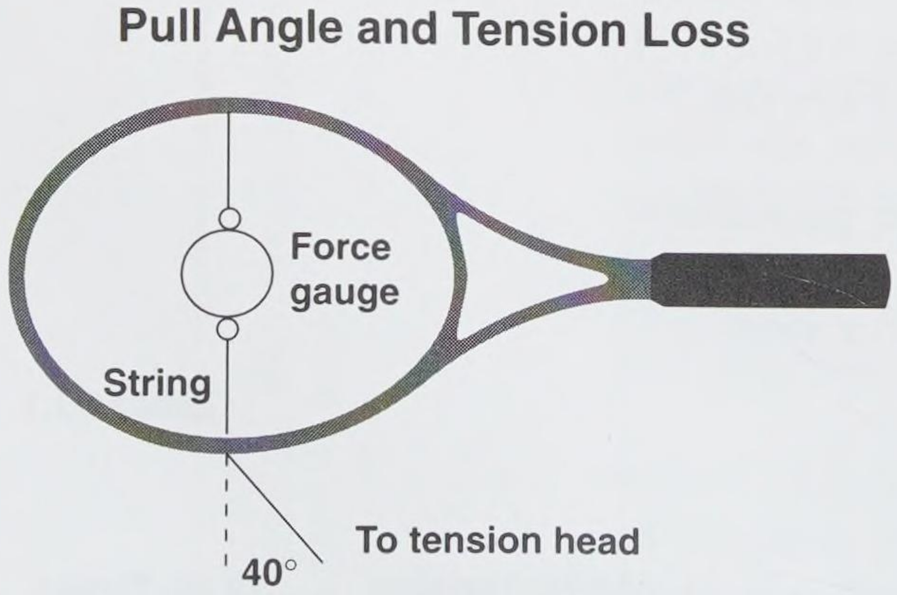
Consistency and Attention to Detail

So, can you get to the same place starting at different tensions? Maybe. The key is understanding where "there" is. If a player likes the tension and string you used last time, that's easy. The key, however, is in reproducing your method. As the graphs show, the difference between clamping off 4, 7 or 10 seconds on different strings will result in a big difference on where those strings settle out compared to each other. The key is consistency. Consistency from string to string and from string job to string job for each customer. You want to clamp off at the same point on the elongation curve every time.

On a spring tension machine, this happens automatically because when the tension head locks out, the string stops elongating and tension relaxation sets in immediately. So clamp time is not as important on this type of machine. But even with this machine, if you prestretched the string prior to stringing, then time once again enters the equation, but in a different way. How long did you prestretch, at what tension (hard to determine on a door knob), and how long has it been since you prestretched? The last is the most important. Over time, a good percentage of the creep elongation and molecular alignment caused by prestretching is lost. So, if the prestretch duration, tension, and time since the prestretch are not all exactly the same each time, you put the string in the racquet (for the same string model), then you are installing a different string each time. Each string job will then have a different stiffness at the same tension.

Figure 33.6

Pull Angle and Tension Loss



When pulling at an angle, not all of the force is concentrated along the line of the string. Some of the force is lost in pulling sideways. Tension is also lost in the added friction caused by the angle.

Source: Dr. Carl Love

Pull Angle and Tension

Another pesky little variable contributes to final tension not being the same as reference tension — the angle of pull. This refers to the angle the tension head makes with the pulled string. Depending on which string you are pulling, as the string comes out of the grommet, it bends at a certain angle as it enters the tension head (Figure 33.6). Pulling at this angle will result in less tension than if the tension head were in a direct line with the string as it emerges from the grommet. Table 33.2 shows the result-

ing tension as measured at two different angles with three different tensions. In point of fact, then, you are pulling each string at a different tension if it is pulled at a different angle. And because each string is at a different tension, when you release the clamp between them, they will equalize somewhere in between.

This is not bad either. This is the way stringing machines are designed (except True Tension machines). Still, some stringers have been known to compensate by locking the tray table in a straight line position for each string pull. This will make the resulting stringbed much stiffer. But you don't have to do this because matching reference tension in the final product is not your goal. It is to duplicate the stringbed stiffness that the player likes.

Tension Head Location and Final Tension

When you pull tension, the length of string between the grommet and the tension head is essentially being prestretched. If the tension head is in a different position on the track each time you pull, then some lengths might be pretensioned and others not. This might not seem important if you pull, clamp, and release very quickly, but in fact, it is. It amounts to a double pull which can be shown to increase stringbed stiffness significantly. As long as you pull from the same spot on the track, you will pretension the same amount of string and adjacent strings will have the same degree of stiffness. If you don't, the further back on the track you pull, the more you prestretch and the tighter will be your stringbed.

Temperature and Tension

What if you string one racquet in your garage in the morning at 45 degrees ambient temperature and one in the afternoon at 105 degrees? If you string them both at 70 pounds, will they result in the same stringbed? The graph in Figure 33.7 shows what happens. At higher temperatures, molecules move more easily relative to each other, so the creep curve is accelerated. At lower temperatures, it slows down.

Conclusion

Tension loss while stringing occurs all over the place. It's like a big leak in the dike. But you don't necessarily have to stick your finger in it to plug it

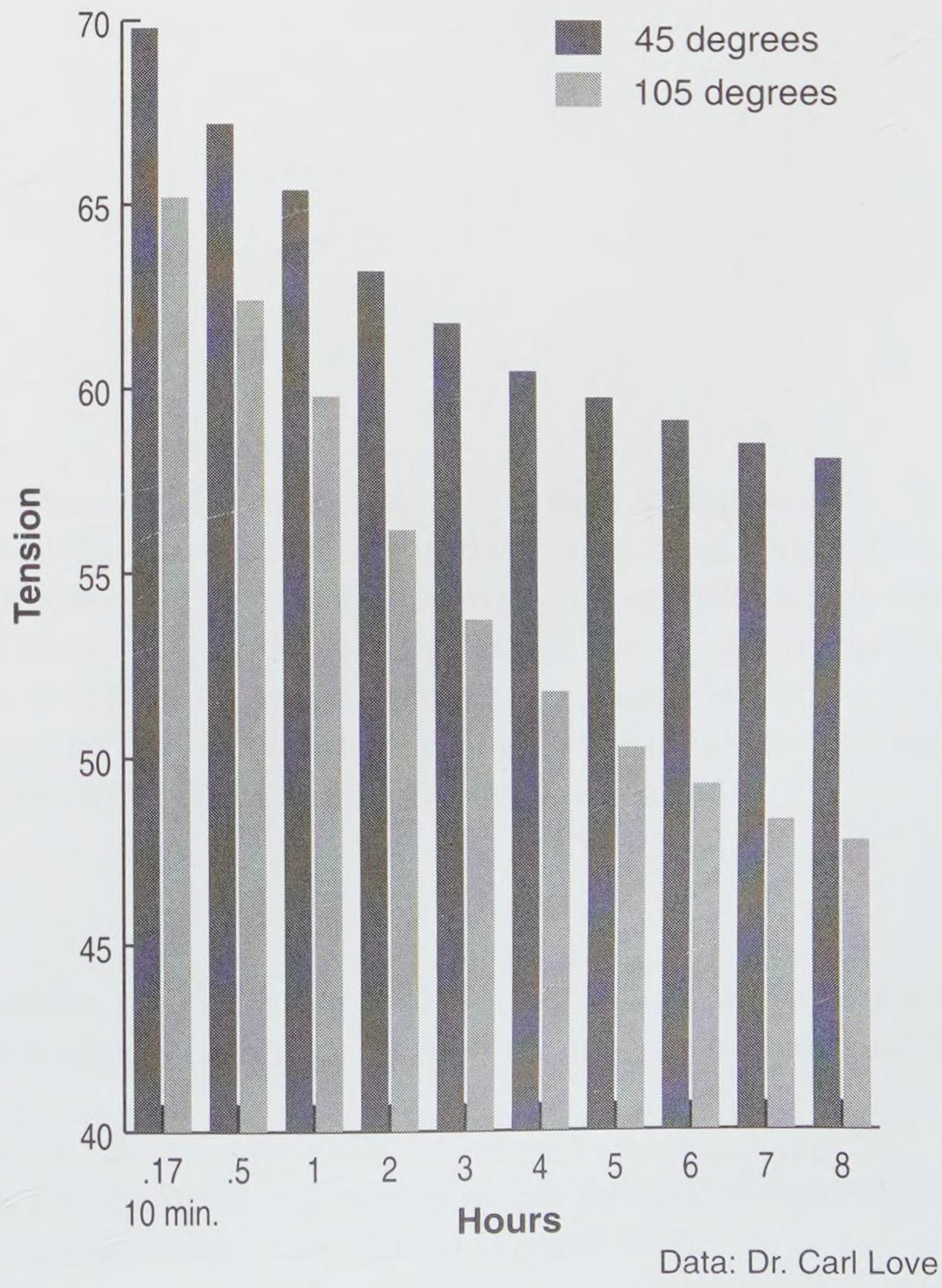
Pull Angle and Tension Loss

Pull Angle	String Tension (lbs) 60	50	42
20°	53.7	46.1	37.0
40°	50.5	44.9	34.8

Source: Dr. Carl Love

Table 33.2

Temperature and Tension Loss



Data: Dr. Carl Love

Figure 33.7

up. Tension loss is not a bad thing (it's natural and unavoidable) as long as you understand it and know how your actions with respect to it can influence the final stringbed stiffness. The goal of stringing is not to be at the reference tension when you finish the job. That would be an unbearably stiff stringbed. The goal is to be at the desired stringbed stiffness produced by setting reference tension at a certain value for a certain string. Stiffness is not tension. Stiffness is combination of string type plus the tension and the time at tension before clamping or the pretstretching procedure used, if any. On a continuous pull machine, time at tension means more elongation and molecular alignment and results in a stiffer stringbed. Same tension, different feel. On a spring tension machine, the issue only arises if you prestretched ahead of time or double pulled. Either way, time is the key element. Any procedure related to elongating a string must follow an exact procedure with each task exactly timed to reproduce the same results, string job after string job. And the same applies to the pull angle, tension head location when pulling, and room temperature. Fancy techniques don't mean anything unless everything else is also kept consistent.

Lindsay Davenport, United States

34

Measuring String Tension

How To Measure It and What It Means

By Rod Cross

It is easy to string a racquet at, say, 60 pounds or 65 pounds and to know that it is strung at that tension, but how do you check the tension after the racquet is strung, say one week later or even one minute later? There are several ways, as experienced stringers will already know. The most common way is to hit the strings with one hand or with another racquet and listen to the sound they make. A high pitch "ping" means the strings are tight and a low pitch means the strings are loose. A better way is to use a racquet diagnostic machine, or a Stringmeter, or maybe a fancy electronic gadget such as the ERT700. But how do you know if these devices give the correct answer? The problem is how can they be tested? What else can they be tested against?

Racquet Diagnostic Machines

When you string a racquet, one end of the string is tied down and the other end can be connected to a spring or a strain gauge to measure the tension. But when both ends of the string are tied to the frame, this method of measuring the tension is no longer available.

The next best method is to push the string sideways with a known force to see how far the stringbed deflects. That is the method used in Pacific and Babolat racquet diagnostic machines (RDC) (Figure 34.1).

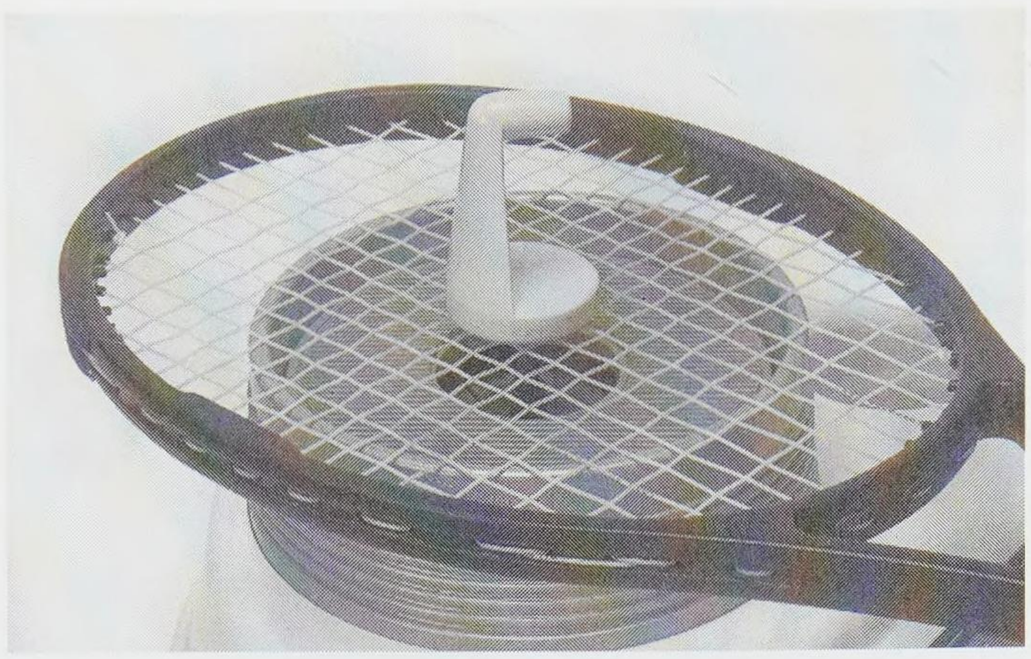


Figure 34.1

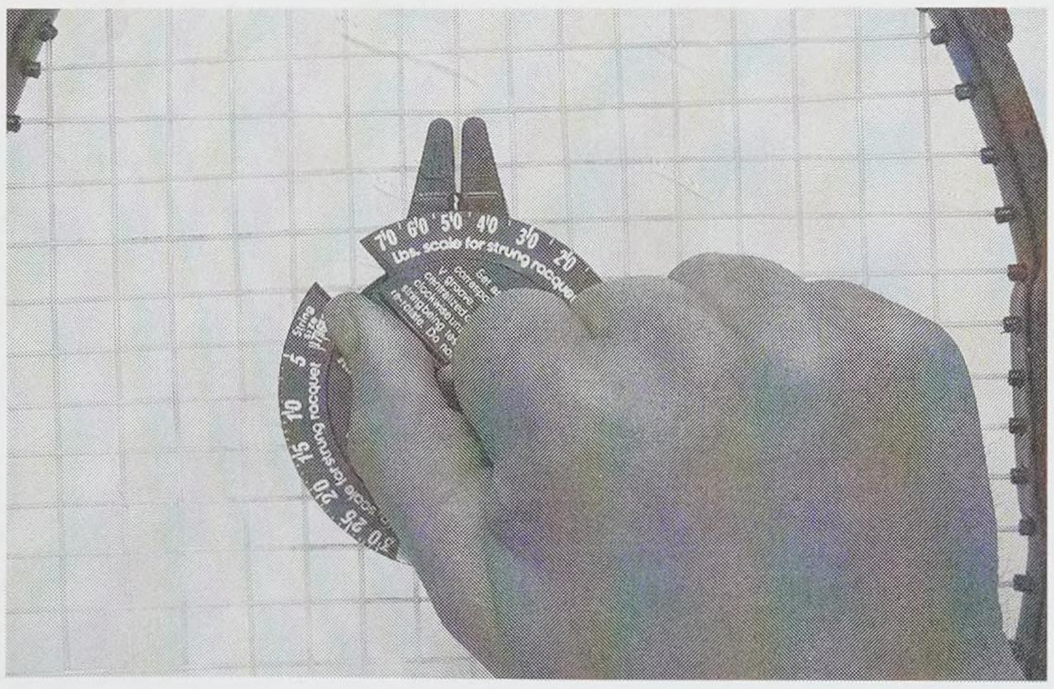
RDC stringbed stiffness measuremen.t

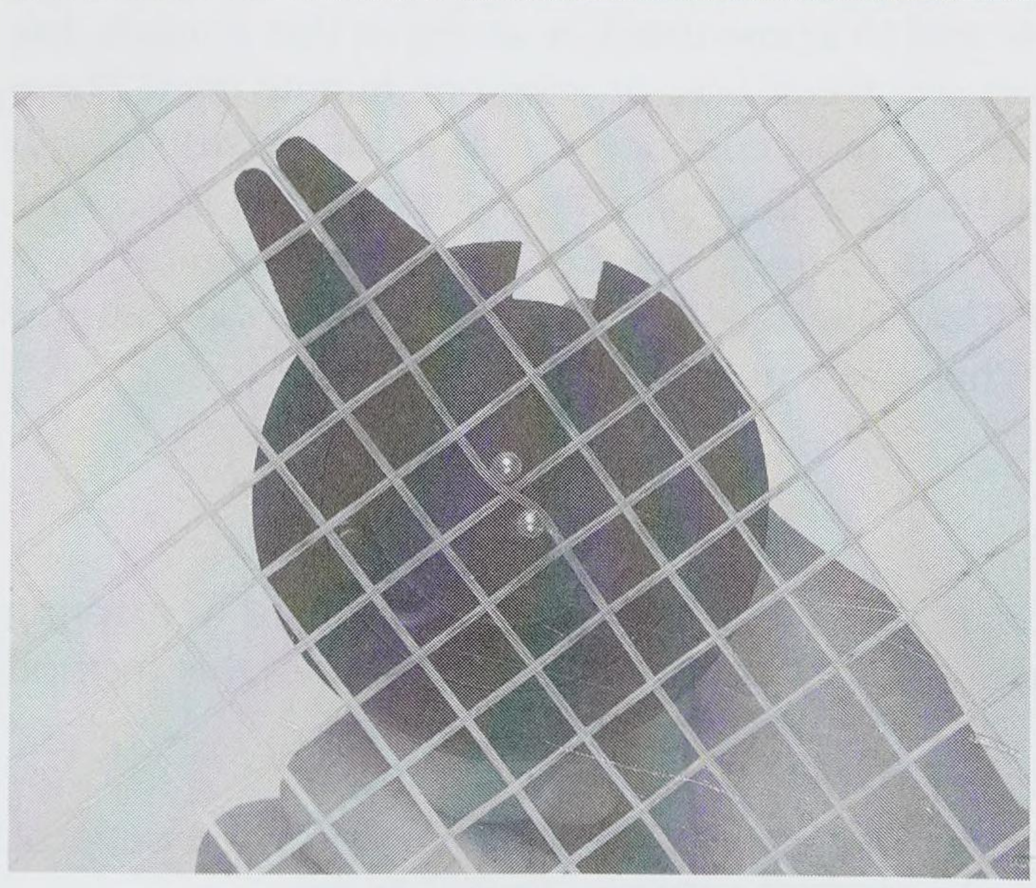
The answer depends on the string tension as well as several other factors such as the length of each string, the number of strings, the stiffness of each string, and the diameter of the object (usually cylindrical) that is used to push the strings sideways. For that reason, a racquet diagnostic machine can't give a reliable measure of the string tension, but it does give a reliable measure of the stringbed stiffness. In fact, the stringbed stiffness is more important than the tension itself, since the stringbed stiffness determines how far the strings will deflect, and how much the ball will squash when you hit a tennis ball. Even so, stringbed stiffness is usually quoted in DA or RA units ("made up" units indexed against the machine's zero point), which is just another way of admitting that the actual stringbed stiffness is not known. (Another problem, of course, is that the actual stringbed stiffness depends on how hard you hit the ball. The stringbed gets stiffer the more it stretches, but that is another story.)

The Stringmeter

Figures 34.2 and 34.3

Top and underside views of Stringmeter recording main string tension.





A racquet diagnostic machine is useful in measuring whether the string tension or the stringbed stiffness has decreased over time, and it will indicate roughly by how much, but it still doesn't give an accurate or absolute measurement of the actual string tension because all the other factors are not properly accounted for. That is why the Stringmeter was invented.

The Stringmeter contains two small cylindrical rods 1/3 of an inch apart that can be inserted anywhere in the stringbed to measure the tension of any desired string. The idea is to rotate the rods so that a particular string is pushed on one side and pushed in the other direction on the other side to form a little kink in the string (Figures 34.2 and 34.3). The rods are connected to a spring-loaded dial that is marked with graduations to read the tension in lb or kg. There is also a small adjustment needed for string diameter, which one does by rotating the position of the scales slightly. The only problem with this is that it is difficult to know whether the answer is correct. One can test the device on a single long string that is pulled to 60 pounds, and it will correctly read 60 pounds, but that is not the same as testing it on a fully strung racquet. In fact, the Stringmeter has three separate scales — one for long straight strings, another in pounds for a strung racquet, and also a kg scale for a strung racquet. The difference is due to the fact that the kink in a single long string has a different shape to the kink when the strings overlap.

Friction between the mains and the cross-strings prevents the string kinking over the full length of the string, with the result that the string kinks only over a short section of the string. A bigger force is needed to kink a short string than a long string, by an amount that is difficult to estimate with any accuracy. Consequently, the pounds and kilogram scales on the Stringmeter are difficult to check to determine if they are correct. Nevertheless, a Stringmeter can be used to determine if the tension has dropped with time, and roughly by how much, and it can be used to measure the tension in every single string if you have the patience. A surprising result is that the tension in the cross-strings is often a lot less than the tension in the mains. The Stringmeter is the only instrument I know of that can give this particular information. In principle, one could use a strain gauge located on every string to measure the tension in each string, but the Stringmeter is a lot more convenient.

The ERT700

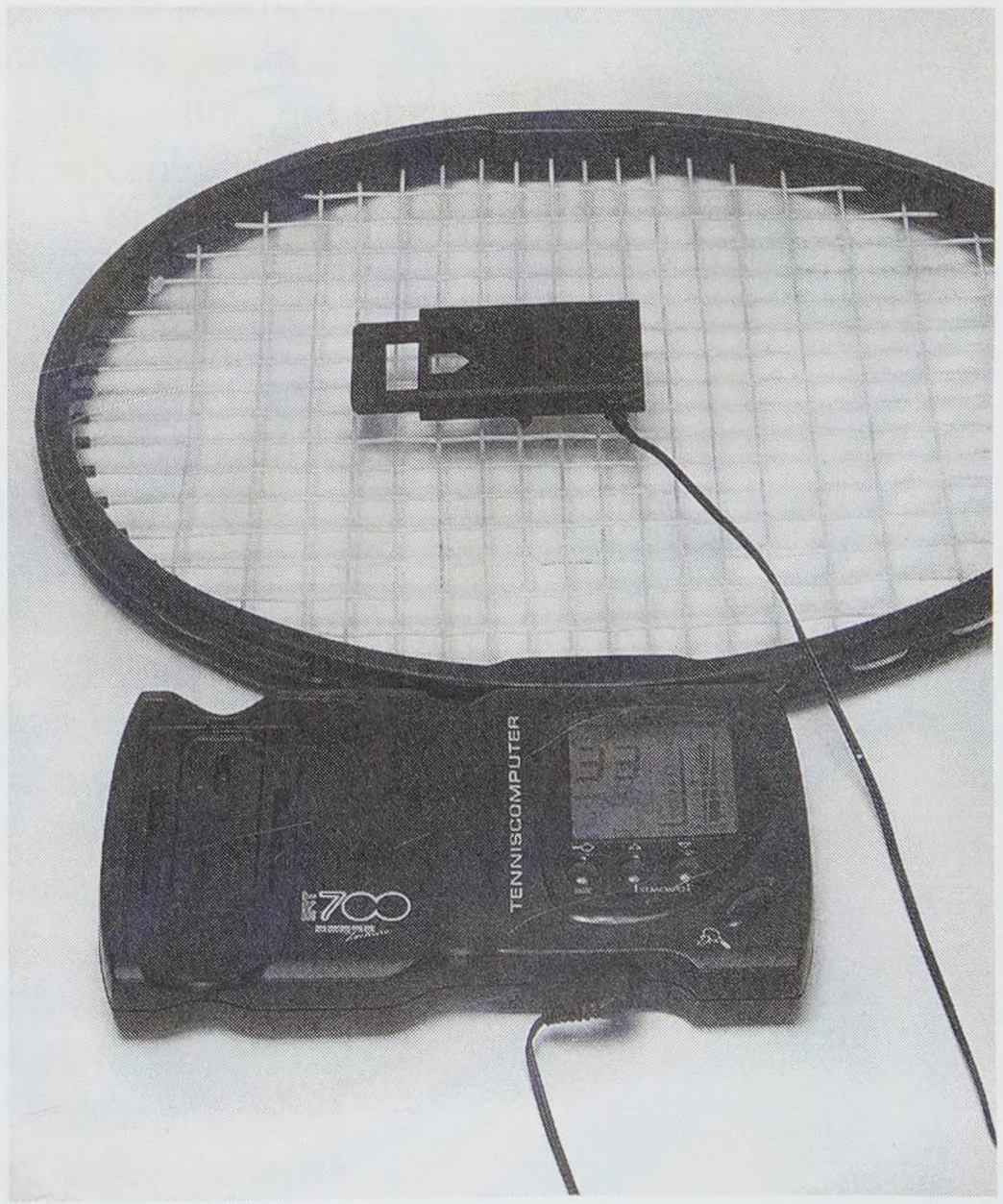
Every other method of measuring string tension measures all the strings at once, so these methods give only the average string tension in all the strings — not the tension in any given string. A high-tech method is used in the ERT700 (Figure 34.4).

It is high-tech since it contains two boxes of electronics and a digital display, but the basic method is fairly simple. The ERT700 forces the strings to vibrate back and forth, in much the same way that they vibrate when you hit them with your hand. The point of doing this is that the vibration frequency depends on the string tension. It is the same as in a piano or guitar or violin. Tuning up a guitar involves adjustment of the tension in each string so that it vibrates at just the right frequency. If you increase the tension it will vibrate faster and the pitch rises. If you drop the tension, the string vibrates more slowly and the pitch drops. The ERT700 detects the frequency of vibration of the whole string plane and assigns a number to the tension. However, the tension indicated by the ERT700 is not the actual tension since the vibration frequency depends on string tension, string length, the area of the stringbed, the mass of each string, the number of strings, and the actual string pattern. The formula for the actual frequency is too complicated to figure out the actual tension.

The ERT700 uses a small plastic box (which they call a ball simulator) that is clipped onto the strings, preferably in the middle of the string plane. The box has a mass of 34 gm, which is a bit less than a standard 57 gm tennis ball, but bigger than the mass of the strings themselves. The strings have a mass typically about 15 or 16 gm. Without the box in place, the strings themselves will vibrate at about 500 Hz or 600 Hz when you hit them, depending on the string tension. When the box is added, the extra mass of the box slows down the vibration frequency to something

Figure 34.4

ERT stringbed stiffness measurement.



between 140 and 180 Hz, again depending on the string tension. Inside the box is a small brass weight attached to a piezo disk. When a voltage is applied to the disk, the piezo causes the brass weight to vibrate in the same way that a piezo buzzer works in a watch, or a computer, or an alarm, to make a buzzing sound. By varying the vibration frequency of the piezo until it matches the frequency of the strings plus the simulator box, a resonance frequency is established and it is used to give readings of string tension, dynamic tension, and percent power vs control. All these numbers are based on one measurement of the frequency, so they are all just different ways of describing the same number.

Provided that one is willing to accept that all the numbers are artificial, then the ERT provides a useful and easy-to-use guide as to whether string tension has dropped over time and by roughly how much. The power reading varies from 20% to 80%, even though a decrease in 10 pounds in string tension will increase the ball speed by less than 1%. In other words, the readings need to be taken with a grain of salt.

Artificial Tension Calibration

The manufacturers of racquet diagnostic machines and the ERT700 have decided that the simplest thing to do is to make sure that the tension indicated by their device is nearly the same as the pull tension, at least immediately after a racquet is strung. As time goes by, the tension drops, and these devices will indicate that the tension drops, and they agree fairly closely with each other, but they all give a reading that is higher than the actual tension. There are four good reasons why the tension immediately after stringing is, in fact, a lot lower than the pull tension, as described below. However, stringers and customers are unlikely to be happy about this. If you string a racquet at a pull tension of, say, 60 pounds, and the actual tension immediately after stringing is only 40 pounds, then what will you tell your customer? If the customer asked for 60 pounds and you give him 40 pounds, then he is not going to be very impressed. That's why these tension measuring devices are calibrated to read the pull tension rather than the actual tension. It keeps everyone happy and no one knows the difference, including the stringer.

Four Reasons for Tension Loss During Stringing

Bond Slipping. If a string is stretched to a tension of 60 pounds, and then clamped at a fixed length, the tension will immediately start to drop since the bonds in the string keep breaking whenever the string is under tension. The weakest bonds break first, but it takes a longer time to break the stronger bonds. The tension will drop by about 8 pounds in the first 15 minutes. After that, the tension drops at a much slower rate. However, the main point is that the tension drops dramatically while a racquet is being strung. After that, the loss in tension is hardly noticeable. Polyester strings drop the most in the first 15 minutes.

Frame Distortion. Another problem is that all racquet frames are light and flexible. Even if you clamp the head at six or more points, the bits that aren't clamped can move during the stringing process. Graphite racquets are made to be stiff in a direction perpendicular to the string plane, but that is done at the expense of making them soft in a direction parallel to the string plane. Every time you add another string, it pulls the frame inwards in the direction of the

string, and stretches the frame outwards in a direction perpendicular to the string. If you add another cross string, the tension in the previous cross-strings drops, and the tension in the mains increases. That is why the tension in the cross-strings is usually a fair bit less than the tension in the mains. The total pulling force of say 10 strings each at 60 pounds is 600 pounds. That is a huge force on the frame, and it is no wonder that the frame distorts even when it is clamped at six or more points. A poorly mounted racquet can expand in width by an inch or more when the mains are strung. That is a terrible thing to do to the frame, but people do it, and then say it doesn't matter since the frame pulls back into shape when the crosses are added.

Friction. Friction is another reason why the string tension after stringing is a lot less than the pull tension. The tension at one end of a string in a racquet is not necessarily the same as the tension at the other end. When the mains are strung, then the tension at each end is the same, since there are no cross-strings to interfere along the length of any given string. But when the crosses are installed, they are woven through the mains. Friction between the mains and the crosses then results in the tension at the pulling end of a cross-string being higher than the tension at the other end. This is easy to demonstrate. Take a three-foot-long string and weave it across the main strings in a fully strung racquet. Clamp a 2-pound weight on the bottom end of the string. Then, holding the racquet so it can't move, lift the other end of the string. In order to lift the weight at the bottom end, you will need to pull with a force of about 4 pounds. That means the tension in the top end is 4 pounds and the tension in the bottom end is only 2 pounds. Or, if the tension at the pull end is 60 pounds, the tension at the other end will be about 30 pounds.

Pull Angle. The fourth reason has to do with the pull angle. The pulling head on some machines has to be a bit lower than the racquet in order to rotate the racquet. The string makes an angle of about 10 degrees with the horizontal between the pulling head and the edge of the racquet frame. It then passes through a grommet hole. Friction in the hole ensures that the tension after the string passes through hole is less than the pull tension, typically by about 2 pounds.

This all adds up to a total tension loss of at least 10 pounds, and maybe a lot more by the time the racquet is strung, depending on the stringing technique.

String Tension: Can Players Tell the Difference?

In 2001, Rob Bower (Lecturer in Sport Science at the University of Technology, Sydney, Australia) and I strung five identical Volkl Pro Comp racquets all with the same string but at five different tensions. We wanted to see how well players could pick the differences in tension just by hitting a ball a few times with each racquet. They were instructed not to hit the strings with their hand or with another racquet since the differences in the sound are easy to identify. A string dampener was inserted in each racquet to deaden the sound, since we wanted the players to pick the differences in the feel or performance, not the differences in the sound. After testing 41 different players it became obvious that they were not very good at picking differences. More than half of them could not pick a difference of 15 lb. We then tested 18 professional satellite players at a tournament in Sydney. For them, we used four identical Topspin 630 Pro racquets, all a bit heavier than the Volkl racquets and more suited to their game. They were not much better than anyone else at picking the tensions.

Eleven of the 18 satellite players could not pick a difference of 11 lb, even though most of them specify the tension to the nearest lb when stringing their racquets.

Though many accomplished players can't pick small differences in string tension, a small minority can. The above players were not asked to identify specific tension, just to choose which was strung looser or tighter than the other. In the process of doing the experiment, we had to measure tension in many different ways – reference tension, pull tension during stringing, stringbed stiffness (not tension) after stringing, actual average stringbed tension (not stiffness) after stringing, actual individual string tensions after stringing, and perceptual tension, which was what the experiment was all about. These are all different concepts but very frequently used interchangeably, lumped into one, and referred to simply as "string tension." To add to the confusion, the available methods to determine these things use different calibrations and measure different parameters, yet often use the same vocabulary. Figuring out what we were measuring was important to our test, but it is also important to the stringer. It is important to know what each device is actually measuring and how it is useful to you. The main thing we discovered is that all the devices do a good job measuring relative changes in racquet tension, but actual tension was much harder to pin down. Fortunately, once the racquet is strung, stringers and players are primarily interested in relative tension changes, not absolute measurements. Unfortunately, many are not aware that is what they are doing.

Determining Actual Stringbed Tension: Method 1

The main thing we needed to be sure about in our experiment was the string tension. The stringing machines were accurately calibrated against a load cell to make sure the pull tension was correct, and exactly the same procedure was used to string each racquet. At the time, we didn't have a Stringmeter or an ERT700 or a portable racquet diagnostic machine, so I invented my own way of measuring the tension at regular intervals before the playtests and afterwards, to make sure the differences in tension stayed about the same as when the racquets were first strung. For that purpose, a small piezo was attached to the strings to measure the vibration frequency of the strings. That was enough to show me that the strings at high tension vibrated at a higher frequency (as I could hear with my own ears) and that the relative differences in the tensions were correctly maintained throughout the tests. The tensions all dropped slightly over a few days, but not enough to worry about.

Determining Actual Stringbed Tension: Method 2

The average tension in all the strings can be calculated directly from the vibration frequency of the strings. The key to this measurement is the fact that the vibration frequency depends on the total area, A, of the string plane and not on its exact shape. In fact, the vibration frequency of the whole string plane is almost exactly the same as that of a single string of length L, where $A = L^2$ (Cross and Bower, 2001). This result indicated that the string tension in each racquet was a lot lower than the pull tensions, as one might expect from the fact that all strings drop rapidly in tension after clamping (see Chapter 33). Nylon strings drop by about 9 lb in just 17 minutes, which is about the time it takes to string a racquet. My tension results were even lower than that, so I

checked the measurements using a microphone to record the sound of the vibrating strings. This time the frequencies and hence the tensions were slightly higher than I found with the piezo. The problem with the piezo was that it weighed 2.2 gm, a lot less than the 15 or 16 gm weight of the strings, but enough to lower the vibration frequency by about 15%. Even so, the tensions were all about 30% lower than the pull tensions in all of the racquets, immediately after stringing them. For example, the racquet strung at 60 lb was actually at a tension of about 42 lb. But still, was the frequency determined tension correct? The frequency calculation gives an average stringbed tension. How does that compare to the tensions in individual strings?

Determining Tension in Each String

The arrangement shown in Figure 34.5 was used to investigate further.

A metal bracket was made up so that it could be attached at the 3 o'clock position together with a load cell to measure the tension in two of the cross-strings. For this experiment I used a 1.30 mm diameter polyester string. After stringing the mains, the first seven cross-strings were installed and the 8th and 9th strings were threaded through a small bracket attached to the load cell. Each string was tensioned to 62 lb. However, the reading on the load cell was then 112 lb. It should have been 2 x 62 = 124 lb since two strings each at 62 lb were pulling on the load cell. The alignment of the pulling head, together with friction in the grommet holes and friction between the mains and crosses all acted to reduce the tension from 124 to 112 lb. As each additional cross-string was added, the reading on the load cell dropped further due to contraction of the frame. The frame was actually supported at 10 points in a Pacific stringing machine, but that was still not enough to prevent the frame contracting slightly. By the time the job was completed, and on removal of the racquet from the stringing machine, the load cell reading had dropped to 76 lb, or an average of 38 lb in each cross-string. The tension in the main strings was not measured due to restricted access near the 12 o'clock position. So here was a baseline actual tension against which to compare both theoretical calculations and tension testing equipment.

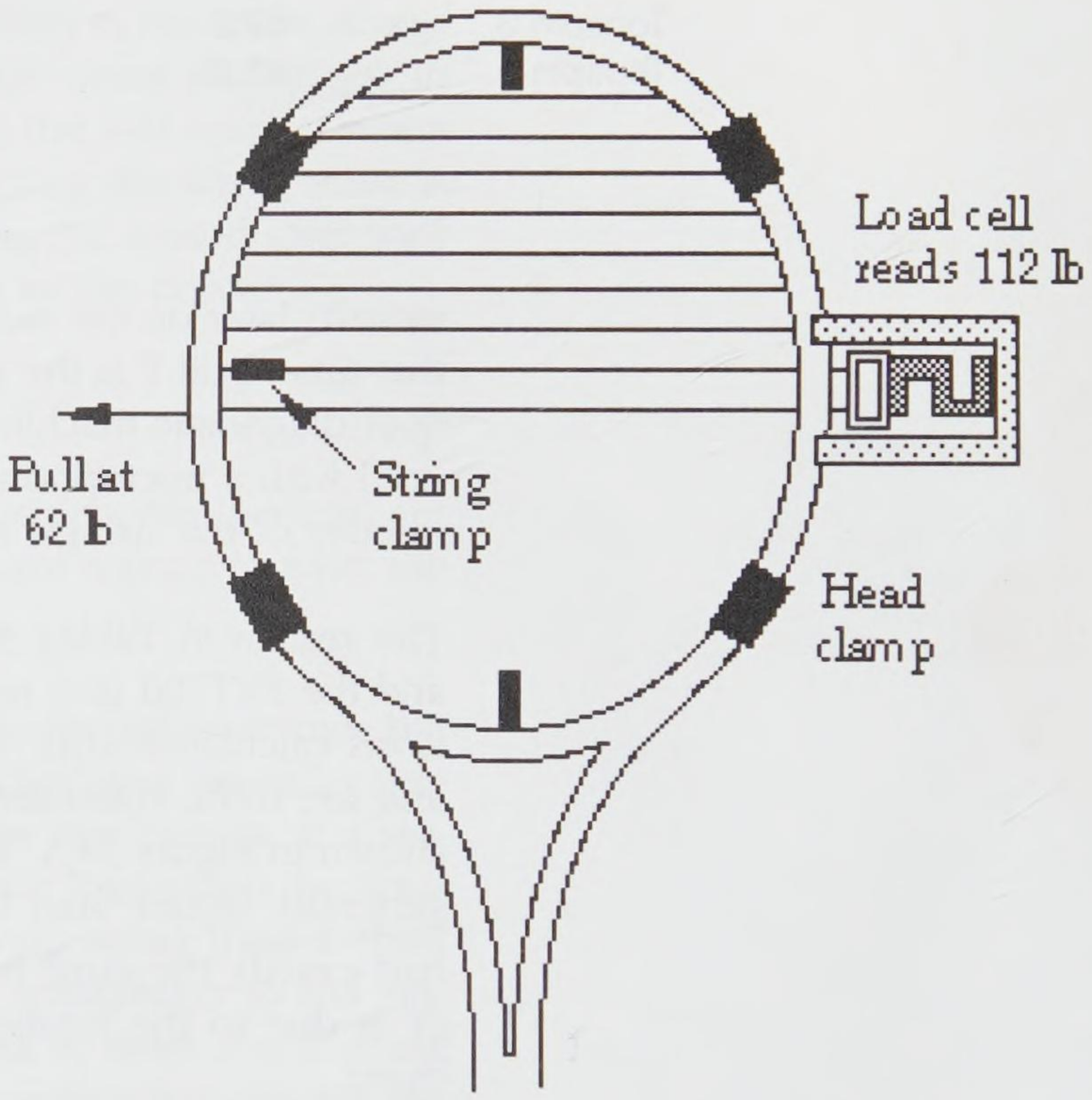


Figure 34.5

Comparing Pull Tension, ERT, Pacific RA, and Calculations

These results seem to be at odds with the folklore that the tension immediately after stringing a racquet might be a few pounds lower than the pull tension, but not much more than that. So I compared Stringmeter and ERT700 measurements with my calculations using frequencies determined with a microphone. The results of the various tests are given in Tables 34.1 and 34.2. Some of the tests were done immediately after stringing and some were done a few

Table 34.1. Volkl racquets immediately after stringing with 1.42 mm nylon.

Racquet	Pull T (lb)	TP (lb)	f(Hz)	TVib (lb)
Volkl 1	39.7	46.3	504	26.7
Volkl 2	46.3	50.7	549	31.7
Volkl 3	50.7	52.9	565	33.5
Volkl 4	55.6	55.1	599	37.7
Volkl 5	62.6	58.4	645	43.6

Table 34.2. Topspin racquets immediately after stringing with 1.27 mm nylon.

Racquet	Pull T (lb)	ERT (lb)	f(Hz)	TVib (lb)
Topspin 1	46.3	50.7	632	32.8
Topspin 2	52.9	52.9	653	35.0
Topspin 3	57.3	57.3	695	39.7
Topspin 4	63.9	63.9	724	43.2

months later on the same racquets, which were put aside and not used during that time. Pull T is the pull tension, TP is the tension indicated in a Pacific racquet diagnostic machine, f (Hz) is the vibration frequency of the strings measured with a microphone, TVib is the tension calculated from the vibration frequency of the strings, ERT is the tension indicated by the ERT700.

The results in Tables 34.1 and 34.2 show that the Pacific diagnostic machine and the ERT700 give tensions that are similar to the pull tensions, but the tensions calculated from the vibration frequencies of the strings are much lower and are more consistent with the measurements made using the arrangement shown in Figure 34.5. The vibration frequencies for the Topspin racquets were generally higher than those for the Volkl racquets, even though both models had exactly the same head size at 630 cm^2 (98 in^2). The difference in frequency is due to the smaller diameter and lighter string used in the Topspin racquets.

Comparing the Stringmeter with Calculations

The Volkl racquets were strung on 30th Nov 2000. The Topspin racquets were strung on 8th Feb 2001. The racquets were put aside after testing the players, and on 11th March 2001, the tension in every string was measured using a Stringmeter. I also re-measured the vibration frequencies of the strings. The results for racquets Volkl 4 and Topspin 4 are shown in Figure 34.6.

In both cases, the tension in the cross-strings was much less than the tension in the mains, a result that can be explained by the fact that the tensions in the cross-strings drop and the tensions in the mains increase due to contraction of the frame when the crosses are installed. The Topspin 4 frequency had dropped to 655 Hz, giving an average stringbed tension of 35.3 lb. The

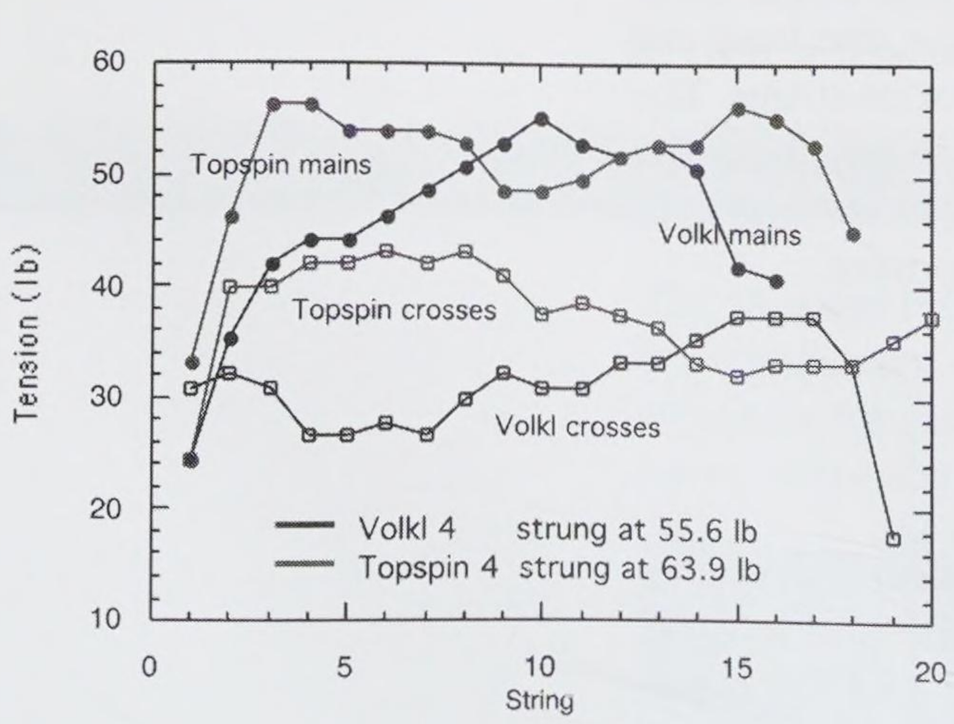


Figure 34.6

Comparing Stringmeter results with calculations.

Volkl 4 frequency had dropped to 558 Hz, giving an average stringbed tension of 32.6 lb. These tensions correspond roughly to the average tension in the cross-strings rather than the average tension in the mains if one can believe the Stringmeter readings shown in Figure 34.6. Theoretically, one would expect that the vibration frequency of the strings should be determined by the average tension of all the strings, not just the cross-strings. Consequently, I decided that either the Stringmeter readings were too high or the tensions deduced from the string vibration frequencies were too low.

Where To Go From Here?

The ERT700 also uses a measurement of the vibration frequency of the strings to determine the average string tension. On 11 March, the ERT700 readings were 50.7 lb for the Topspin 4 racquet and 48.5 lb for the Volkl 4 racquet. These tensions are close to the average Stringmeter tension in the main strings rather than the cross-strings, and appear to be too high since the results in Table 34.2 indicate that the ERT700 is calibrated to read the pull tension when a racquet is freshly strung. All three methods of measuring the string tension indicated that the tension dropped in all racquets during the period after they were strung, but the actual tensions were proving hard to pin down.

The Stringmeter

When a Stringmeter is used to measure the tension in a string, two prongs are positioned on either side of the string then the prongs are rotated to twist the string sideways (Figure 34.3).

Most of the length of the string is locked into position by the other strings, but a small section of the string about 1-inch long gets stretched and twisted, especially in the section about 1/2-inch long surrounding the two prongs. If a single long string is tested this way, the Stringmeter reading is lower than the actual tension since a smaller force is needed to twist a long string than a short string. That is why a separate scale is provided on the Stringmeter to test single strings. Furthermore, a much larger force is needed to twist a stiff string than a soft string. If one tests a steel wire under tension with a Stringmeter, the reading on the Stringmeter is much higher than the actual tension.

Apparatus was set up to test the Stringmeter as shown in Figure 34.7.

A single string about 14-inches long was tensioned to either 40 or 60 lb, as indicated on an accurate load cell. Two short parallel bars, each perpendicular to the string, were positioned a distance d apart so that the string passed through a small hole in each bar. A Stringmeter was then used to measure the tension in the string between the bars as the distance d was varied from 150 mm down to 15 mm. The sideways movement of the string was thereby restricted to only a short section of the string of length d.

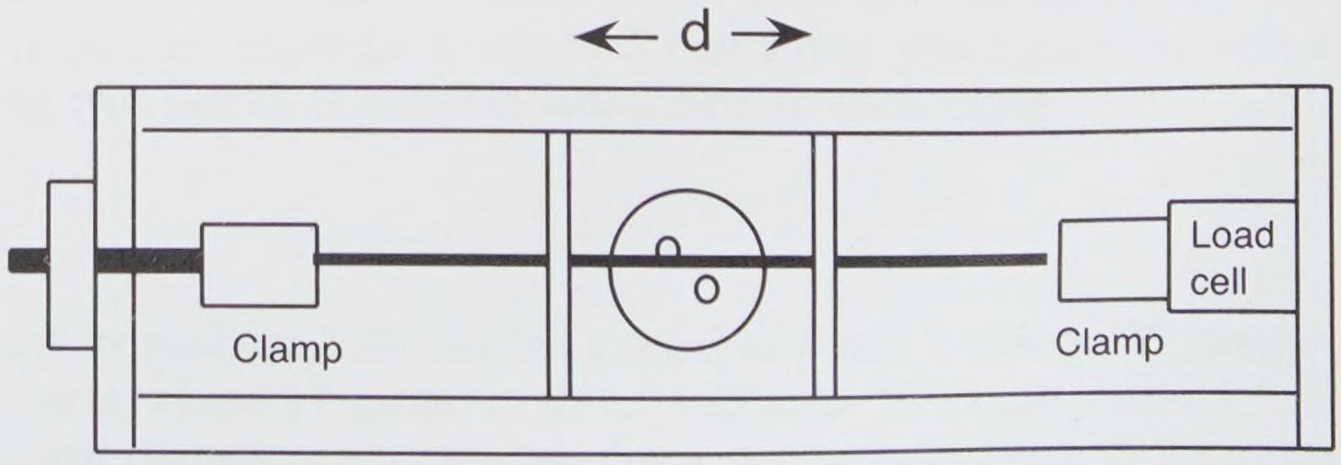


Figure 34.7

Apparatus to test Stringmeter measurements

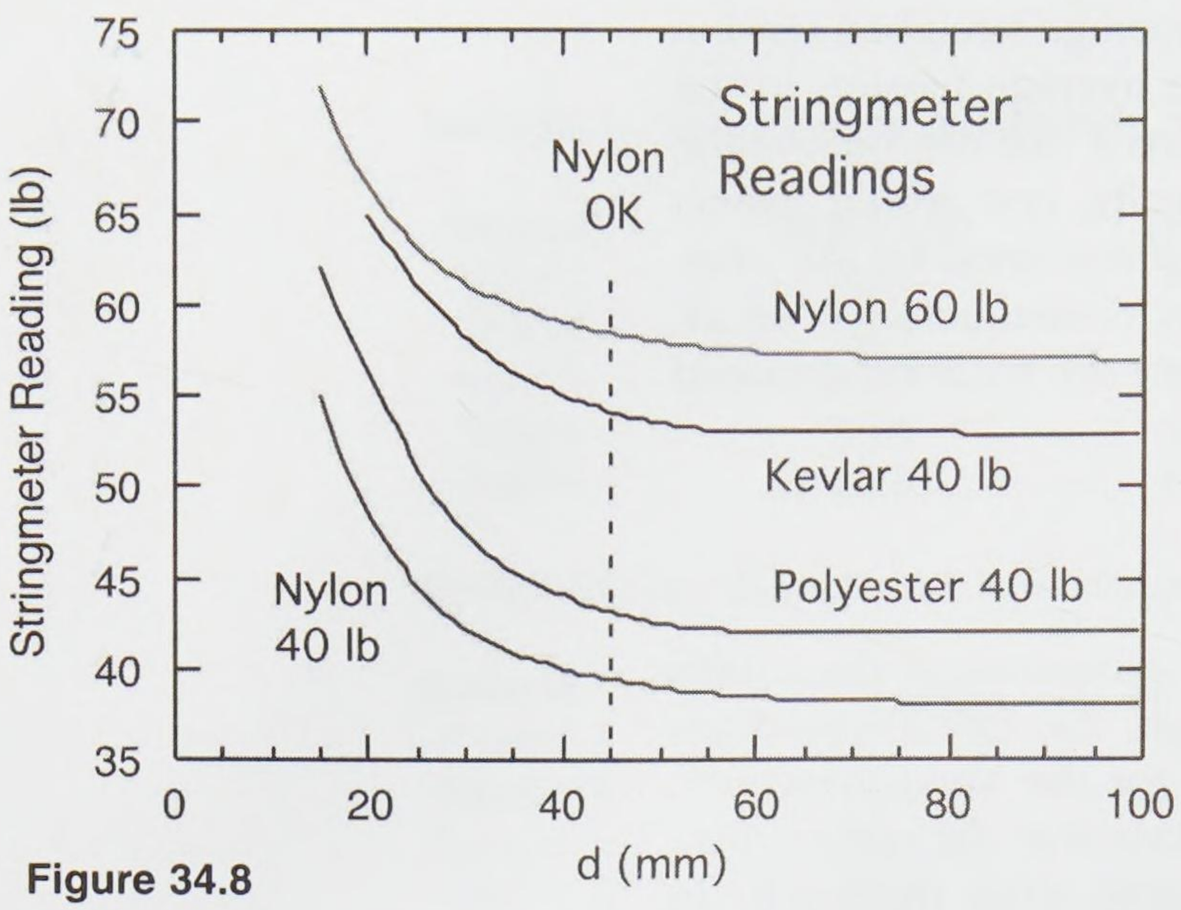


Figure 34.8

As shown in Figure 34.8, the Stringmeter reading increased as d decreased. At large d the reading for a nylon string was smaller than the actual tension, but for a kevlar string, the reading was larger than the actual tension. At a distance of about 40 to 50 mm, corresponding to the actual distance over which the string is twisted in a racquet, the Stringmeter reading was almost spot-on for the nylon string, but was way too high for a kevlar string.

Conclusion

My conclusion is that the old-fashioned Stringmeter is still the best instrument available to measure actual string tension, provided that it is only used on relatively soft strings such as nylon and natural gut, and provided that that is what is useful to you. But at the same time, it is more difficult to determine what has happened to the entire stringbed.

The other methods also indicate whether the tension in a racquet has dropped, and roughly by how much, but the actual values are not correct. The ERT700 and racquet diagnostic devices read too high.

All the devices are good at telling relative differences in a racquet over time or between two racquets. As long as you understand what each device is telling you, and how each differs from the other, then you will be able to make intelligent decisions when it comes to stringing tensions.

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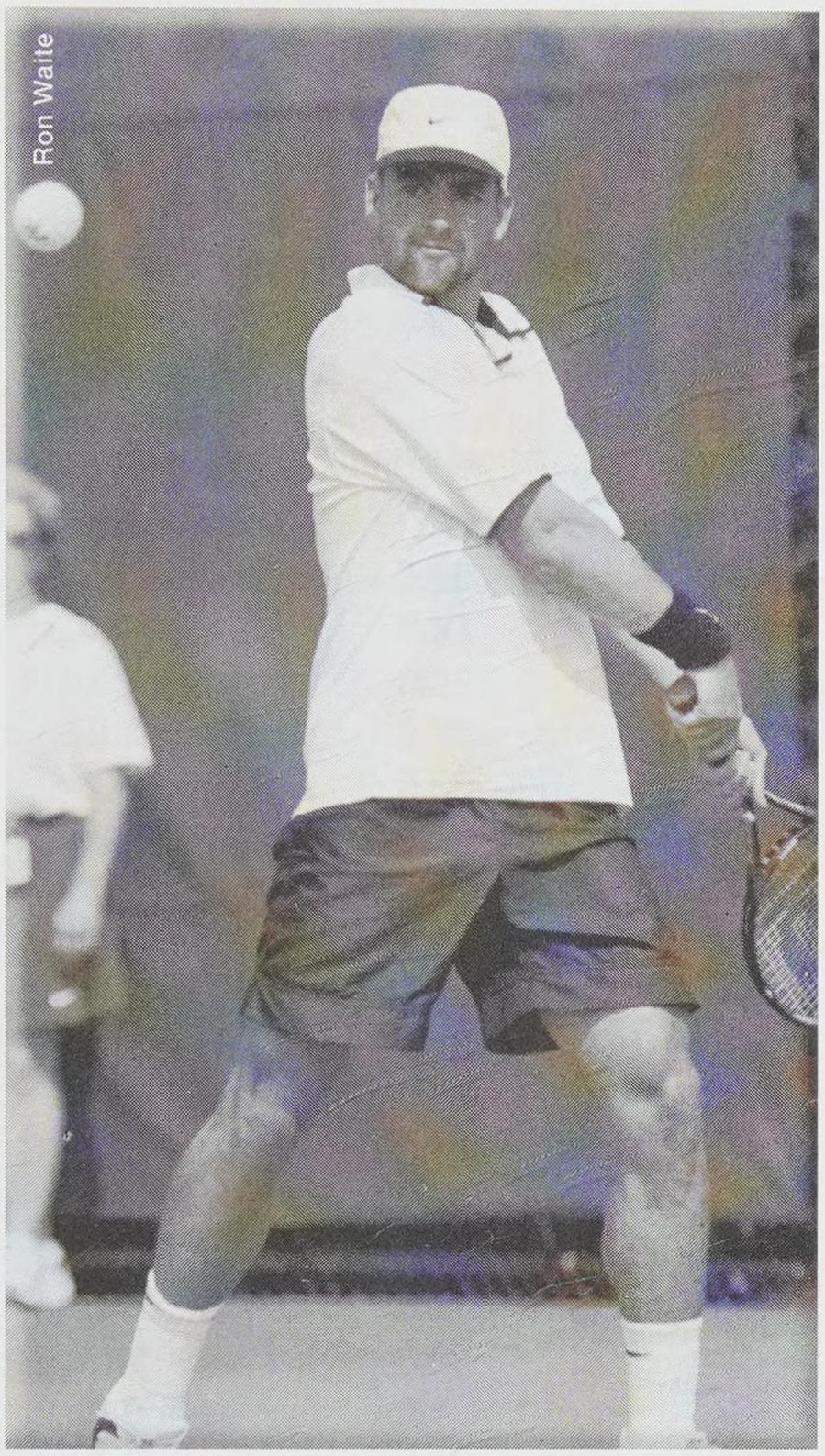


Nicolas Kiefer, Germany

35

String Gauges

Are Thin Strings Softer or Stiffer?

By Rod Cross

Summary

Any given tennis string is normally available in several different diameters. The thinner versions might be expected to be more elastic and to lose tension faster over time. Of 18 different string pairs tested, 8 of the thin versions were dynamically stiffer and 8 maintained tension better than the corresponding thick versions. These effects were investigated by a controlled experiment where a thin string was compared with a thick version of the same string made by placing two pieces of the thin string next to each other.

Introduction

Tennis strings are available in a range of diameters from about 1.20 mm (18 gauge) to about 1.40 mm (15 gauge). Manufacturers have devised their own gauge system where 17 gauge is about 1.25 mm, 16 gauge is about 1.30 mm and 15L gauge is about 1.35 mm. The nominal diameters vary slightly from one manufacturer to the next. It is commonly assumed that thinner strings are softer or more elastic, but this is not necessarily the case.

A sample of 18 different pairs of 16 and 17 gauge strings were tested. As demonstrated in Table 35.1, eight of the thinner strings were found to be dynamically stiffer than the thicker versions.

The parameter of most relevance in a tennis string is the increase in elongation for a given increase in tension, starting at the tension at which a racquet is normally strung. This parameter can be termed the dynamic stiffness, and it is this

Table 35.1

Dynamic Stiffness, Elongation, and Tension Loss for 16-17 String Gauge Pairs

Numbers shown in bold are measurements that are contrary to conventional wisdom.

Brand	String Name	Diameter mm	Dynamic Stiffness (k) lb/in	Stretch (mm) at 62 lb	Tension Loss* lbs
Babolat	Super Fine Play 16	1.32	208	35.0	11.3
Babolat	Super Fine Play 17	1.26	202	**34.5**	**10.5**
Babolat	VS Power 16	1.29	108	26.0	7.5
Babolat	VS Power 17	1.24	100	29.2	7.8
Gamma	Live Wire 16	1.31	177	33.5	11.2
Gamma	Live Wire 17	1.27	**180**	34.0	13.5
Gamma	Live Wire XP 16	1.32	161	32.0	12.8
Gamma	Live Wire XP 17	1.27	**183**	**29.8**	13.0
Gamma	Synthetic Gut 16	1.30	214	31.5	8.5
Gamma	Synthetic Gut 17	1.24	211	**31.0**	9.1
Gamma	TNT 16	1.32	188	33.0	8.9
Gamma	TNT 17	1.25	**207**	34.3	10.2
Gamma	TNT 18	1.16	**197**	43.0	12.5
Gamma	TNT Pro Plus 16	1.31	187	38.5	10.8
Gamma	TNT Pro Plus 17L	1.23	163	40.5	10.9
Gosen	OG-Sheep Micro 16	1.33	209	35.0	11.0
Gosen	OG-Sheep Micro 17	1.30	207	31.0	9.3
Prince	DNA Helix 16	1.30	192	31.0	9.4
Prince	DNA Helix 17	1.25	192	32.8	9.9
Prince	Perfection 16	1.29	206	27.5	11.4
Prince	Perfection 17	1.25	204	27.5	**11.3**
Prince	Sweet Perfection 16	1.295	183	34.0	12.2
Prince	Sweet Perfection 17	1.25	178	37.0	**11.7**
Prince	Synthetic Gut 16 w/Duraflex	1.29	202	33.5	8.9
Prince	Synthetic Gut 17 w/Dura flex	1.25	198	32.0	10.1
Prince	Synthetic Gut Original 16	1.32	194	41.5	10.8
Prince	Synthetic Gut Original 17	1.24	**212**	**30.0**	**9.1**
Prince	Synthetic Gut Soft 16	1.29	213	30.0	8.7
Prince	Synthetic Gut Soft 17	1.23	207	32.8	9.3
Tecnifibre	NRG2 SPL 16	1.34	170	31.5	8.4
Tecnifibre	NRG2 SPL 17	1.23	**184**	**30.0**	**8.3**
Wilson	Extreme Control 16	1.29	210	29.0	8.8
Wilson	Extreme Control 17	1.23	196	29.5	9.2
Wilson	Sensation 16	1.32	183	29.0	9.0
Wilson	Sensation 17	1.25	213	29.8	10.1
Wilson	Sensation Ice 16	1.32	194	34.0	11.4
Wilson	Sensation Ice 17	1.25	**204**	**33.0**	**11.3**
Wilson	Sensation NXT 16	1.32	208	29.0	10.5
Wilson	Sensation NXT 17	1.24	202	29.8	**10.4**
Wilson	Syn Gut Extreme Control 16	1.30	198	29.5	9.3
Wilson	Synth Gut Extreme Control 17	1.25	**218**	**29.4**	**8.3**

*Tension loss is total loss in string 1000 seconds after being pulled to 62 lbs and then impacted 10 times with a force equivalent to 120 mph serve.

parameter that was observed to be larger for 8 of the 18 thin strings (a string with a higher dynamic stiffness means that the string elongates less for a given tension rise than does another). Neither the elongation nor the dynamic stiffness, at tensions lower than that at which the racquet is strung, are of any consequence regarding the performance of tennis strings. So don't go by the stretchiness of a string during stringing. This occurs in a tension range below the strung tension and is irrelevant. When you hit the ball, the tension rises rapidly above the strung tension, and the string stretches by an amount that depends on the dynamic stiffness. Typically, a nylon string that is soft when you string it is about twice as stiff when you hit the ball. The reason that it is twice as stiff is only partly because it is stretched faster. The main reason is that the string does not stretch as easily at tensions above the normal strung tension, even if you stretch it slowly.

The stiffness of the string plane in a racquet refers to the deflection in a direction perpendicular to the string plane when a force is applied perpendicular to the string plane. For small deflections, this is independent of the dynamic stiffness of the strings and would be the same for any type of string provided the string tension, string length and number of strings remained the same. For the larger deflections encountered in practice, the string plane stiffness does depend on the dynamic stiffness since the string tension increases significantly during an impact with a ball, typically by a factor of about two for a fast serve. The increase in tension is smaller for soft strings such as natural gut, and larger for stiff strings such as kevlar.

A graph of tension, T, vs elongation, x, for small and large diameter strings is shown in Figure 35.1a. For any given string, a graph of T vs x will be a smooth curve, where the slope (i.e., steepness of the curve) at low T is typically about a factor of two smaller than at high T, at least for nylon or polyester strings. Strings made from natural gut are more linear.

For purposes of illustration, the curve for a nylon or polyester string can be approximated by two linear segments, as in Figure 35.1b, showing the simplified graph of a thin and thick string (the normal shape of the curves is illustrated in Figure 35.1a). If one assumes that the material properties of the thin and thick strings are the same, and if the strings are the same length, then for any given elongation x, T will be proportional to the cross-sectional area of the string. At any given tension, the extension of a thin string will therefore be larger than that of a thick string. (Note: a tension vs elongation curve is different than a stress vs strain curve, though they look the same. The former plots how tension changes with a given elongation in any given piece of string — thick or thin. The stress vs strain curve looks similar but it takes the size of the string into account — factoring in the cross-sectional area and length of the string. As a result, the stress vs strain curve will be the same for any string length or diameter of a given material. It measures the property of the material of a standardized dimension. The tension vs elongation curve measures the behavior of string of a particular dimension. Therefore, the same level of stress (tension divided by cross-sectional area, i.e., pounds per square inch) will occur at different tensions for different gauges. Both gauges behave the same at the same stress; it's just that the thinner string achieves that stress at a much lower tension. The two terms are incorrectly used interchangably in common discourse.)

In practice, racquets are strung at a tension of about 55 pounds, regardless of the string diameter. As a result, thick strings can operate in a region where x

Figure 35.1

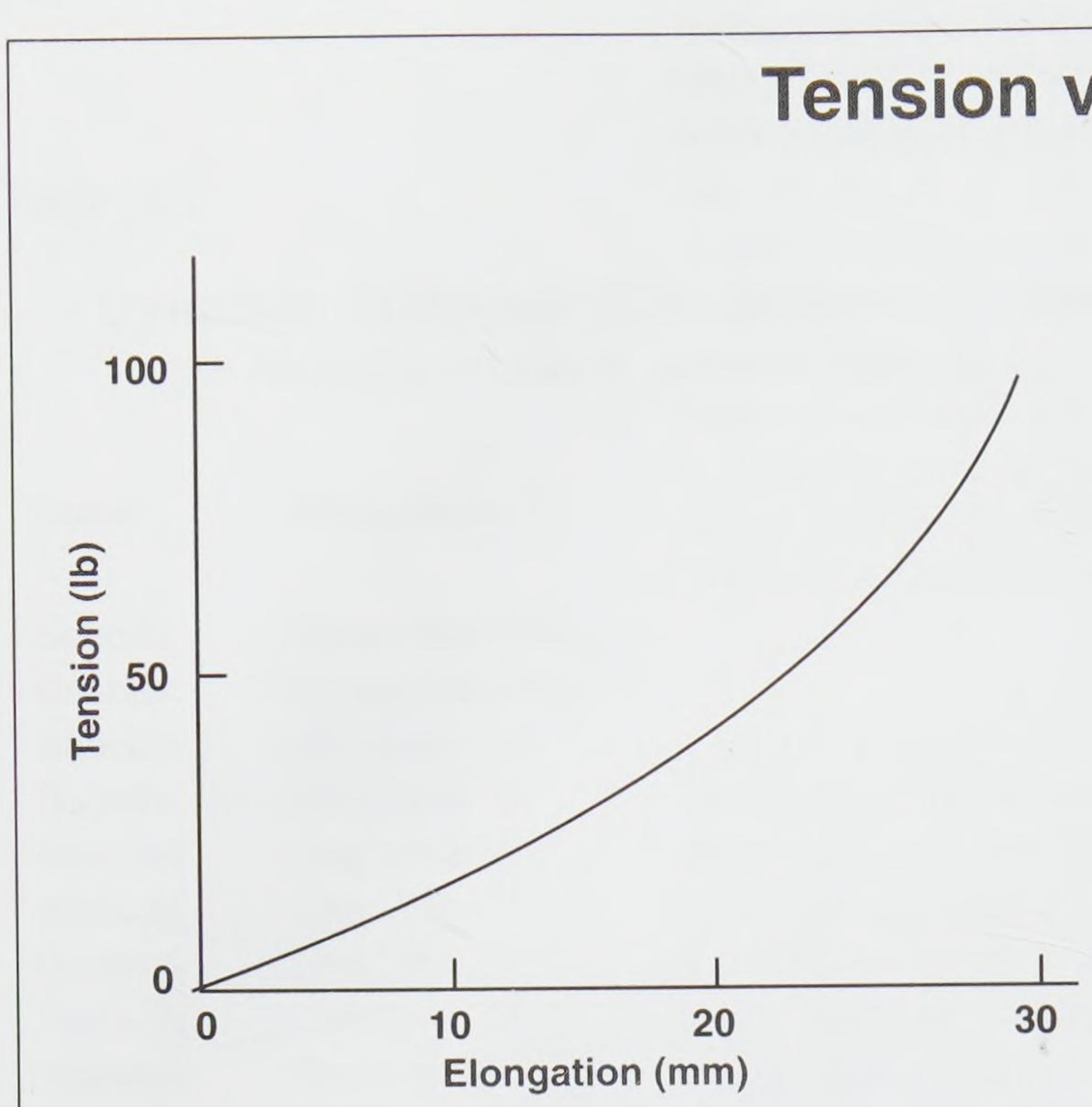


Figure 35.1a

A typical tension vs elongation curve. The string gets stiffer the more it is stretched, indicated by the steeper slope of the line.

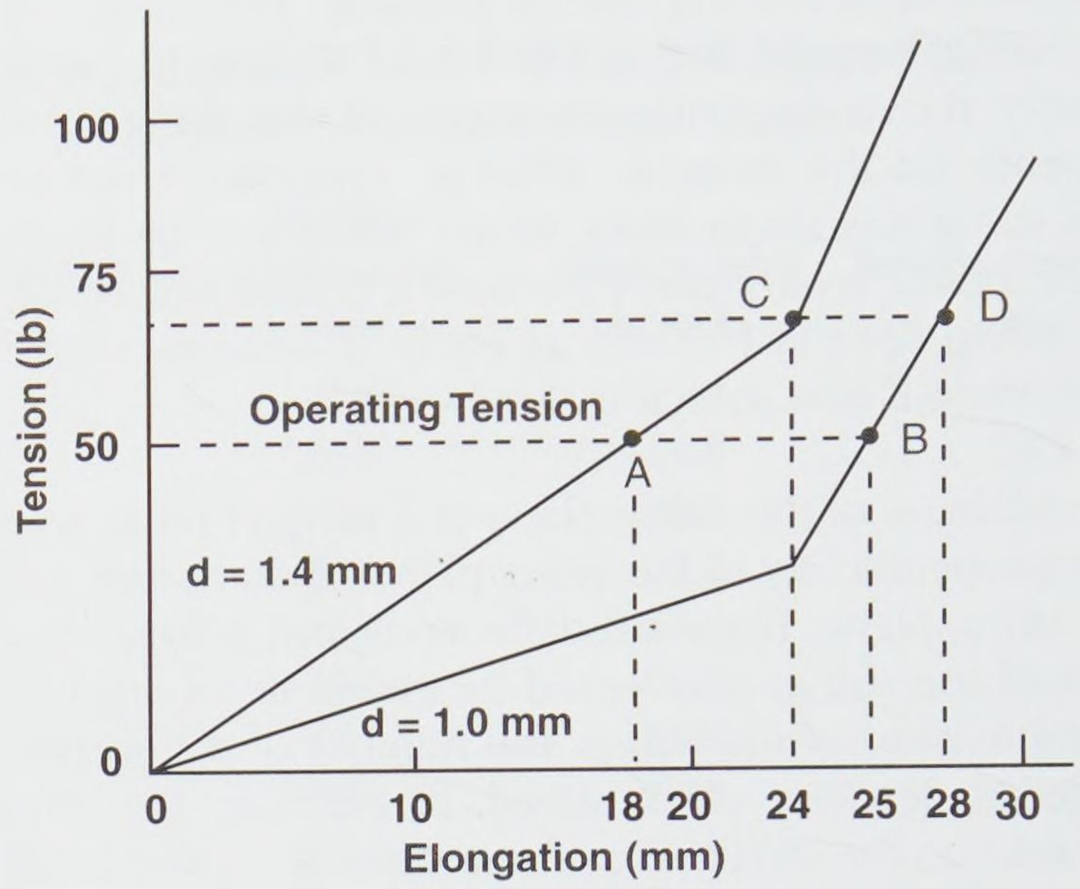


Figure 35.1b

A simplified version of the tension vs elongation curves for two nylon strings of different diameter.

As shown in Figure 35.1b, when the thick string is strung at 50 lb, it stretches by 18 mm (point A). When the thin string is strung at 50 lb, it stretches by 25 mm (point B). Up to that point, the thin string is softer than the thick string, since it stretches futher, which is what everyone would expect.

Now, suppose a ball hits the string and the tension suddenly rises from 50 lb to 70 lb and then back to 50 lb, all during the 5 ms impact with the ball. If it hits the thick string, the string stretches from point A to point C, or an extra 6 mm. If the ball hits the thin string, it stretches from point B to point D, or an extra 3 mm. This time, the thick string stretches more than the thin string, so the thin string is stiffer.

In this example, the dynamic stiffness of the thin string is twice as large as the thick string since it stretches by an extra 3 mm, while the thick string stretches an extra 6 mm (for the same increase in tension from 50 to 70 lb).

and the dynamic stiffness k = dT/dx (i.e., change in tension divided by the change in length) are both small, while thin strings can operate in a region where x and dT/dx are both large. In other words, the dynamic stiffness of a thin string may be larger than that of a thicker string of the same material, depending on the shape of the stress vs strain curve and the operating tension. In practice, the string tension can rise by a large factor during an impact, in which case an average dynamic stiffness can be defined as DT/Dx, where DT is the increase in tension and Dx is the increase in elongation.

Controlled Experiment

Most strings are constructed as composite materials with an inner core and an outer protective coating to enhance durability. When the string diameter is varied, the relative dimensions of the core and the coating may not remain fixed. Nevertheless, a string that is marketed under the same name would be expected to have similar proportions of core to coating thickness, and the thinner gauge would normally be expected to be more elastic and stretch further at a given static tension. Indeed, this was the case for most of the 18 string pairs tested. The exceptions may be due to a change in coating thickness or core composition, despite each string pair having the same name. It might even be possible to manufacture a 17 gauge string simply by stretching a 16 gauge version.

Since the precise composition and manufacturing techniques used to construct different gauge strings is unknown, a controlled experiment was devised to investigate the effects of varying the string diameter. For this purpose, the properties of a single 18 gauge nylon string (string A) were compared with a string (B) of twice the cross-sectional area mounted in the test device as two lengths of string A in parallel. Strings A and B were both tensioned, in separate experiments, to 62 pounds and then clamped at a fixed length (320 mm). The tension in each string decreased with time due to stress relaxation. The tension in string A dropped by 9.2 lb to 52.8 lb after 1000 seconds. The tension in string B dropped by 11.9 lb to 50.1 lb after 1000 seconds. This result is consistent with the common observation that thin strings often hold tension better than thick strings, despite the fact that the stress in a thin string is larger. It is also consistent with the observation that when string A was tensioned to 31 lb, the tension dropped by 6.1 lb to 24.9 lb after 1000 seconds. When string B was tensioned to 62 lb, each "half" of the string was tensioned to 31 lb. If the tension in each half drops by 6.1 lb after 1000 seconds, then the total tension in string B would drop by 12.2 lb, essentially as observed.

The dynamic stiffness of strings A and B was measured immediately after the 1000 second delay, by impacting ten times with a hammer as described in Chapter 30. In theory, one should be able to determine the dynamic stiffness of a string simply by measuring its elongation curve under quasi-static conditions using an Instron or similar materials testing device (these devices are like stringing machines — they pull relatively slowly and at a constant rate of elongation. That's why the rate of pulling is always specified in testing. These rates are typically 10 mm to 500 mm per minute.). In practice, this is not as accurate or as relevant as an impact method since the elongation curve depends on the rate of stretch. For example, if a string is stretched by an additional 1 mm and then clamped, the tension may increase by say 2 lb while the string is being stretched, but it immediately starts to decrease as soon as the string is clamped, typically by about 0.4 lb in the first two seconds. This problem can be avoided if the string is stretched sufficiently rapidly (a hammer impact stretches the string at a rate of about 1000 inches per minute).

For the impact tests, the impact duration was about 29 milliseconds for each string, and the impact energy was 1.63 Joules per impact (equivalent to the energy on one string in a 120 mph serve). The increase in tension during each impact was 37.8 lb on average for string A and 43.1 lb for string B. Calculations revealed that string A had a dynamic stiffness k = 196.6 lb/in. averaged over 10 impacts, at an average tension of 49.7 lb. For string B, k = 235.5 lb/in aver-

aged over 10 impacts, at an average tension of 48.2 lb. An average tension is quoted since the tension decreased slightly following each impact. In this case, the thinner string was softer, but not by the factor of two that one would expect if the stress vs strain curve were linear.

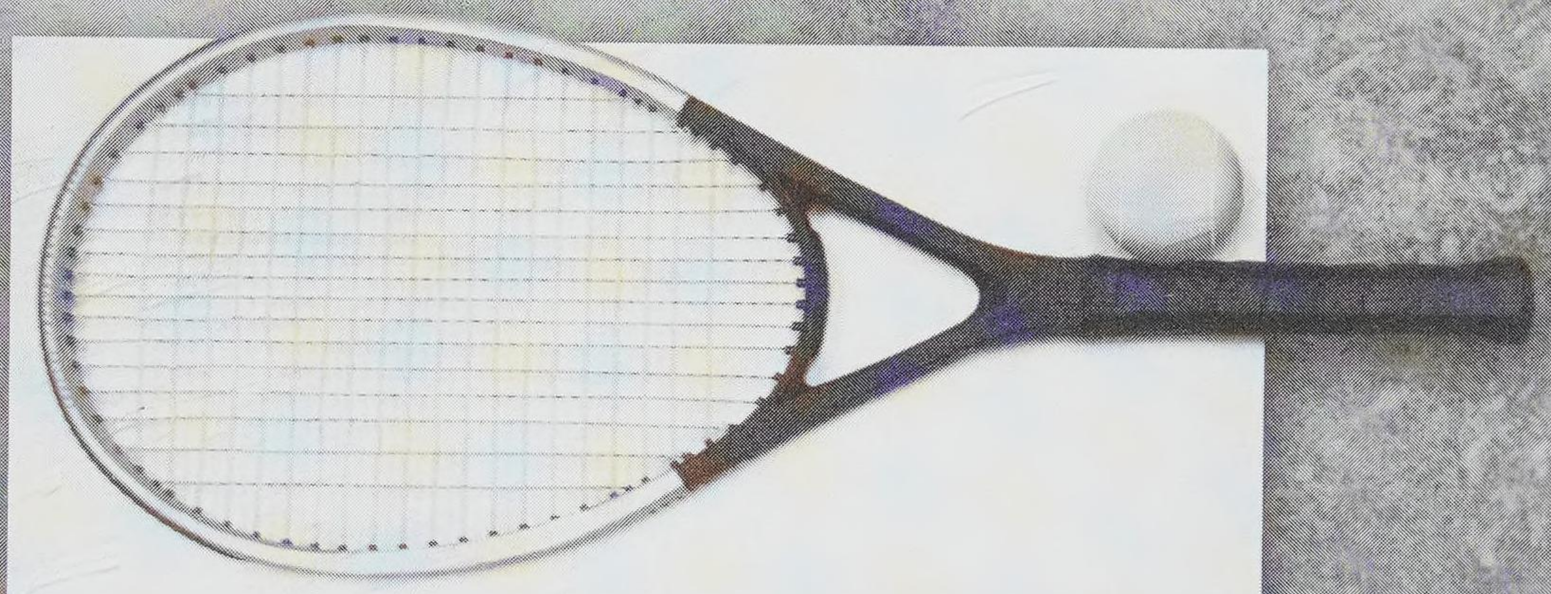
To understand this, consider a simple spring. Suppose it stretches one inch when you pull it with a force of 25 lb. Suppose also that it stretches by 2 inches when you pull it with a force of 50 lb. Such a spring is said to be linear, since when you double the force, you double the amount it stretches. Now suppose you put two such springs in parallel, and pull with a force of 50 lb. Then there is a force of 25 lb acting on each spring, so they each stretch by 1 inch. The two springs in parallel are twice as stiff as only spring since they stretch only one inch at 50 lb, whereas one spring alone stretches by 2 inches at 50 lb. Similarly, two identical tennis strings in parallel would be twice as stiff as one string alone, if the strings behaved as linear springs. In fact, it turned out that strings A and B (i.e., two A strings in parallel) had about the same stiffness, which means that the strings were not linear. For string A, one can infer that the dynamic stiffness at a tension of about 48.4 lb is a factor of 1.7 larger than the dynamic stiffness at a tension of 24.2 lb. If it were a factor of two higher, then the dynamic stiffness of strings A and B would have been the same. If the stiffness at 48.4 lb was the same as the stiffness at 24.2 lb (i.e., the strings were linear) then string B would be twice as stiff as string A.

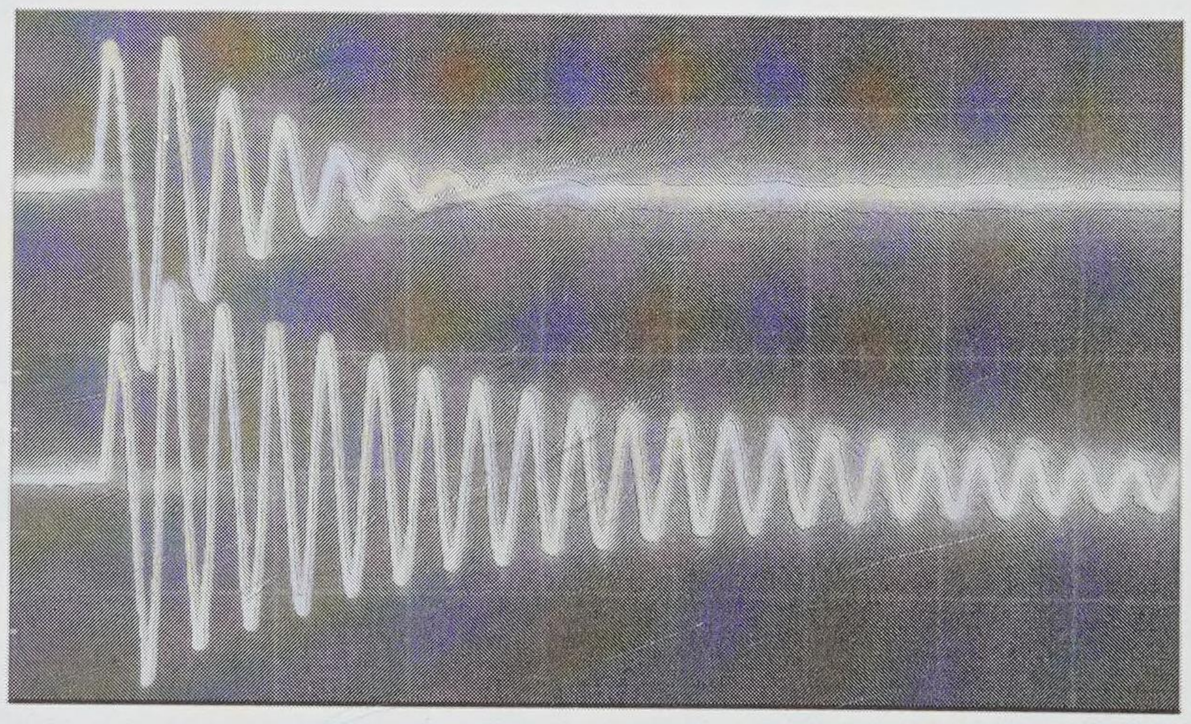
Conclusions

A surprising result of testing thin and thick versions of commercially available tennis strings is that thin strings can be dynamically stiffer and they can maintain tension better than the corresponding thick versions. In this chapter, it has been shown how this can arise. Two linear strings connected in parallel will be twice as stiff as one. Tennis strings are nonlinear and have the property that the stiffness increases with tension. Consequently, the stiffness of two parallel strings can be larger, smaller, or the same as that of a single string depending on the stress vs strain curve and the operating tension. Similarly, the rate at which a string loses tension is a nonlinear function of tension. The rate of tension loss increases with tension, but the loss rate may not increase in proportion to the tension. As a result, two strings in parallel can lose tension at a faster rate than a single string.

PART FOUR

BALLS & COURTS





Tara Snyder, United States

36

The Tennis Ball

Rules and Types

By Howard Brody

Why should you care about the tennis ball? Both you and your opponent are going to be using the same ball. How can it possibly give you an advantage over your opponent? How can a different ball change your enjoyment of the game?

Let's start off by assuming that the balls you are using meet all the International Tennis Federation (ITF) and USTA specifications as to rebound, compression, size, weight, etc as spelled out in Rule 3 and Appendix 1 of the Rules of Tennis. Then what choices do you have? Each manufacturer has several lines of balls of different quality, with names like "Championship," "ATP," "Grand Prix," etc. Within each brand and line there may be several types of balls, such as "Heavy Duty," "Regular Duty," "All Court," "Hard Court," etc. The extra duty or heavy-duty balls are designed to last longer on abrasive courts, while the regular duty balls are usually designed for softer courts.

For recreational use you may use any type of ball on any type of court and be within the rules of tennis if the balls are ITF approved (it will say ITF or USTA Approved on the can or box).

Pressurized and Pressureless Balls

Most balls are fabricated with an internal air pressure that gives them their bounce and feel when you hit them. Unfortunately, the rubber walls of most balls are slightly permeable to air, so balls are sold in pressure cans, to keep the internal air from leaking away and the ball going dead while the can might is sitting for months on the shelf in the store. Once the can is opened and the balls are exposed to normal atmospheric pressure, the air begins to seep out through the walls. The balls then begin to lose their bounce and after a period of time become soft and "dead."

There are also some balls that have an internal air pressure that is the same as the ambient air pressure (they are called pressureless or non-pressurized). Therefore, the air cannot leak out and cause the ball to go dead. This latter category gets its bounce, stiffness and resiliency predominantly from its rubber shell. Pressureless balls are popular in Europe where balls, because of their high cost, are expected to last for many weeks.

Note that all balls that meet ITF and USTA standards are comparably hard when you slowly compress them. At present, there are no rules on the dynamic stiffnesss or hardness of a ball. (In a dynamic situation, the ball is compressed quickly, as if it were being hit.) You can easily tell which balls are pressureless, since they are usually sold in a cardboard box and not in a pressure can. Tretorn has a hybrid ball that has material other than air inside of the ball to give it some of its dynamic properties.

High Altitude Balls

In addition, there are high altitude balls. There are two separate and distinct reasons why pressurized tennis balls play differently at high altitude, compared to the way they play at or near sea level.

1.) The coefficient of restitution (COR, or how lively the ball is when hit or when it bounces) of a pressurized tennis ball depends on the internal pressure relative to the outside (atmospheric or ambient) pressure. At high altitude, the outside, atmospheric pressure is reduced. This results in an increase in the pressure difference between the internal pressure of a normal tennis ball and atmospheric pressure. The larger this difference in pressure, the greater will be the COR (the ball will be livelier). This increase in COR will affect both the bounce of the ball on the court (it will bounce higher) and the speed of the ball as it comes off of the strings (it will give you more power).

2.) The flight of the ball through the air (its trajectory) is influenced by the force of air resistance. The more air molecules the ball runs into, the more the ball will slow down. At high altitude, the ball will run into less air molecules (the air is thinner), so it will not slow down as much (the ball will "fly" a bit more).

Reducing the internal pressure in a ball (so that it does not bounce as high when dropped at sea level) will produce a ball that, at high altitude, has a COR that is similar to the COR of a regular ball at sea level. This reduction in bounce height is presently part of Rule 3 of the Rules of Tennis.

Using a larger ball at high altitude will result in an air resistance force closer to the force encountered by a standard ball at sea level and should lead to ball trajectories that are closer to the ones observed at sea level.

It then makes sense that the Rules of Tennis (Rule 3 specifically) be modified to recommend that a larger pressurized ball (Ball Type 3) with reduced rebound height be used when tennis is played at high altitude. If such a ball is not available, then a larger ball (Type 3) with normal internal pressure can be used. If non-pressurized balls are to be used, it is recommended that they be Type 3 balls, if available.

The official ITF Rules of Tennis specify many of the ball's parameters, including the diameter, rebound height (when dropped from 100 inches) and the static compressibility. Some players who play on slow, soft courts (such as red clay) prefer harder tennis balls (less compressible) because they tend to play faster (not lose as much speed on the bounce). Starting in January of 2002, the ITF is allowing a ball that is harder than the standard ball to be used in tennis. These balls are called Type 1 balls.

Big Balls

Many players and spectators feel that the game of tennis has gotten too fast and the serve is becoming too dominant, particularly on fast courts, such as grass. The proposed solution to this problem is to allow the use of a larger tennis ball (6 percent greater diameter) on fast courts. The larger ball will hit more air molecules in its flight, increasing its air resistance, and slowing the larger ball down more than a standard size ball. These larger balls can be used on any court in recreational tennis and are intended for men's professional tournaments on fast surface courts such as grass. These lager balls are called Type 3 balls.

There is an interesting sidelight to this larger ball idea. When you watch some of the top players (such as Pete Sampras) prepare to serve, they ask for several balls, then reject all but one. On what basis are they selecting that ball? They know that a ball whose cover is fluffed up acts as if it had a greater cross section. It will have more air resistance and will slow down more in flight, giving their opponent more time to get to and return their serve. A ball with a felt cover that is somewhat matted down will get to (or past) their opponent faster, which they feel gives them a competitive advantage. In addition, all tennis balls are not exactly the same size to begin with. The Rules of Tennis allow a little over a tenth of an inch variation or latitude in the diameter of an approved tennis ball. So the players select the ball, which to them, appears to be smallest, and will play the fastest on their serve.

When tennis balls are placed in a wind tunnel and their air resistance measured, the results depend very much on the fuzziness and condition of the felt cover on the ball. (In fact, before one test, the ball was shaved with an electric razor.) As the ball fluffs up, then begins to wear down, clear changes in the ball's performance can be noted. That is why, in major tennis matches, the balls are changed every nine games. However,

Air Resistance And The Big Ball

The force of air resistance is determined by the number of air molecules that the ball runs into, which is proportional to the density of air (the number of air molecules/cubic centimeter). This density of molecules is linearly dependent on the air pressure, and therefore both the density and pressure are the same function of altitude. The pressure and the density are proportional to the sea level value multiplied by a function of the altitude. This function is:

e to the negative mg/kT multiplied by the altitude h, where e is the base of the natural logarithm and equal to 2.8 .

$$e^{-(mg/kT)h} \qquad (36.1)$$

When the values of m (the mass of a molecule of air), g (9.8), k (Boltzmann's constant) and T (degrees Kelvin) are put in, the result is e to the negative height, h, above sea level (in kilometers) divided by 9.

$$e^{-(h/9)} \qquad (36.2)$$

This means that at a height of one (1.0) kilometer = 3281 feet, the density (pressure) will be $e^{-1/9} = 0.895$ times the density (pressure) of the sea level value. At a height of 2.0 km (6562 feet) the density will be $e^{-2/9} = 0.80$ times the density of air molecules at sea level.

A ball that has a diameter 6.5% greater than a standard ball (13.4% greater cross sectional area) will have the same air resistance force on it at a height of 1.13 km (3700 feet) as a standard ball at sea level.

$$e^{-(1.13/9)} = 1/1.134$$

A 6.5% larger ball will then have the same trajectory at 3700 feet as a standard ball would have at sea level.

(All of the above assumes that the temperature is the constant with altitude)

there are no rules in the Rules of Tennis concerning the aerodynamic properties of the felt cover on a ball, other than it be a fabric.

When the bounce of these different type of balls is examined in actual play using high speed photography, it is discovered that the harder balls (Type 1) tend to bounce a bit lower than the standard (Type 2) balls and the larger balls (Type 3) tend to bounce a bit higher than the standard (Type 2) balls.

Mats Wilander, Sweden

37

Ball Testing for Bounce

Testing Ball Properties for Perfect Bounce

By Rod Cross

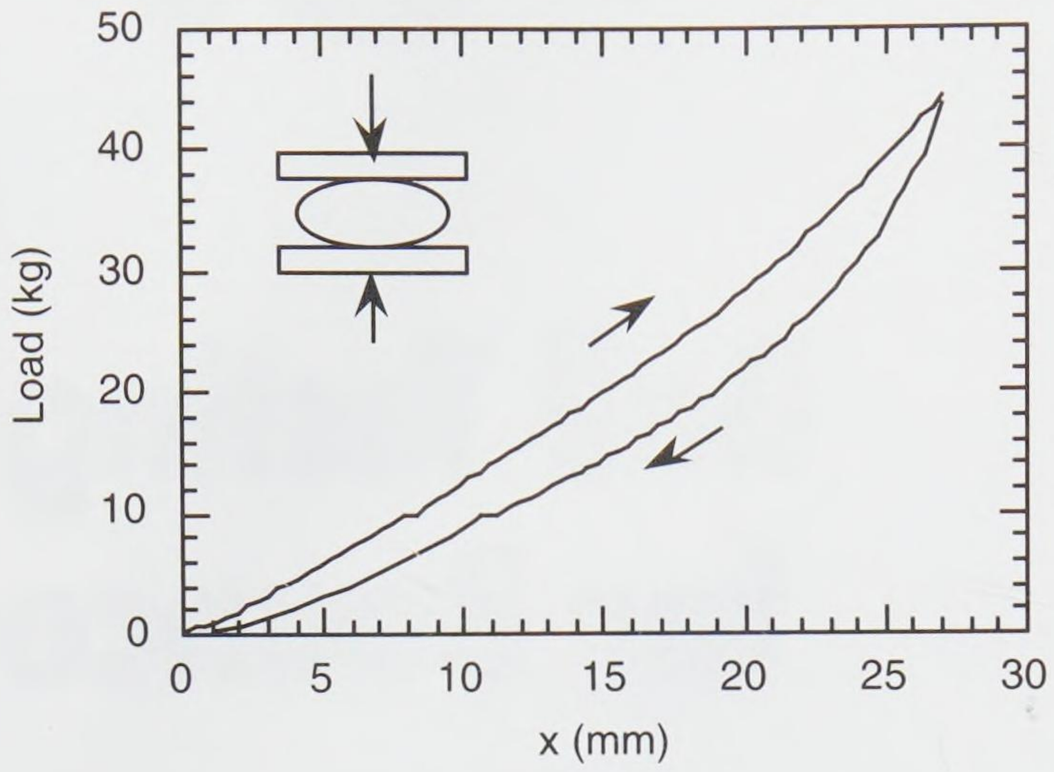
The rules of tennis are very specific about the properties of a tennis ball. In addition to having the correct weight and diameter and color (white or yellow), a ball must bounce to the correct height when dropped onto a concrete slab from a height of 100 inches. Furthermore, an approved ball must squash by 1/4 of an inch ± 0.04 inch (6.5 mm ± 1 mm) when subject to a load of 18 lb (8.2 kg). If the ball squashes more than the specified amount, it will be too soft and it won't bounce very well. If it squashes less than the approved amount it will be too stiff and it will bounce too high. Given all these restrictions, one might expect that a ball will behave in a very predictable manner. In fact, a tennis ball is full of interesting surprises as described below.

Compression Test

The modern way to test the stiffness of a ball is to place it between two parallel metal disks and to push the disks together using a materials testing machine. These days, such machines are computer controlled and cost about $10,000 or more. The machine compresses the ball at a controlled rate and gives a plot of force vs compression. A typical result is shown in Figure 37.1. The plot is not a straight line since a tennis ball gets stiffer the more it compresses. Furthermore, when the force applied to the ball is reduced to zero, the decompression curve lies below the compression curve. The machine used to compress the ball was instructed to pause at 10 kg for 5 seconds on the way up (10 kg is the specified 8.2 kg load plus a specified preload of 1.8 kg) and to pause at 10 kg for 10 seconds on the way down. This is written into the rules to give the person using the machine suf-

ficient time to take a reading. The rule is no longer strictly necessary since a computer can take readings 100 times a second if it is instructed to do so. In the old days, the reading was taken with a manually operated Stevens machine (and is still done so in some countries), but the rules haven't yet caught up with modern technology.

Figure 37.1



The curve shown in Figure 37.1 is a typical hysteresis curve, meaning that the curve for an increasing force is different from the curve for a decreasing force. At a load of 10 kg, the compression is 8 mm on the way up and 10.5 mm on the way down (after the specified waiting times). This ball passed the test since it compressed by the approved amount (6.5 mm ± 1 mm) when the load increased by 8.2 kg (from 1.8 kg to 10 kg). It also passed the specified test on the way down.

The area between the two curves represents the energy lost in the ball. The area between the top curve and the horizontal axis represents the work required to compress the ball, and it therefore represents the energy stored in the ball at maximum compression. The area between the bottom curve and the horizontal axis represents the energy given back by the ball. The difference is the energy lost. The area under the top curve in Figure 37.1 is 5.03 Joules. The area under the bottom curve is 3.87 Joules. The energy lost is therefore 5.03 - 3.87 =1.16 Joules, which represents 23% of the stored energy (5.03 J).

There is an interesting problem with this result, since it is not consistent with the 100 inch bounce test. When a tennis ball is dropped from a height of 100 inches it must bounce to a height between 53 and 58 inches. The height of a ball above the ground is a measure of its potential energy. Consequently, if the ball bounces to 55% of its drop height it loses 45% of its energy during the bounce. Therein lies the problem. The compression test indicates that a ball should lose only 23% of its energy when it squashes. If it lost only 23% of its energy during a bounce it would bounce a lot higher than 58 inches.

Slow vs Fast Compression

One difference between the drop test and the squash test is the time taken to squash the ball. When a ball is dropped on a hard surface, it spends only about 0.0045 seconds in contact with the surface. The ball is squashed over a period of about 30 seconds when measuring its compression. Another difference is that the ball contacts only one surface when it is dropped, but it is squashed between two surfaces in a compresion test. A third difference is that the compression test squashes the ball by just over one inch, but a ball dropped from 100 inches compresses by about 3/4 inch.

The behaviour of a tennis ball during a rapid compression is not the same as that for a slow compression. The force and compression can be measured during an actual ball bounce by dropping the ball on a pressure sensitive plate (Cross, 1999). Typical results are shown in Figure 37.2 (force vs time) and Figure 37.3 (force vs compression) for a ball incident vertically downwards at

5.2 m/s or 13.5 m/s onto a hard surface. The surface was a ceramic piezoelectric block which generates a voltage proportional to the force on the block. The hysteresis curve for a rapid compression is much fatter than one for a slow compression, meaning that the ball loses more of its stored elastic energy when it compresses and decompresses rapidly.

The results in Figures 37.2 and 37.3 are typical of all tennis balls. There are only very slight differences between different balls. At low ball speeds, the area enclosed by the hysteresis curve is about 35% of the maximum stored energy and the ball bounces to about 65% of its drop height. As the ball speed increases, the peak force on the ball increases, the impact duration decreases and the ball loses a greater fraction of its impact energy.

The main reason for the increased energy loss is that a tennis ball is much stiffer when it is compressed rapidly and hence more work is needed to compress it, and more energy is stored in the ball for any given compression. However, the wall of the ball buckles after a short time delay and the ball then becomes softer. When the wall buckles it loses a lot of its stored elastic energy. The wall also buckles during a slow compression but it buckles at a much earlier stage when there is very little elastic energy stored in the ball.

During a rapid compression a small region of the ball in contact with the surface compresses locally, with the result that the force on the ball increases by up to half its maximum value before the wall starts to buckle. It is much harder to compress a piece of rubber than to bend it, and the more it is

Figure 37.2a

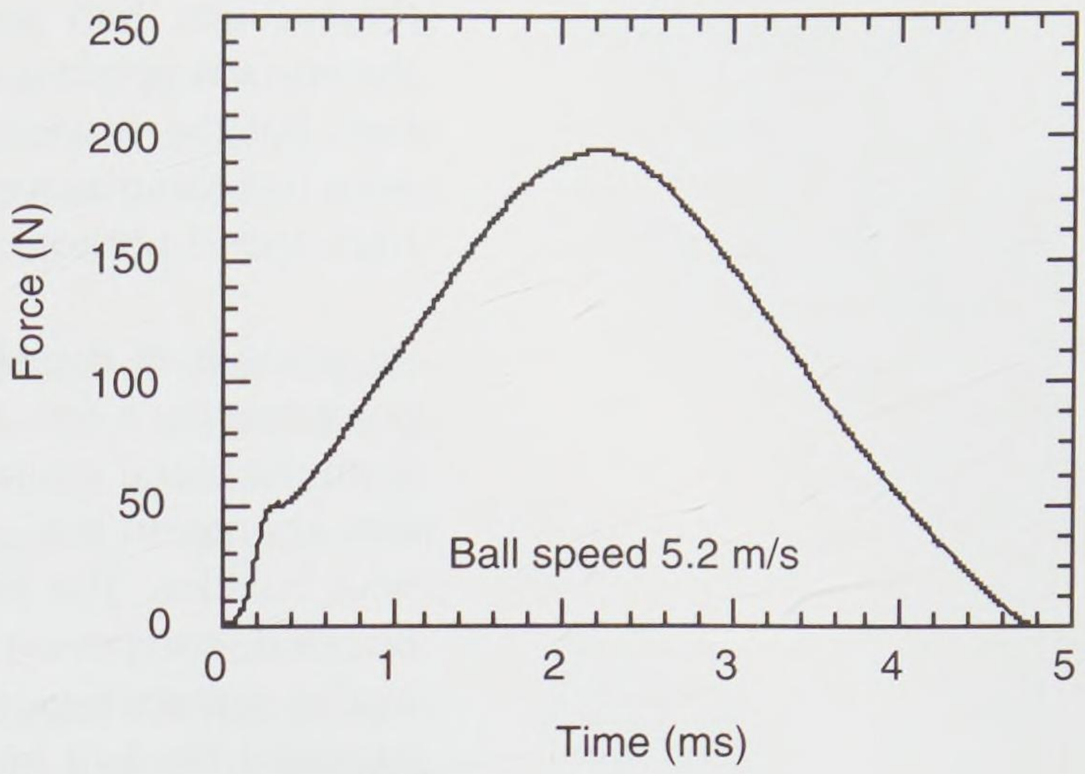


Figure 37.2b

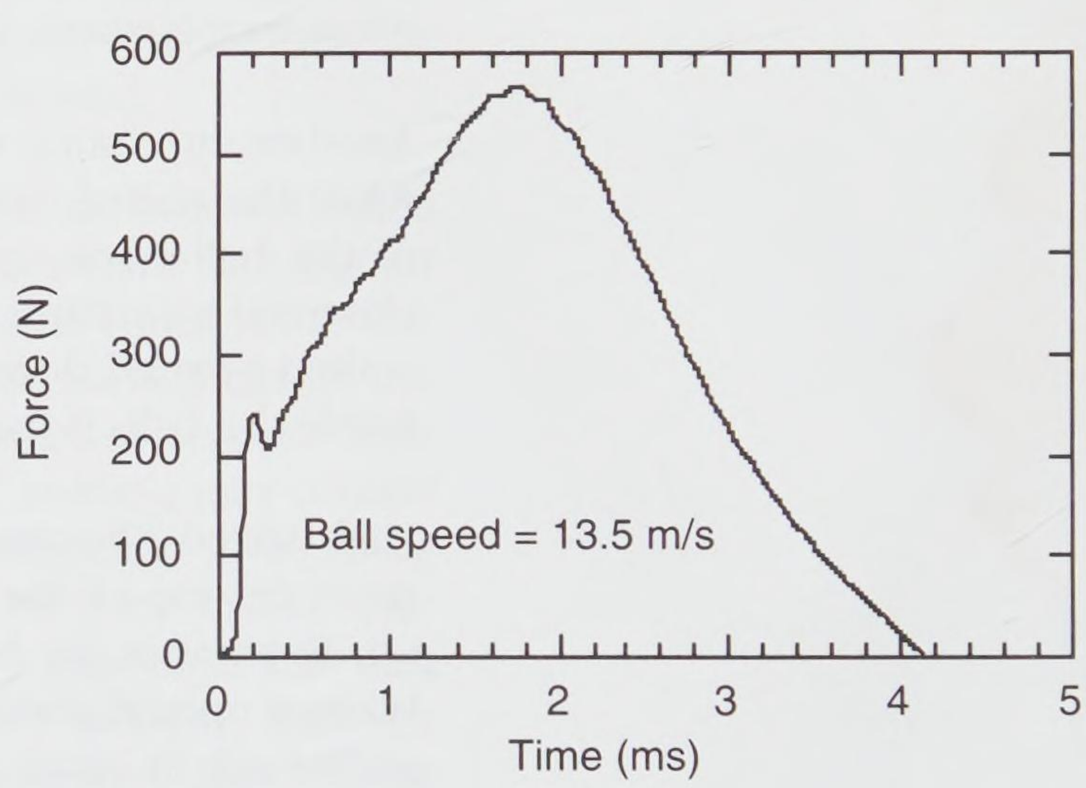


Figure 37.3a

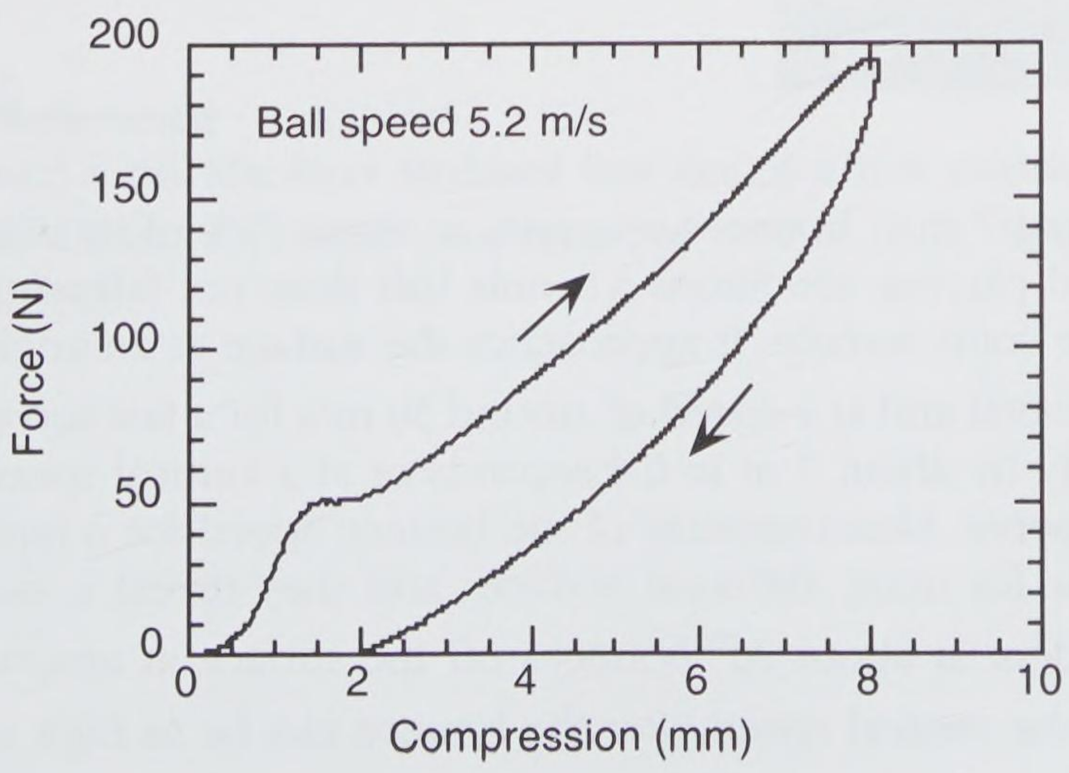
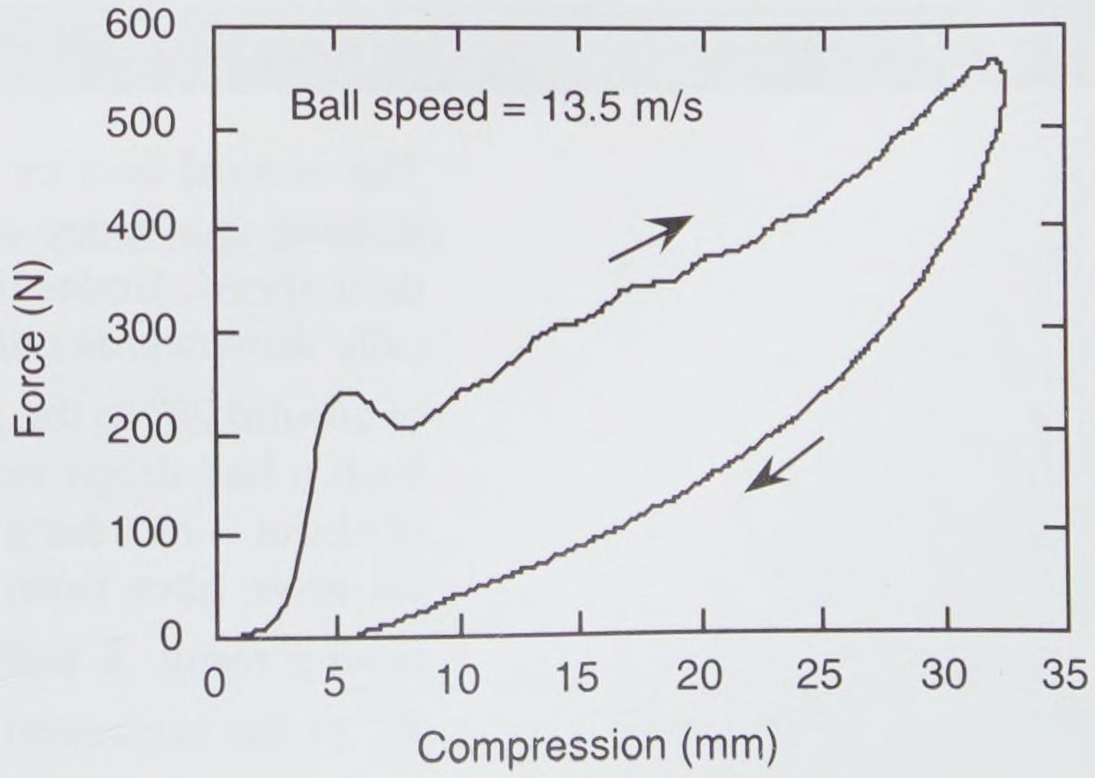


Figure 37.3b



compressed the stiffer it gets. As a result, the force on the ball increases exponentially with time during the first 0.2 ms of the bounce. The ball is about 6 times stiffer than normal just before the wall buckles. The initial contact with the surface sends a transverse bending wave around the circumference of the ball, but the wave travels relatively slowly around the ball. The wave can be seen travelling around the ball using a small piezo disk glued to the ball, or by high speed photography.

As a result of the slow wave speed, the rubber wall in the contact area can compress by a few mm before the wave travels any significant distance away from the initial contact point. As shown in Figs. 37.2a and 37.2b, there is a sudden change in ball stiffness after about 0.2 ms, which is the time at which the wall buckles. The ball incident at 13.5 m/s travelled 2.7 mm before the wall buckled, compressing the rubber and cloth to about half its original thickness in the contact region. During a slow compression, this time delay is insignificant and the ball buckles at a much lower force and at a much smaller value of the ball compression. This effect can be seen by comparing Figs 37.2a and 37.2b. At the lower ball speed, the wall buckles when the force on the ball is 50N. At the higher ball speed, the wall buckles when the force on the ball is 230N. Buckling occurs not at a fixed value of the force but after the bending wave travels about 1 cm away from the initial contact point.

Another interesting effect is that the bottom of the ball can bounce upwards while the rest of the ball is still coming down. As a result, the bottom section of the ball pops up inside the ball like a small bubble. This effect can be observed by cutting a tennis ball in half with a hacksaw. If one half of the ball is then pressed down on a hard surface by hand, the contact region pops up inside the ball. If the ball is pushed hard enough, it turns itself inside-out.

High speed photography shows that the top of the ball also forms a dimple since the top of the ball continues downwards while the sides are bouncing up. Even after the ball has bounced up off the surface, the top continues to bounce up and down for a short while, as the dimple pops out of the ball and pops back in again at a rate of about 500 cycles per second (two cycles during the 4 ms impact period). This generates a short ping which is part of the sound generated by the ball when it bounces. Some balls vibrate more than others and sound different when they bounce, but all approved tennis balls bounce to essentially the same height and take essentially the same time to bounce off the surface, regardless of whether they are the pressurised or pressureless variety.

Bounce of Spinning Ball

The vertical bounce test shows that a tennis ball incident vertically on a hard surface, at a speed of about 7 m/s, bounces upwards at about 75% of its incident speed. Under normal playing conditions a tennis ball does not fall vertically downwards onto the court surface. It approaches the surface at an angle of around 20° to the horizontal and at a speed of around 30 m/s for a fast serve. Such a ball drops vertically by about 3 m in 0.4 seconds or at a vertical speed of about 7 m/s for a fast serve. Measurements of the bounce speed for a typical serve have been made for many different surfaces and they reveal a surprising result. A ball incident at about 20° bounces off the surface at around 20° to the horizontal but the vertical speed after the bounce can be as high as

90% of the vertical speed before the bounce. In other words, the ball behaves more like a superball than a tennis ball in terms of its vertical bounce properties. The ball bounces much higher than one would expect from the standard 100 inch drop test. At 7 m/s, a ball should bounce to a height of about 55 inches or about waist height. In fact, some balls incident vertically at around 7 m/s bounce to head height. This is a typical result for a kick serve with lots of topsin.

There is as yet no generally accepted explanation for the higher than expected bounce. One possibility is that a ball may sweep some sand ahead of it to form a small ramp and is then deflected upwards by the ramp. However, high bounces are also observed on hardcourts without any sand. In that case, ball spin may supply an explanation, as follows.

A ball bouncing at an oblique angle bounces with topspin even if it has no spin before it bounces. If the ball is served with topspin then the amount of topspin increases after the ball bounces. For example, a ball can be served so that it spins at about 60 revolutions/sec before it hits the court, and it will then spin at around 100 revolutions/sec after it bounces. The spin increases from 60 to 100 rev/sec during the 4.5 ms time that the ball is in contact with the court surface. If we assume that the average spin during this time is 80 rev/sec then the ball rotates by 80 x 0.0045 = 0.36 revolutions during the bounce.

The point on the bottom of the ball that first contacts the surface will rotate during the bounce so that it is nearer the top of the ball when the ball lifts off the surface. That point will start to squash when the ball first contacts the surface but it will not squash very far because it rotates off the surface before the wall has had time to collapse. As shown above, the wall starts to collapse after 0.2 ms. As the ball rotates, each new section of the ball coming into contact with the surface will also take 0.2 ms to collapse. Consequently it might be possible for the ball to keep rotating and not collapse at all if the ball is spinning fast enough. The ball is very stiff before the wall collapses. If the wall does not collapse at all, or if it does not have time to fully collapse, then the ball will compress by only a small amount and it will lose very little energy during the bounce. A spinning ball might therefore bounce much higher than a non-spinning ball. Detailed measurements would need to be made to see if this explanation is correct and whether it applies to all court surfaces or only some surfaces. An alternative explanation for the high COR is described in Chapter 39.

Reference

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Monica Seles, United States

38

The Bounce of the Ball

An Introduction

By Howard Brody

The bounce of a tennis ball on a tennis court is a complicated physical situation, but by making some simplifying assumptions, the problem can be easily analyzed. These assumptions include keeping the ball a hollow sphere (no deformation), having the court hard (no court deformation), and having the friction force (the force on the ball parallel to the surface) obey the simple law of friction, as typically taught in an introductory physics course.

When a ball interacts with a court surface by bouncing, there are three forces acting on the ball, a normal force (perpendicular to the surface), a friction force (parallel to the surface), and the force of gravity. When the forces involved are examined, it turns out that the normal and friction forces are at least an order of magnitude greater than the force of gravity on the ball, so the gravity force can be neglected during the few milliseconds that the ball is in contact with the court's surface. The ball's motion is then broken up into a normal or vertical velocity and a parallel or horizontal velocity. The problem is then solved for each motion separately, and then the velocity components are recombined to get a final result.

Vertical Velocity

When a tennis ball bounces on a court, it does not return to its original height. In fact the Rules of Tennis state that when a tennis ball is dropped from a height of 100 inches onto a hard (concrete) surface it must bounce

to a height between 53 and 58 inches. This ratio of heights (between 0.53 and 0.58) is the ratio of energy returned to the ball after the bounce to the energy the ball had just before it hit the ground. There is a term called the coefficient of restitution (COR), first used by Newton and Wren in the sixteen hundreds, that can also be used to describe the collision of the ball with the ground. The COR is the ratio of the ball's vertical velocity just after hitting the ground to its vertical velocity just before its impact with the ground. The COR is the square root of the rebound height ratio, so for a ball to be approved, the COR must be between .728 and .762 (which is the square root of 0.53 and 0.58). Note, this 53 to 58 inch return is onto a hard surface and for a vertical drop of 100 inches only. There are other surfaces that may produce a different rebound height for a "legal" ball, since some surfaces are dead, and absorb energy. If the ball is dropped from less than 100 inches, the COR will be slightly higher. In the analysis that follows, we will make the assumption that the COR of a ball is the same whether it is just dropped or whether it is rebounding at an angle as part of its trajectory and it has a horizontal velocity as well. In other words, we are assuming the rebound height ratio and the COR of a ball are independent of the horizontal velocity of the ball.

When a tennis ball hits the ground and rebounds back upward, the direction of the ball's velocity reverses, and a force is required to do this. The actual change in velocity is the difference between the incident and rebounding velocities, and it is equal to the incident velocity times (1 + COR). The ball has a momentum equal to its mass times its velocity, and Newton's Second Law says that an impulse (force x time) is required to change the momentum of an object. The magnitude and direction of the impulse is equal to the change in momentum of the object. For a bouncing ball, the ground produces a vertical force (called the NORMAL force) equal to the change in the ball's vertical momentum (which is mv(1 + COR)) divided by the time of the ball's contact with the ground.

Horizontal Velocity

When a ball bounces, there is a frictional force parallel to the court on the bottom of the ball. The rougher the court surface, the larger is the friction force. The roughness can be characterized by a parameter called the coefficient of friction (COF). The magnitude of the friction force is also proportional to the magnitude of the normal force. The bigger the normal force, the larger is the frictional force. Combining these two effects gives the actual friction force on the bottom of the ball, and that friction force is equal to the COF times the normal force (COF x N). The change in the ball's horizontal momentum will be equal to the frictional force multiplied by the contact time of the ball on the surface (the horizontal impulse). When you put this together with the formula for the Normal force in the previous section, you get that the change in the horizontal velocity of the ball, due to its bounce, is COF(1 + COR) times the vertical velocity component of the ball before the bounce.

Since there is only one horizontal force on the ball when it is in contact with the court surface, and that force acts on the bottom of the ball, there is a net torque on the ball. This torque caused by the ball skidding or sliding on the surface, produces a topspin on the ball. The result is that most shots, after the bounce are found to have a fairly large amount of topspin on them. If the ball strikes the ground at a large angle (the ball has cleared the net by a huge margin) and the COF is high, it is possible for the ball to begin rolling before it

leaves the surface (see Note 1). Once the ball begins to roll instead of sliding or skidding, the frictional force is greatly reduced. When a ball goes into a biting or rolling mode, it will have lost 40% of its original horizontal velocity component. On a typical serve, the ball will lose about 30% of its original velocity component parallel to the ground.

Fast Courts — Slow Courts

Think about the slow clay of Roland Garos or the fast grass at Wimbledon. What is it that makes these courts fast or slow? How does this "speed" change the way you play your game?

It is quite clear that the only way a court can influence the speed of the game (the speed of the ball) is for the ball to bounce. There are several very different effects that influence the ball when it bounces. The ball may pick up bits of clay and become heavier. This will slow down the game because a heavier ball comes off of the racquet with less speed, but the more massive ball will slow down less due to air resistance. These two effects tend to cancel each other out. The bounce may fluff up the ball a bit and therefore it will have more air resistance. This will slow down the speed of the ball in its flight. However, these are not the effects that are normally considered when the causes of fast or slow courts are discussed.

It is the friction between the ball and the court that determines the speed (or PACE) of a court. If the friction force is small, then the ball will lose little of its horizontal velocity component when it bounces, and you will have a fast court. When the friction force is large, the ball will lose a larger fraction of its horizontal velocity component, and the court will be considered slow. If the COR (rebound height) is the same for both a fast and slow court, because the horizontal components of the velocity will be different, on a fast court the ball will rebound with a lower angle than the angle it comes off at on a slow court. It is also clear that on a fast court, the player will have less time to get to the ball and prepare for a shot than on a slow court.

Surprisingly, on ground strokes leaving your opponent's racquet at 60 miles an hour, the ball will arrive at your baseline only 0.07 seconds earlier on a fast court compared to a slow court. On a 110 mile an hour serve, the difference in time between a ball coming to you on a clay court and a grass court is just a few hundredths of a second! This seems to fly in the face of all that you thought you knew about fast and slow courts. How can these sort of numbers agree with our idea that there is a perceptible slowdown in play on clay courts? Why is it that when you work out the mathematics, the ball may be moving 18% slower when it reaches the baseline on a slow court than on a fast court, yet it takes only 5% longer to get there?

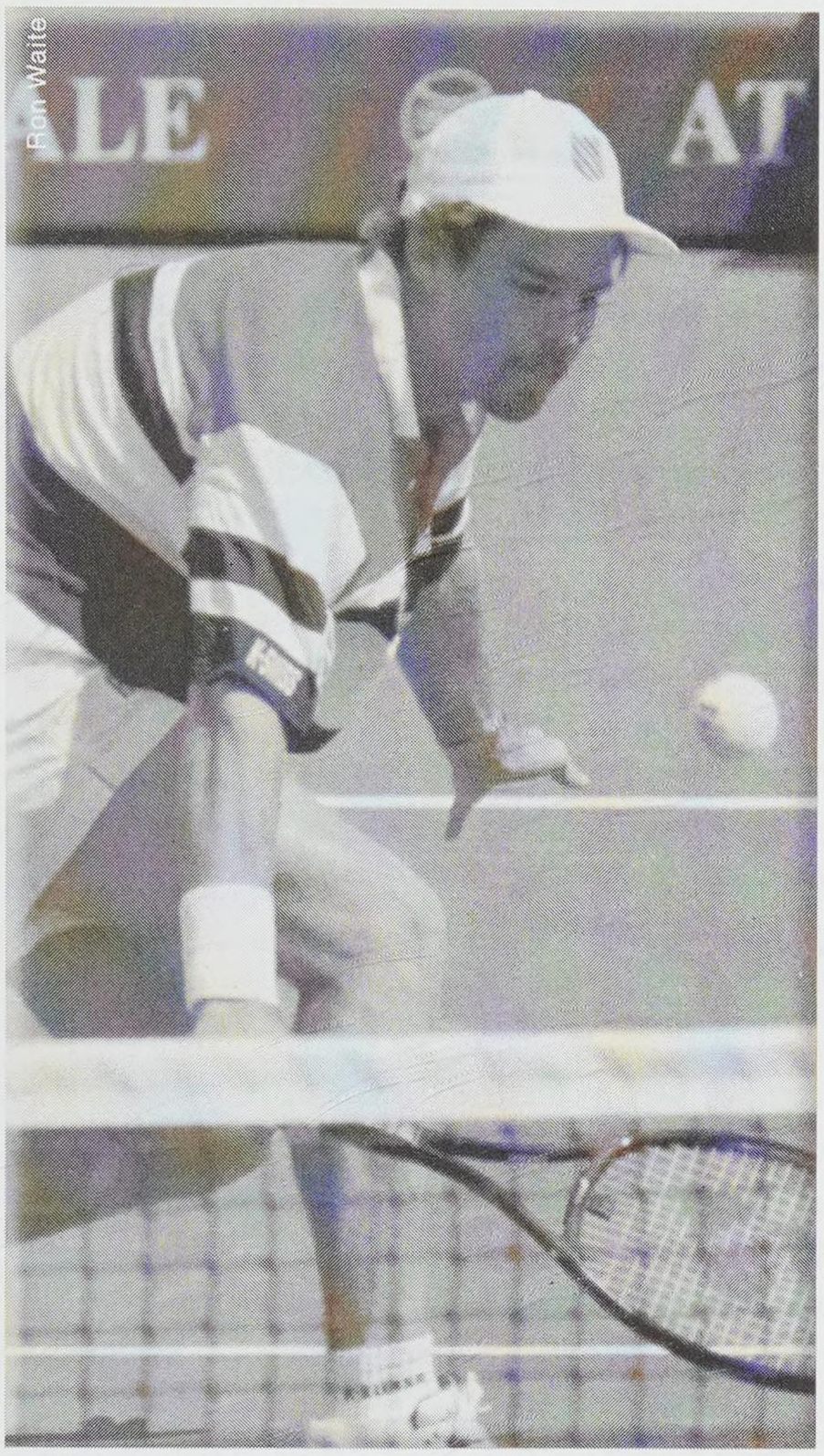
In going from baseline to baseline, the ball spends most of its transit time in the air before it bounces. Any slowdown due to the bounce can only affect the ball during the very short distance that the ball moves between the bounce and the baseline. Therefore, an appreciable loss of speed at the bounce will not lead to an appreciable increase in the time it takes the ball to get to the player.

This still does not seem to answer the question of why the entire game seems to slow down on clay courts? To understand this, we must investigate the ball-

racquet dynamics. For a groundstroke, the speed at which a ball leaves the racquet is determined not only by the racquet head speed but also by the speed of the incoming ball. When you work out the physics of the interaction, about 25% of the outgoing ball speed is determined by the speed of the ball coming toward the racquet. When a ball bounces on clay, there is an appreciable loss of speed, so the return of that shot will be reduced in speed (unless the player makes a very conscious effort to swing the racquet much harder). The result is that on a clay court, with the players swinging with their normal strokes, the ball speed BEFORE the bounce is reduced, compared to a fast court. This leads to a general slowdown of the game. The shots are leaving the racquets at lower speed and then slowing down more when they bounce.

Notes

1. Balls do not roll when they bounce since they are not perfectly rigid. Instead, the bottom of the ball bites the surface. However, biting and rolling are almost the same for a tennis ball. The differences are explained in Chapter 39.



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Ball Spin and Bounce Off the Court

Friction, Sliding, Rolling, Biting and Spinning Balls

By Rod Cross

Summary

The main object of the game of tennis is to hit the ball over the net and into the opponent's court in such a way that the opponent finds it difficult or is unable to return the ball. The ball needs to bounce at the right speed, angle and spin, not only off the racquet, but also off the court. Players learn how to do this by trial and error and years of practice and coaching. In this chapter, we examine the main factors that determine the way a ball bounces off the court from a physics point of view.

Introduction

Tennis players rarely stop to think about what happens during the brief instant when a ball bounces off the court or off the strings. They just let it happen, and they know from experience what happens when they hit the ball a certain way. The ball spends only 0.005 seconds on the court or on the strings while it is bouncing, so there is nothing the player can do during or after that time to alter the fate of the ball. In that respect, the bounce is predetermined. During a bounce, the ball changes speed, it changes direction and it changes the rate at which it spins. All these things change very rapidly. For example, a ball served at 100 mph accelerates from 0 to 100 mph in 0.005 seconds or about two thousand times faster than a sports car. Interesting things happen under such

extreme conditions. The force on a ball served at 120 mph is enough to lift two people off the ground; it is enough to squash the ball in half; and it is enough to stretch the string plane backwards by an inch or more. It happens so fast that it is impossible to see by eye, but it can be captured and measured using the right instruments.

When a ball bounces off a surface, the ball always reverses its direction of motion perpendicular to the surface, and it usually slows down in a direction parallel to the surface. For example, a ball headed downwards at an angle to the court bounces up and away from the surface. If it is traveling from left to right before it bounces, it usually keeps traveling left to right after the bounce at a reduced speed. A ball with lots of backspin can sometimes bounce backwards, which is an extreme example of slowing down in a direction parallel to the surface. A ball with lots of topspin can sometimes shoot forwards at a greater speed than it had before the bounce, but this is a rare event. It can happen in a topspin lob but it never happens with a serve or a normal groundstroke.

Motion of the ball in a direction perpendicular to a surface is easy to explain. On the court, the ball just squashes and pushes itself back off the surface as it expands back to its original shape. It is actually the surface that pushes on the ball, but the ball pushes with an equal and opposite force on the surface. It is also easy to explain why the ball slows down in a direction parallel to the surface. It is due to the fact that friction acts backwards on the ball while the ball is moving forwards in contact with the surface. All objects moving along a surface experience a friction force that acts backwards on the object. It is for this reason that any object that slides or rolls along a horizontal surface eventually comes to rest if it is moving under its own steam (without an onboard motor to keep it going).

The most difficult part of explaining the bounce of a tennis ball involves estimating or calculating the magnitude of the friction force. Most things are easy to explain in words. But scientific explanations involve more than just words. In order to determine if the explanation is correct, it has to be tested against calculations and measurements. The problem here is that there are three different types of friction:

- Sliding friction acts only when an object is sliding.
- Rolling friction is much smaller and acts only when an object is rolling.
- Static friction acts only while an object is at rest on a surface and when some other force is applied in an attempt to push or pull the object along the surface.

If you try to push an elephant along the ground, the elephant will remain at rest due to static friction between its feet and the ground. But you can push a car along a smooth horizontal surface since the friction force on a rolling wheel is relatively small.

It turns out that when a ball bounces, static friction is sometimes more important than the other types of friction, since the bottom of a ball can actually come to rest while it is bouncing. The bottom of the ball always comes to rest in the vertical direction when the ball bounces, but it can also come to rest in the horizontal direction. In that case, we say that the ball grips or bites the surface. Even though the bottom of the ball comes to a complete stop, the whole

ball doesn't come to a complete stop. If that is hard to visualize, consider what happens to your foot when you are walking. You move one leg forward, plant it on the ground and move the rest of your body forward. Your front foot comes to a complete stop until you lift first your heel and then your toes off the ground. The horizontal force that allows you to walk forwards arises from static friction on your foot. Sliding friction arises when you step on a banana skin.

A ball incident at a low angle on a surface usually slides throughout the bounce. An example is a fast serve without much topspin. At higher angles or when the ball is incident with a lot of topspin, it usually slides at the start and then bites before it bounces off the surface. To complicate matters, one part of the ball can bite the surface while other parts are stretching or compressing and sliding along the surface. If tennis balls bounced in a really simple manner, this chapter would be very short. But balls can do surprising things when they bounce, which gives them some personality and it also makes them harder to explain. An example is a basketball. When a basketball bounces, the friction force at the bottom of the ball changes direction six times during the bounce. When a tennis ball bounces, the friction force sometimes reverses direction once, sometimes twice, and sometimes not at all, depending on if and when it bites the surface.

Ball Spin

If a ball bounces off the court or off the strings of a racquet, it almost always changes its rate of spin. The only case where it doesn't change its spin is when it bounces at right angles to the surface. For example, if a ball is dropped without spin onto the court or onto the strings of a racquet, then the ball will bounce without spin. If the ball is thrown or hit onto the court without spin, but at some angle to the surface, then it bounces with topspin. The faster the ball is incident on the court, the higher it bounces and the faster it spins. An interesting question is how one can make the ball spin as fast as possible. One way is to throw or hit the ball as fast as possible, but what is the best angle to throw it, and does the angle depend on whether the surface is fast (like grass) or slow (like clay)? And what angle is needed to get the most spin off the strings? Topspin is imparted when the racquet head rises upwards to contact the ball. An insect on the strings sees the ball coming in at an angle and going out with a different spin than it had coming in. The question is, can one get more spin when the head rises at say 30 degrees to the horizontal, or is 45 degrees better and does it make much difference if the head is tilted forwards? These questions are answered in Chapter 41. In this chapter, we concentrate mainly on the bounce off a court.

Friction Force

A ball incident at some angle on the court will usually bounce with reduced speed. The speed reduction is typically between 30% and 60%. For example, a ball served at 100 mph will slow down through the air and hit the court at about 75 mph. It then bounces off the court at about 50 mph. If it had no spin when it hit the court, then it will bounce with topspin, spinning at about 6,000 rpm or 100 revolutions per second. The speed reduction and the extra spin both arise from the same force acting on the ball. The force is due to friction between the ball and the court and it acts on the bottom of the ball. The friction force acts in a direction parallel to the surface, and it acts backwards on

the ball if the ball is moving forwards, slowing it down. The friction force also exerts a torque on the ball, causing it to rotate or to spin. The bigger the backwards force, the more the ball slows down and the bigger the spin. Consequently, the way to get the ball to spin faster is to make sure the friction force is as large as possible. The same is true for spin off the strings. However, the price to pay for extra spin is that the ball does not come off the strings as fast as a ball hit without spin.

The friction force, F, depends on three factors:

- The force, N, exerted by the ball pushing downwards onto the court. That depends on how fast the ball hits the court and it also depends on the angle of incidence. If the ball slams vertically into the court, then N will be large. If the ball is incident at the same speed but at a small angle to the court, then N will be smaller, since the vertical speed of the ball will be smaller.

- The smoothness or roughness of the court surface. That is described by the coefficient of sliding friction COF. For a fast court like grass, COF is about 0.6. For a slow court like clay, COF is about 0.8. For all other courts, COF is about 0.7. The friction force is given by F = COF x N. At least, that is the case if the ball slides along the court while it is bouncing. It comes as a surprise to most people that the horizontal friction force does not depend on the horizontal speed of the ball but it does depend on the vertical speed.

- Whether the ball slides throughout the bounce or whether it bites the court during the bounce.

Sliding means simply that the bottom of of the ball slides along the court. It can do that throughout the whole bounce period or just at the start. If the incident ball has sufficient topspin, or if it acquires sufficient spin during the bounce, then the bottom of the ball can actually come to rest on the court. That occurs if the bottom of the ball spins backwards at the same speed that the whole ball moves forwards. In that case we say that the ball "bites" or grips or sticks to the court. If the ball starts to bite during the bounce, then F drops gradually to zero and then it reverses direction as explained in Chapter 40. It can even reverse direction twice if it starts to bite early in the bounce. This complicates the situation and makes it difficult to calculate exactly what will happen. However, measurements and calculations of bouncing balls have helped to clarify the situation and reasonably good predictions can be made, as described later.

The critical factors that determine whether the ball slides or bites are:

- the angle of incidence
- the coefficient of sliding friction, and
- the amount of spin on the incident ball.

A ball incident on the court at an angle of less than about 16 degrees to the horizontal and without any spin will slide throughout the bounce unless the court is really rough or covered in coarse sandpaper. A ball served at high speed is incident on the court at an angle around 12 degrees to the horizontal, and it slides throughout the bounce. A ball incident at an angle larger than about 20 degrees usually bites the court, especially if it has a lot of topspin. Typical cases where the ball bites are a slow topspin serve or a groundstroke hit high over the net or with a lot of topspin.

When a player hits a ball, the angle of incidence of the ball on the strings is usually around 45 degrees or more. The coefficient of sliding friction with the strings is a lot smaller than that with the court. The ball will bite if the angle of incidence (between the path of the ball and the string plane) is greater than about 45 degrees, provided the ball is not spinning when it hits the strings. In fact, a ball bouncing off the court with topspin is usually incident on the strings with backspin. That makes it harder to hit the ball back with topspin and it means that the ball is less likely to bite the strings. Most players prefer to say that the strings bite the ball rather than the other way around, but the effect is exactly the same in that the region of the ball in contact with the strings stops sliding across the string plane if it bites the strings.

Rolling

When a ball is pushed along the ground it doesn't slide and it doesn't bite, but it rolls. When a ball bounces it can slide or it can bite, but it doesn't roll. However, the effect of biting on a tennis ball is similar to rolling. The effect is much the same when the friction force is small or when the friction force reverses direction towards the end of the bounce, since the average friction force during the latter part of the bounce is close to zero. When Howard Brody first calculated in 1984 how a ball would bounce, he assumed that the ball can either slide or roll, but it wouldn't bite. Howard was correct in assuming that the ball can slide, but he was not correct in assuming that the ball would roll. Nevertheless, it is sometimes a good approximation to assume that a tennis ball rolls, and it makes calculations much easier. In fact, no one has ever calculated what happens when a tennis ball bites. Calculations have been done for a solid ball that bites, but a tennis ball is hollow and that makes an important difference. We therefore have only three choices at present. We can use Howard's bounce model, or we can use the model developed for a solid ball or we can be guided by experimental measurements of tennis balls.

An object that rolls along a surface travels a lot further than an object that slides along the surface, since the friction force for rolling is much smaller. You can roll a tennis ball from one end of the court to the other easily, but you can't slide anything that far. If you slide your racquet along the court, it will probably stop before it gets to the net. You can't slide a tennis ball the whole length of a court either. It might start to slide when you push it, but it will start rolling almost immediately. The same thing happens to a bowling ball in a bowling alley. A bowling ball starts sliding down the alley but it will start rolling before it gets to the end. The reason that a ball can roll so far is that the bottom of the ball does not push horizontally on the surface and the surface does not push back horizontally on the ball. Any point on the circumference of the ball comes to rest when it gets to the bottom because it rotates backwards around the axis of rotation at the same speed as the axis moves forwards. A ball that is completely at rest on a horizontal surface remains at rest because there is no horizontal force on the ball. Similarly, if a ball is rolling along a surface, there is almost no horizontal force acting on the ball since the bottom of the ball is at rest.

When a ball is rolling, the horizontal speed of the ball is the same as the rotation speed of any point on the circumference around the axis. Suppose that a point on the circumference takes a time T to complete one revolution. The rotation speed around the axis = circumference/time = $2\pi R/T = 2\pi fR$ where $f = 1/T$ is the rotation frequency. For example, if the ball spins at f = 100 revolutions/sec, then $T = 1/f = 0.01$ sec. A tennis ball has a radius R = 33 mm and

a circumference $2\pi R$ = 207 mm. If a ball is spinning at 100 revolutions/sec, then any point on the circumference spins around the axis at a speed of 207 x 100 = 20,700 mm/s = 20.7 m/s. If the ball is rolling along a horizontal surface, then it travels along the surface at a horizontal speed $v = 2\pi fR$ = 20.7 m/s. The numbers here are typical of the speeds at which a ball spins and slides during a bounce. Even though a ball does not roll when it bounces, it always comes close to rolling by the end of the bounce. If you know the horizontal speed of the ball when it bounces, you can estimate how fast it spins to within about 30% by assuming that it was rolling when it bounced.

Suppose that a ball without spin is incident at some angle on the court, and it starts sliding along the court at, say, 20 m/s. Friction at the bottom of the ball will slow it down and it will also make the ball start to spin. The further the ball slides the slower it slides and the faster it spins. If the ball speed drops to a point where $v = 2\pi fR$, then the bottom of the ball will come to rest. If the COF is small enough, or if the angle of incidence is small enough, then the ball will bounce off the court before the bottom of the ball comes to rest. Conversely, if the COF is large enough, or if the angle of incidence is large enough, then the bottom of the ball will come to rest before it bounces. If that is the case, then any increase in the COF or roughness of the surface will have only a small effect on the final speed or the spin of the bouncing ball. That would also explain why thin strings and thick strings make very little difference or no difference at all to the ball spin off the strings (see Chapter 41).

A simple rule of thumb is that when a ball is incident on the court at a low angle, it slides throughout the bounce and the spin is about 10 or 20% lower than one would expect for a rolling ball. If the ball is incident at angles greater than about 20 degrees, it will bite the surface and the spin will be about 10 to 20% higher than one would expect for a rolling ball. At least, this is the case for a tennis ball. For example, a tennis ball that bounces with a horizontal speed of 20 m/s will be spinning at about 100 revolutions/sec after it bounces. If the ball bounces with a horizontal speed of 10 m/s, then it will spin at about 50 revolutions/sec after it bounces. We can subtract 10 or 20% from those figures if the ball was incident at a low angle, and we can add 10 or 20% if the ball was incident at a high angle.

Spaghetti Strings

When a ball bites, the bottom of a ball comes to rest during the bounce but the top of the ball keeps going. An elastic ball will tend to lean forwards and then spring back as the ball bounces off the surface. Elastic energy is always stored during the bounce due to compression in the vertical direction, which allows the ball to spring back vertically and bounce off the surface. If elastic energy is also stored in the bottom of the ball as a result of stretching in the horizontal direction, then the bottom of the ball can spring back horizontally as the ball lifts off the surface. This extra kick in the horizontal direction means that the ball can spin a lot faster when it bounces. This effect is particularly noticeable with a superball. A superball bounces higher than a tennis ball, and it also spins much faster when it bounces, since it is almost perfectly elastic in both the vertical and the horizontal directions.

In the mid 1970's, a gardener in Germany named Werner Fischer came up with a bright idea to make a tennis ball spin as fast as a superball (US Patent 4,273,331). The idea was to make the strings almost perfectly elastic in a direc-

tion parallel to the surface of the stringbed. That way, elastic energy was stored in the strings as well as the ball. The ball was given extra spin when the strings sprung back to their original position. One way to do that is to string the mains and crosses separately so that they are not woven or interlaced. The mains can then slide freely over the cross strings. Fischer used plastic spaghetti tubing over the strings to reduce friction, and this system was therefore called spaghetti stringing, or double stringing, since the mains were tied together and slid over the crosses. It was quickly banned by the International Tennis Federation since it allowed a player to put excessive spin on the ball. Nevertheless, some movement of the strings occurs even when the strings are interlaced. It is unlikely that this will put any extra spin on the ball since the strings don't move very far and since they don't spring back freely. It is common to see top players spend half their time between points pulling their strings back into position. If the main strings don't spring back freely, then energy is wasted in pushing them sideways and the energy is not recovered. An interesting question concerns strings that don't move within the string plane. The question is whether they don't move at all or whether they do move and then spring back to where they were while the ball bounces off the strings. High speed photography shows that the strings do move.

Ball Speed and COR

We mentioned above that a ball bouncing off the court or off the strings changes speed in a direction parallel to the surface. It also changes speed in a direction perpendicular to the surface. The ball itself does not necessarily travel parallel or perpendicular to the surface, and is usually incident at an angle of around 20 degrees on the court or around 70 degrees on the strings. Nevertheless, the ball will have a certain speed in a direction parallel to the surface and a different speed perpendicular to the surface. For example, suppose that a ball is served from a height of 9 feet and it lands 0.5 sec later on the service line 60 feet from the server. The ball drops in a vertical direction at an average speed of 9/0.5 = 18 feet/sec (12.3 mph) and it travels in a horizontal direction at an average speed 60/0.5 = 120 feet/sec (81.8 mph). If the ball traveled in a straight line from the racquet to the court, it would travel a distance of 60.7 feet. It actually travels in a curved path of length about 62 feet, in which case the average speed along that path is 62/0.5 = 124 feet/sec (84.5 mph).

In practice, the ball accelerates in the vertical direction due to the pull of gravity. In the above example, it would hit the court at a vertical speed of about 15 mph. The ball slows down through the air and would hit the court at a horizontal speed of around 70 mph. Suppose the ball bounces off the court with a vertical speed of 12 mph and a horizontal speed of 42 mph. The ratio of the vertical speed after the bounce to the vertical speed before the bounce is called the Coefficient of Restitution or COR. In this case COR = 12/15 = 0.80.

The COR is reasonably constant for a given court, regardless of the ball speed or the angle of incidence. However, it can vary on an uneven or patchy surface like grass. On grass courts, the ball doesn't bounce as well, and the COR is about 0.7, but it depends on which patch of grass the ball hits and whether the grass is new or worn. Values of the COR as high as 0.9 have been observed on some grass surfaces and also on hard court surfaces, particularly at low angles of incidence. The court speed in the vertical direction can be just as important as the court speed in the horizontal direction. When people say that grass courts are fast and clay courts are slow, they are referring to the speed

in the horizontal direction. But grass courts are usually slow in the vertical direction and clay courts are fast in the vertical direction. In other words, a ball bouncing on clay will bounce faster in the vertical direction so it bounces higher. This can be a big effect. For example, if one court has a COR = 0.9, and another has a COR = 0.6, then the vertical speed off the COR = 0.9 surface is 50% larger and the ball will bounce 2.25 times higher. The bounce height is proportional to the COR squared.

Horizontal Speed of a Court

The ratio of the horizontal speed after the bounce to the horizontal speed before the bounce doesn't have a special name because it depends on the angle of incidence. In the above example, the ratio is 42/70 = 0.60. On a fast court the ratio is usually a bit higher and on a slow court the ratio can be lower, but in practice, the ratio is usually between 0.40 and 0.75 on all courts and at all angles of incidence of practical interest. In theory the ratio could be 0.9 or more at very low angles of incidence, but a ball never hits the court at angles less than about 10 degrees. The ratio is sufficiently important that it ought to have a special name. In this chapter, we will call it "Court Speed," since that is what it describes, but it does vary with the angle of incidence.

The International Tennis Federation (ITF) uses a different word and a different definition to describe the speed of a court, called Pace. The formula is slightly complicated, but if the ball slides throughout the bounce, then Pace = 100(1 - COF) where COF is the coefficient of sliding friction. For a grass court, COF is about 0.6 and hence Pace = 100 x 0.4 = 40. For a clay court COF is about 0.8 and hence Pace will be about 100 x 0.2 = 20. The advantage of this definition is that it doesn't depend on the angle of incidence. A disadvantage is that it does not give a direct indication of how much the ball slows down. Furthermore, two courts can have the same Pace but the ball will slow down by different amounts if the COR on each court is different.

The "Court Speed" defined above depends on four factors, not just one. It depends on

- the COF
- the COR
- the angle of incidence
- the spin of the incident ball.

The amount of spin on a ball that bounces off the court depends on all these factors and it also depends on the ball speed. The spin of a ball bouncing off the strings of a racquet depends on all of the above and it also depends on the speed of the racquet, the direction of motion of the racquet and the angle of the racquet face.

Measurements of Court Speed

Every tennis player knows that the grass courts at Wimbledon are fast and that the clay courts used for the French Open are slow. Just how fast or just how slow is hard to measure. Not impossible, but not as easy as you might think. The ITF has sponsored the development of a device called the Sestee to measure court speed. It works by projecting a ball at 30 m/s and at 16 degrees to the court, and it uses infrared beams to measure the incident and bounce

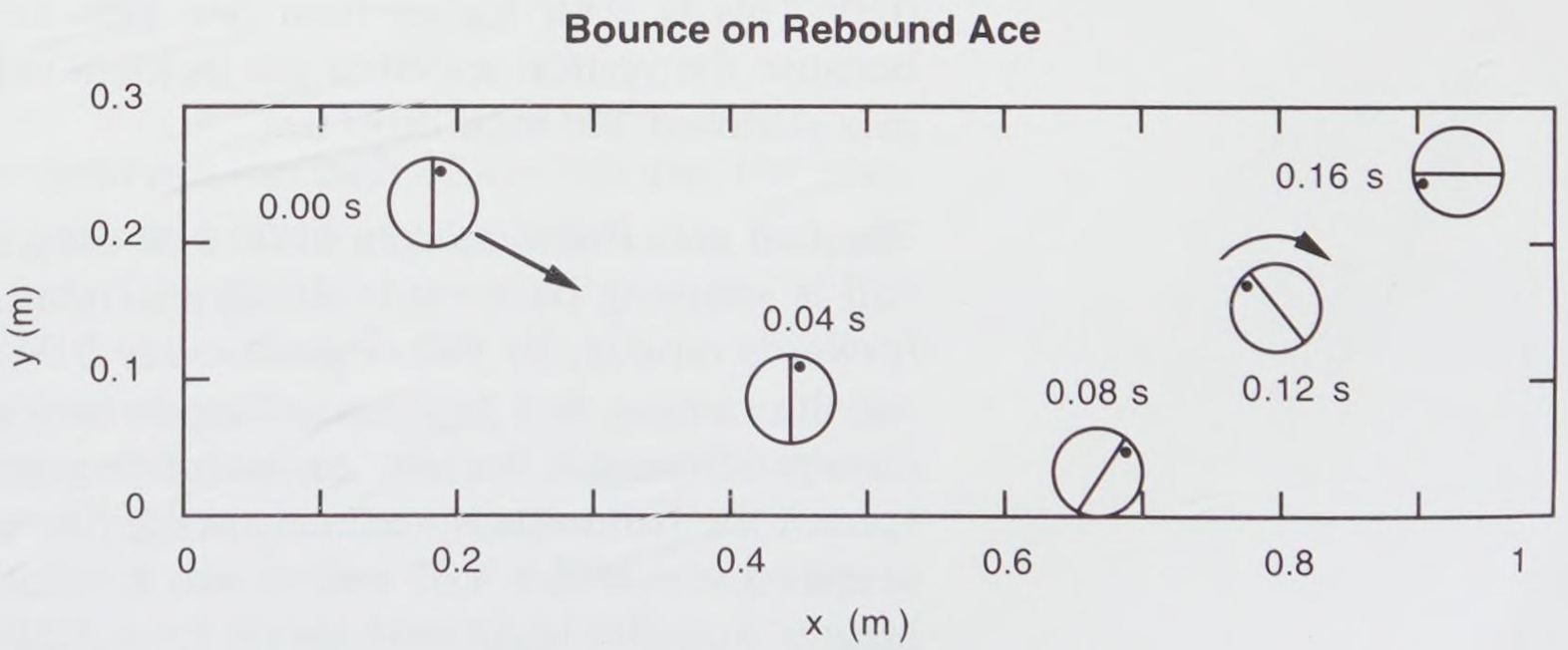
speeds and angles. It is very accurate, it costs around $50,000 and it needs a trained operator to use it. Tennis clubs can't afford to buy one. Others have tried to measure court speed by dragging a tennis ball or a piece of rubber across the court to measure the friction force. This is not as accurate but it is a lot cheaper.

There is another way to measure court speed that is both relatively cheap and accurate, and that is to film the bounce of a ball with a video camera. In fact, it is something that you could easily do yourself if you wanted to. It could also help resolve expensive legal issues that sometimes crop up when there is a dispute between a court builder and a club that feels that the builder let them down. The best sort of camera is a digital video camera since the images can be downloaded to a computer with a video card in order to measure the speed of the ball and the corresponding incident and rebound angles. If you focus close up to the ball, you can also measure how fast it is spinning, provided there is a mark or line drawn on the ball to measure the rotation angle from one frame to the next. However, if you focus up close, then you can measure the ball speed only if the ball is traveling slowly.

Video cameras capture images at 25 or 30 frames/sec which means that the time between each image is 1/25 or 1/30 sec (40 ms or 33.3 ms). If a ball is incident at 30 m/s, then it travels 1.2 m in 40 ms so the camera needs to view several meters of the court in order to measure the ball speed. The ball will then be too small and too blurry to measure its spin. But, if you throw the ball at low speed, you can focus up closer and measure both the spin and the speed of the ball. A fast shutter speed is needed to avoid blurry images. A video camera with a sports mode setting will have a suitably fast shutter speed. An example is shown in Figure 39.1. This is a composite drawing copied from actual video film so that the images are a bit easier to interpret.

Figure 39.1

Illustration rendered from video film of a ball bouncing at low speed on a Rebound Ace surface.



The drawing shows the position of the ball in each frame, which is 40 ms later than the previous frame. The first frame is labeled as time zero and the next as 0.04 s etc. The ball was filmed as it passed in front of a large screen with vertical and horizontal lines to calibrate the distance scales. A straight line was drawn with a felt pen across a diameter of the ball, with a dot at one end to distinguish one end of the line

t	x	y
0.00	0.181	0.230
0.04	0.446	0.080
0.08	0.669	0.031
0.12	0.798	0.153
0.16	0.929	0.253

v_{x1} = (0.446 - 0.181)/0.04 = 6.62 m/s

v_{x2} = (0.798 - 0.669)/0.04 = 3.22 m/s

v_{y1} = (0.230 - 0.080)/0.04 = 3.75 m/s

v_{y2} = (0.153 - 0.031)/0.04 = 3.05 m/s

Rotation speed = 300° every 0.04 s = 20.9 revolutions/sec

Incident angle = 29.5^{o} Rebound angle = 43.4^{o}

from the other. The ball was thrown without spin so that it bounced on a factory sample of Rebound Ace glued to a slab of concrete. Rebound Ace is the surface used for the Australian Open and is similar to the Deco Turf used for the US Open in that it has a 1 mm thick acrylic and sand mixture on top of a spongy rubber backing.

The position of the center of the ball is given in Figure 39.1, together with calculations of the ball speed before and after the bounce.

The calculations in the figure are not highly accurate since they are based on only two positions of the ball. A more accurate result would be obtained by plotting a few more images and then using a best fit line drawn through all the data points. The speeds in the horizontal direction are calculated from the x coordinates of the ball. For example, the ball moved from x = 0.181 m at time zero to 0.446 m at 0.04 s so it moved in the horizontal direction at a speed v_{x1} = 6.62 m/s before the bounce. After the bounce it traveled at a horizontal speed v_{x2} = 3.22 m/s. The speeds in the vertical direction are calculated from the y coordinates of the ball. The ball was incident in the vertical direction at v_{y1} = 3.75 m/s and it bounced at speed v_{y2} = 3.05 m/s in the vertical direction.

The court speed can be defined by the ratio v_{x2}/v_{x1} which for this bounce is 3.22/6.62 = 0.49. In other words, the horizontal speed after the bounce is a fraction under half the horizontal speed before the bounce. At other angles of incidence this ratio might be different, but at least we know the court speed for an angle of incidence $\theta_1 = 29.5^\circ$. The angle of incidence is given by $\tan(\theta_1) = v_{y1}/v_{x1}$ = 3.75/6.62 = 0.566.

The coefficient of restitution for this bounce is COR = v_{y2}/v_{y1} = 3.05/3.75 = 0.81. This is a bit higher than one gets for a purely vertical bounce, partly because the vertical speed of the incident ball is slower than the vertical speed in a standard 100 inch drop test.

The ball was found to spin at 20.9 rev/sec after the bounce. It looks like the ball is spinning backwards slowly in Figure 39.1 but it was actually spinning forwards rapidly, by 300 degrees every 0.04 sec. This was checked by switching the camera to a high speed mode to record at 100 frames/sec. Not many cameras have this feature. An interesting result is found if one compares the spin of the ball to its horizontal speed. Any point on the circumference rotates at speed $v = 2\pi Rf$ = 4.33 m/s where R = 0.033 m and f = 20.9 rev/sec. This is greater than the horizontal speed v_{x2} = 3.22 m/s, which means that any point on the ball in contact with the surface is sliding backwards on the surface by the time the ball leaves the surface. The ball started sliding forwards on the surface at the start of the bounce since it had no spin to start with, but then it acquired enough spin during the bounce to slide backwards. The ball did not roll. Instead, it gripped the surface for a while and then it started sliding backwards.

Measurements of Court Speed on Different Surfaces

Results obtained at other angles of incidence are shown in Figure 39.2, together with results obtained on three other surfaces. One surface was a smooth slab of concrete. The other two were made by bonding sheets of P800 and P150 emery paper to two other smooth concrete slabs. P800 is a fine grade and P150

is relatively coarse, but one can rub a hand firmly across the surface without cutting the skin. P150 means that there are about 150 particles per inch on the emery paper. Results on real courts are described below. The advantage of these "artificial" court surfaces is that the surfaces are more uniform than a real court, so there is less variability to contend with. In addition, the hard concrete surfaces are not deformed by the ball, so that is one less variable to worry about. It is not easy figuring out how a ball bounces. Having only a few variables makes the task a bit easier.

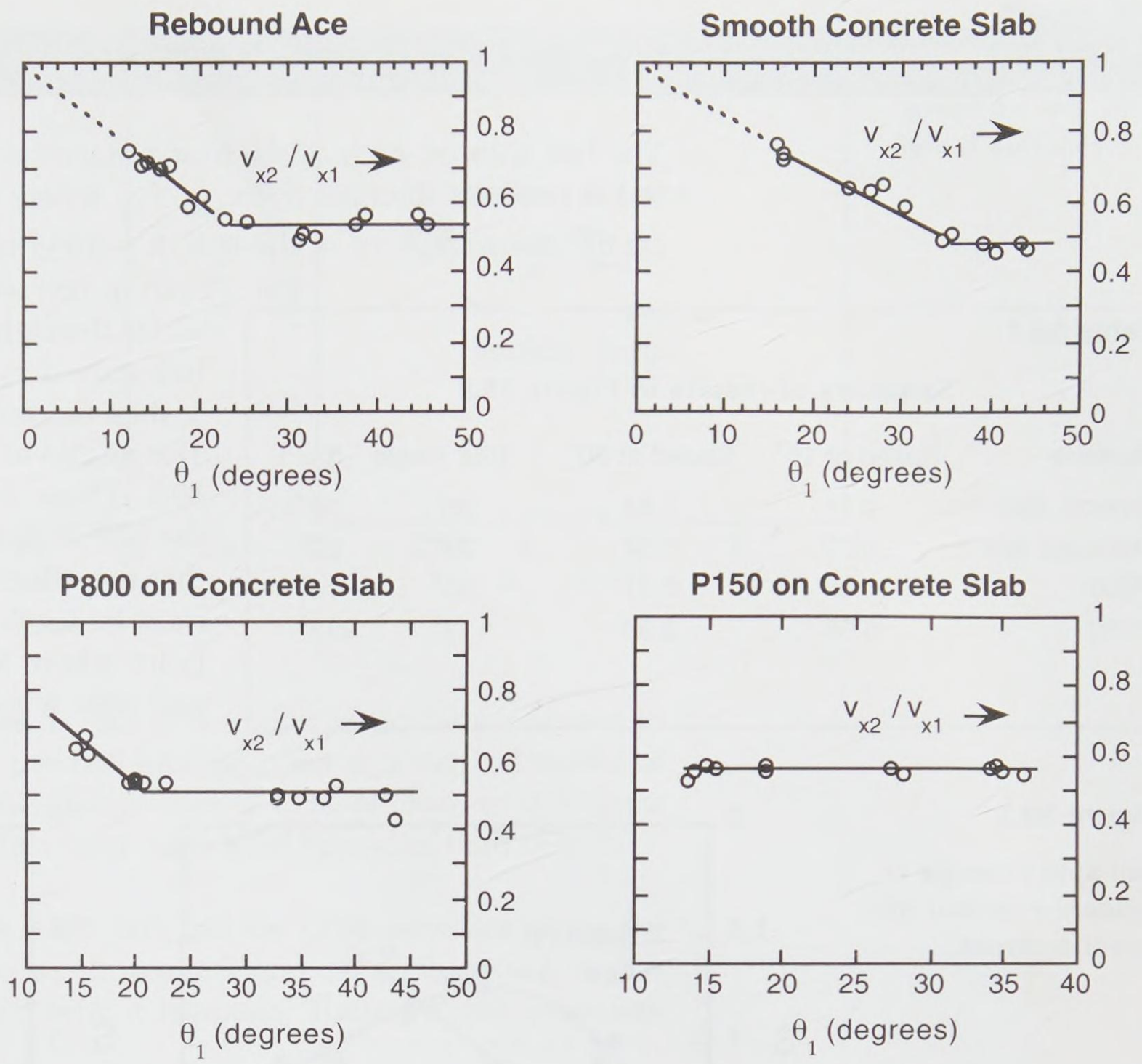


Figure 39.2

Court speed vs angle of incidence θ_1 for four different surfaces.

Each circle in Figure 39.2 represents a single bounce, filmed by throwing a ball at a speed of about 7 m/s at various angles onto each surface so that the ball spin could also be measured. The dashed lines in Figure 39.2 represent the expected result at very low angles of incidence, and the solid lines are included to show the transition from sliding at low angles to biting at high angles. θ_1 is the angle of incidence (between the path of the ball and the horizontal). All the results shown in Figure 39.2 were obtained by using at least three images of the ball before and after the bounce.

The most obvious result is that the court speed is large at low angles of incidence, and it is smaller at high angles of incidence. At low angles, the speed decreases as the angle increases, but at high angles the speed remains constant as the angle increases. The difference is due to the fact that the ball slides throughout the bounce at low angles, and it bites at high angles. On the rougher P150 surface, the speed remained constant at all angles which means that the transition from sliding to biting occurred at an angle of incidence less that 13°. On the smooth concrete slab, the ball started to bite only when the angle of incidence was increased to 35°.

Have a good look at Figure 39.2 and see if you can decide which surface was the fastest and which was the slowest. The speed of each surface is summarized in Table 39.1, showing the speed (i.e., v_{x2} / v_{x1}) at angles of incidence of 16 degrees and 30 degrees. At 16°, the fastest surface was the smooth concrete slab and the slowest was the rough P150 surface, as one might expect. But at 30°, the fastest surface was P150 and the slowest was the smooth concrete slab.

Measurements of Ball Spin

The ball spin on each of the four surfaces is shown in Figure 39.3. This is plotted in terms of the ratio $S = v_{spin}/v_{x2}$, where $v_{spin} = 2\pi Rf$ is the speed of a point on the circumference of the ball, R = 0.033 m is the ball radius, and f is the ball spin in rev/sec. If S is less than 1, then the ball slides throughout the bounce. If S = 1, then the ball started to roll during the bounce, but if S > 1, then the ball bit the surface when it bounced. The angles at which S = 1 are shown in Table 39.1. These angles are all a fraction lower than the bite angles shown in the table, which means that the effect on the court speed due to biting must be similar to the effect due to sliding at the point where S = 1, and the difference is important only when S is greater than about 1.15.

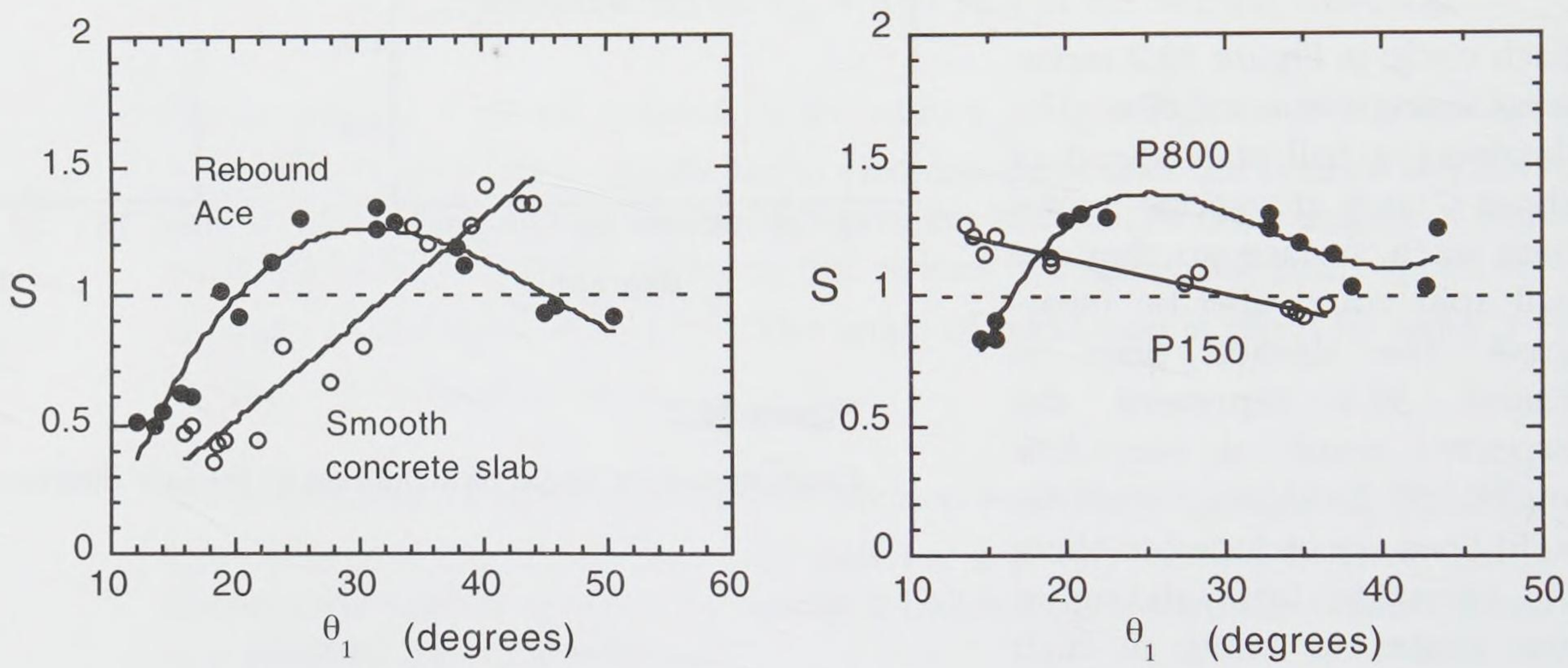
Table 39.1

Summary of results in Figure 39.2

Surface	Speed at 16°	Speed at 30°	Bite Angle	S = 1
Smooth slab	0.74	0.49	35°	32°
Rebound Ace	0.70	0.54	23°	20°
P800	0.65	0.51	20°	16°
P150	0.56	0.56	<13°	<13°

Figure 39.3

Ball spin vs angle of incidence on four different surfaces.



Measurements of Vertical Speed (COR)

Measurements of the COR for bounces on the smoothest and roughest of the four surfaces are shown in Figure 39.4. A correction was made when analyzing the data to account for the acceleration due to gravity in the vertical direction. This affects calculations of ball speed in the vertical direction but no such correction is needed to calculate the ball speed in the horizontal direction. In Figure 39.4, the COR is plotted as a function of v_{y1}, since it is well known that the COR decreases as v_{y1} increases, at least for a ball incident vertically on a surface. That is easy to understand since the ball squashes more in a high speed impact and it loses a greater fraction of its stored elastic energy. The ball bounces higher as the incident speed increases, but the ratio of the rebound speed to the incident speed decreases as the incident speed increases.

When the ball was dropped vertically from various heights onto each of the four surfaces, the COR was found to decrease slightly as the drop height increased, but the effect was relatively small, and the COR was about 0.79 for all drop heights on all surfaces. Figure 39.4 shows the results for a vertical drop on the P150 surface. A larger drop in the COR would occur if the ball was inci-

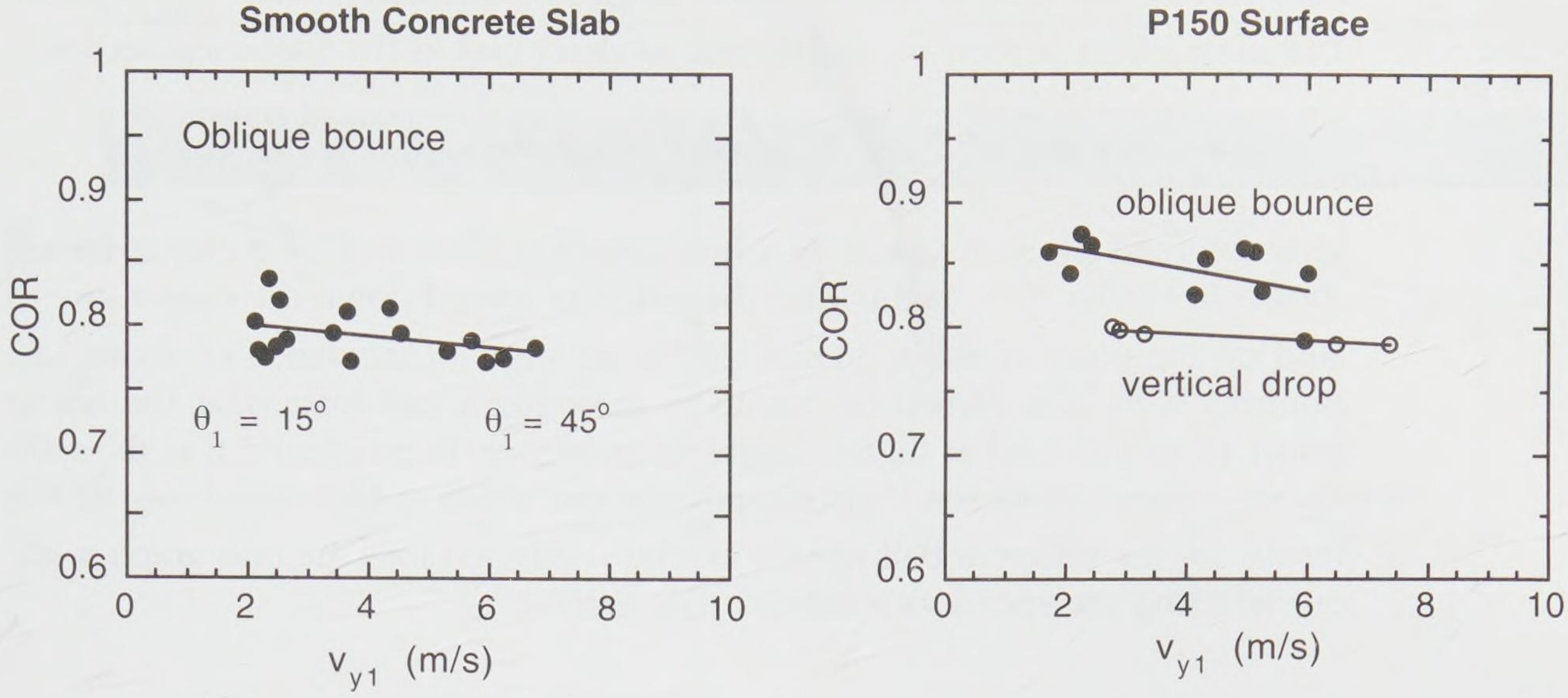


Figure 39.4

COR vs vertical speed on two different surfaces.

dent vertically at much higher speeds, but this is not particularly relevant in tennis. Even for a very fast serve, the ball hits the court at a vertical speed of only about 8 m/s. An approved ball should have a COR of close to 0.75 for a 100 inch vertical drop, but the balls tested were a bit bouncier than this.

Of greater interest and significance is the fact that the COR increases when the ball is incident at an oblique angle, as shown in Figure 39.4. This result is surprising, and no one knows for sure why it happens. There are several possible explanations, such as:

- The ball forms a depression in the surface and is deflected upwards by the ramping slope of the depression.

- The ball spins so fast that each new section of the ball contacting the surface moves off the surface before it has time to compress very far.

- Some of the kinetic energy due to horizontal motion of the ball gets channeled into the vertical direction.

While each of these effects might make a contribution in some cases, the first two effects cannot be important in the present case, since concrete is too rigid for the ball to form a depression and since the COR was enhanced even at low ball speeds. The third explanation looks more promising. Suppose that you tie a string on each side of a ball and pull horizontally. The ball will stretch in a horizontal direction. Now place the stretched ball on a horizontal surface and cut both strings at the same time. The ball will spring back to its original spherical shape and bounce up off the surface. That is an interesting effect since it means that a force can be applied in a horizontal direction and the result is motion in the vertical direction. A similar effect can be engineered under different circumstances using levers or pulleys but a tennis ball does it by springing back into shape.

A ball can be expected to stretch horizontally when it bounces, since the bottom of the ball will slow down faster than the top of the ball. The horizontal stretch would be bigger on a rough surface than on a smooth surface and would be more noticeable if the horizontal speed of the ball is a lot larger than the vertical speed. This is consistent with the results in Figure 39.4, since the COR is larger on the rough P150 surface than on the smooth concrete surface.

Furthermore the COR is largest at low angles of incidence where v_{y1} is small. The same effect is seen on real courts, as described in the following section.

Measurements of Court Speed on Actual Courts

Measurements of court speed on a Rebound Ace court and on a clay court are shown in Figure 39.5. In this case the ball was served down the center line to land on the court at about 30 m/s and at an angle of incidence of about 12°. Bounces were also filmed by hitting or throwing a ball from near the net at about 15 m/s to land at higher angles of incidence. It was found that the ratio v_{x2}/v_{x1} does not depend significantly on the incident ball speed, so all the results for the different ball speeds can be combined into the one graph without affecting the speed vs incident angle graphs.

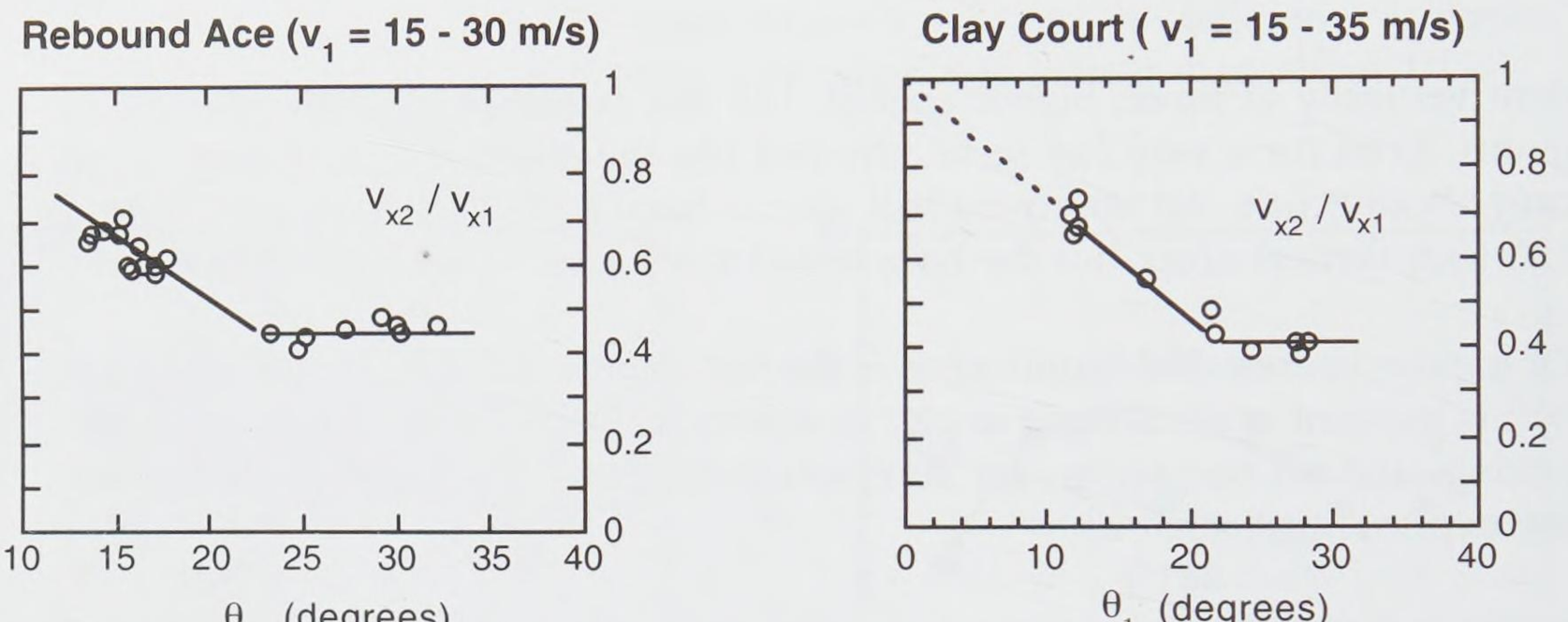


Figure 39.5

Court speed of real courts at real ball speeds

The ball was filmed as it passed in front of two vertical ball cans 1.00 m apart in order to calibrate the horizontal and vertical scales on the film. The cans were placed along the center line, the camera axis was at right angles to the center line, and the ball was projected parallel to the center line so that an accurate measurement could be made of the height of the ball in each frame (Figure 39.6).

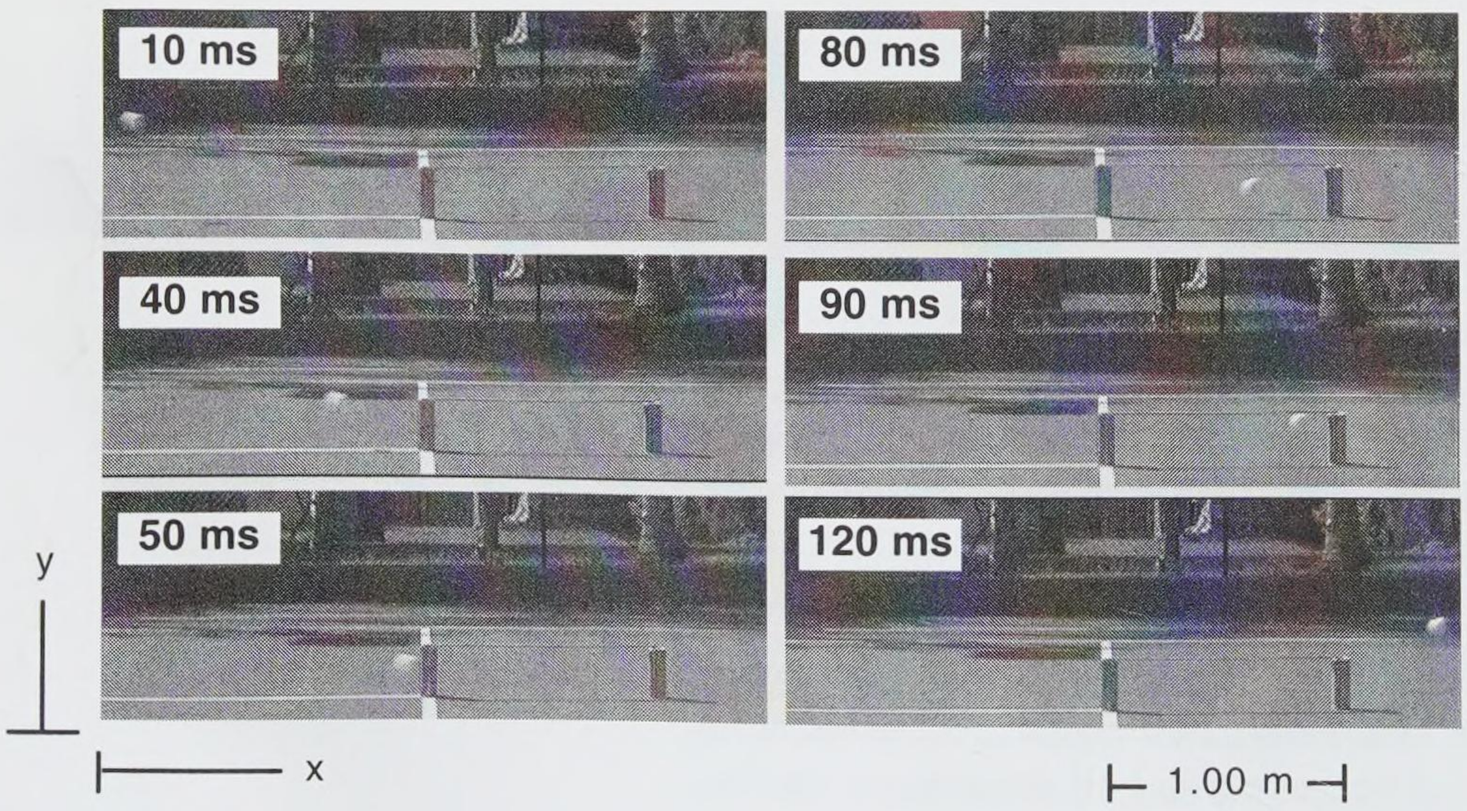


Figure 39.6

Video film of a 90 mph serve on Rebound Ace. The three frames on the left show the ball approaching. The right frames show the ball after it bounced. Filmed at 100 frames/sec with 2 ms exposure each frame. The shadow of the ball provides some depth information, but this was not needed.

These results show that clay is indeed a slow surface but the Rebound Ace court tested was not much faster. The result for Rebound Ace in Figure 39.2 differs from the result in Figure 39.5 since the court had a rougher finish due to the extra sand content. The speed of Rebound Ace can be varied by changing the amount and size of the sand in the acrylic and by sweeping the acrylic with a broom when it is applied. The broom leaves small ridges, with the result that sweeping across the court generates a slower surface than sweeping along the court.

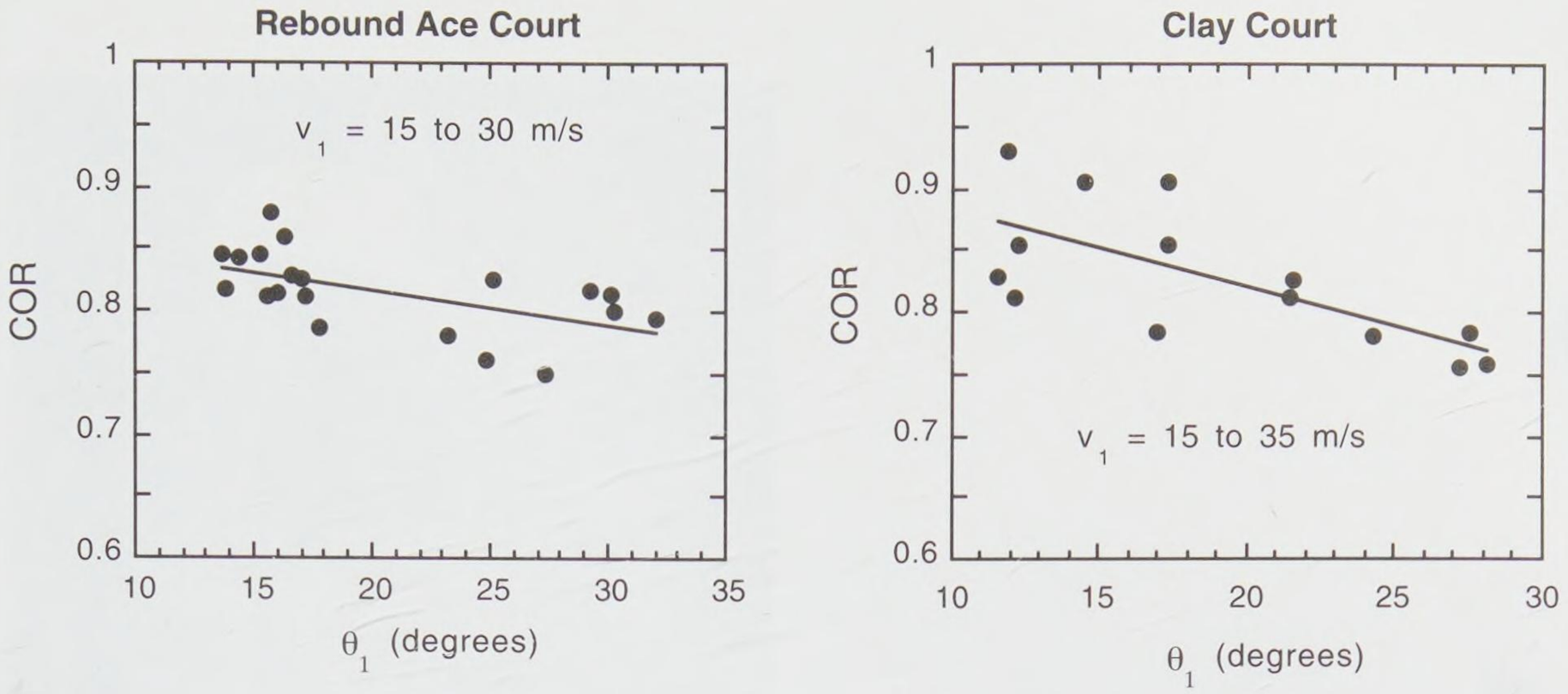


Figure 39.7

COR vs angle of incidence on real courts.

Measurements of the COR for these two courts are shown in Figure 39.7. The COR varies quite a bit from one bounce to the next, so grass courts are not the only courts with a variable bounce. Both courts have COR values greater than the value 0.79 observed for a purely vertical bounce on these courts, especially at low angles of incidence. A COR greater than 0.9 is normally observed only with a superball. If the COR was 0.79, a fast serve would be expected to bounce just above waist height. But since the COR is sometimes as large as 0.9, the ball will sometimes bounce around shoulder or head height even without any deliberate topspin. A head-high bounce is usually the result of a kick serve with lots of topspin, but it can also happen in a "flat" serve if the receiver stands far enough behind the baseline.

Measurements of the Coefficient of Friction

The coefficient of sliding friction is usually represented by the Greek letter μ and it can be calculated from the speed and COR measurements obtained at low angles of incidence. The relevant formula is

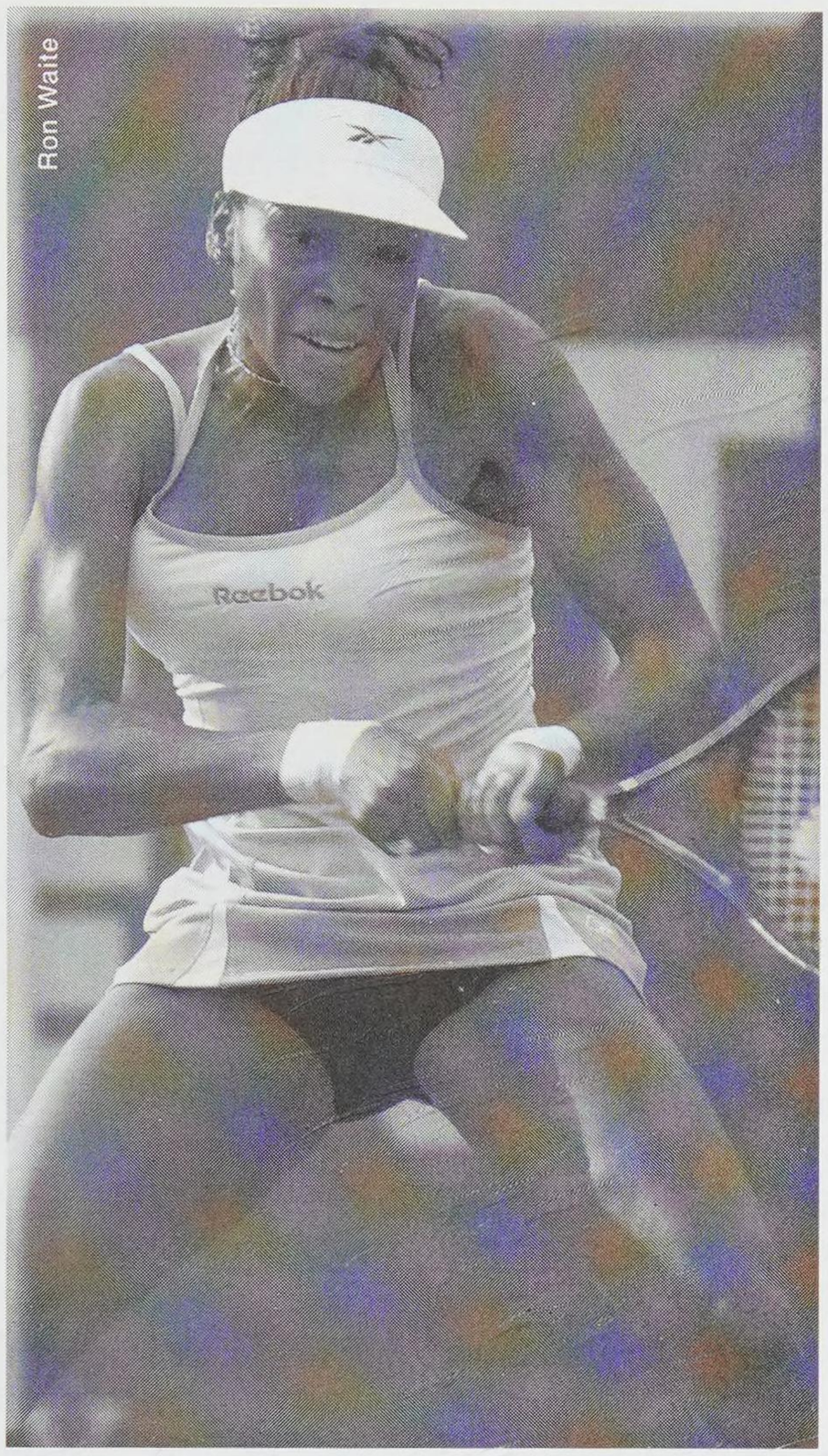
$$v_{x2}/v_{x1} = 1 - \mu(1 + e)\tan(\theta_1) \qquad (39.1)$$

where e is the COR. A value for μ can be calculated for each and every bounce provided the ball slides throughout the bounce. The average value of μ and e for each of the above surfaces is given in Table 39.2. A reliable value for μ could not be found for the P150 surface since the ball did not slide throughout the bounce even when the angle of incidence was only 13°. If the ball bites, then the above formula underestimates the true value of μ. This table highlights the fact that surfaces which are slow in the horizontal direction are usually fast in the vertical direction and vice-versa, but grass is a notable exception (depending on the length of the grass). Artificial grass courts have values of μ and e typically about 0.68 and 0.82 respectively.

Table 39.2

Summary of Court Speed Results

Surface	μ	e
Smooth concrete	0.42	0.79
Smooth Rebound Ace	0.62	0.80
P800 on concrete	0.73	0.82
P150 on concrete	>1.0	0.85
Rebound Ace court	0.70	0.83
Clay court	0.80	0.85



Venus Williams, United States

40

Ball Bite and Bounce

How A Ball Bites the Court When It Bounces

By Rod Cross

Introduction

For a long time, people involved in the physics and engineering of sport assumed that when a ball bounces, it either slides throughout the bounce, or it starts to roll when it acquires enough spin. The assumption about sliding is correct, but the assumption about rolling has recently been found to be incorrect. When balls bounce they bite the surface instead of rolling. As a result, balls that bite spin faster than expected, they slow down more than expected and the friction force at the bottom of the ball does not suddenly drop to zero. Instead, the friction force decreases slowly to zero and then reverses direction due to the fact that the ball starts vibrating in a horizontal direction during the bounce. In this section we describe experimental results showing that real balls bite when they bounce.

Apparatus

The arrangement used to measure the friction force on a bouncing ball is shown in Figure 40.1. A wood block was supported on two cylindrical rollers so that the block could move freely in the horizontal direction with almost no frictional resistance. A ball incident obliquely on the block bounced off the block at an oblique angle, exerting a force on the block only during the brief period of the impact. The horizontal component of the force

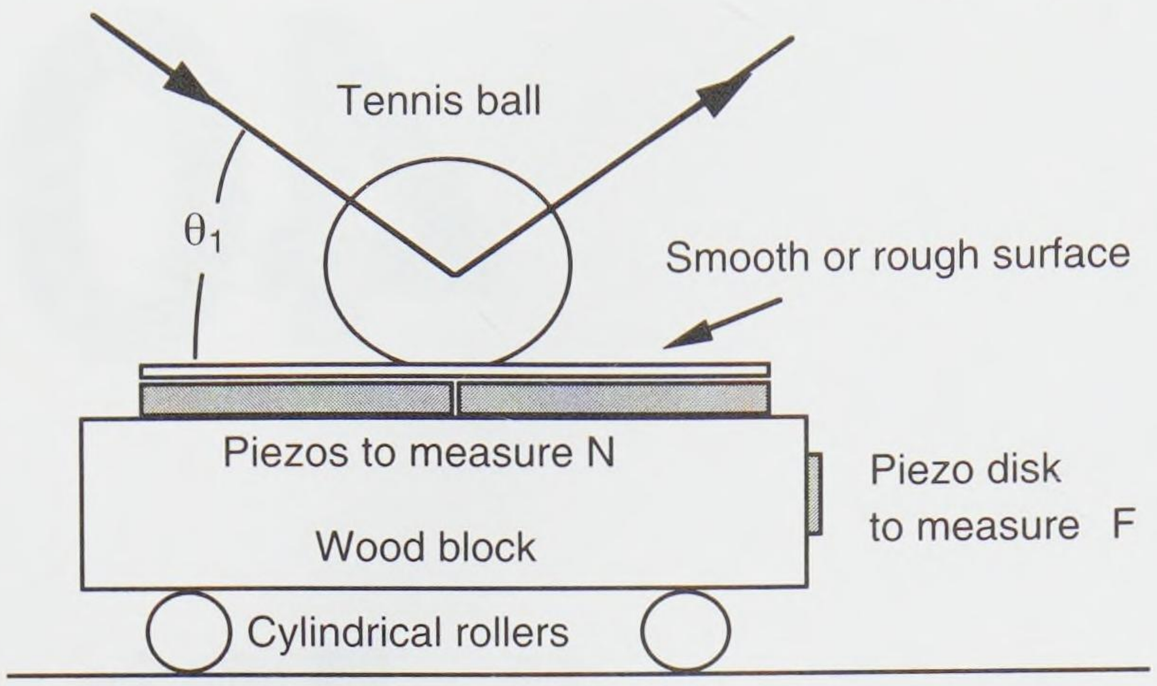


Figure 40.1

Apparatus used to measure the friction force on a bouncing ball.

on the block is equal in magnitude and opposite in direction to the friction force on the ball. The horizontal force caused the block to accelerate in the horizontal direction (from left to right) and it caused the ball to slow down. A 19 mm diameter, 0.3 mm thick ceramic piezo disk was fastened to one end of the block as a simple accelerometer. The output voltage from the piezo is directly proportional to the acceleration of the block so it provided a direct measurement of the time-varying friction force acting on the bottom of the ball. For the low speed bounces studied in this experiment, a block mass of 340 gm (about six times the mass of a tennis ball) was used to obtain an adequate output signal from the accelerometer.

Two large area ceramic piezo blocks were mounted on top of the wood block to measure the vertical force on the block. Each of the large piezos was 51 mm square and 4 mm thick and were connected electrically in parallel to act as a single, large surface area (51 x 102 mm) force plate. The upper surface of the plate was mechanically protected using 0.3 mm thick circuit board attached directly to the plate with double-sided adhesive tape. This surface was quite smooth and is referred to below as the low friction surface. For some experiments, fine grade (P800) emery paper was taped firmly to the circuit board to study the bounce off a surface with a higher coefficient of friction than the smooth circuit board surface. It is referred to below as the high friction surface.

Bounces at several different angles on both surfaces were filmed at 100 frames/sec with a digital video camera to determine the ball speed, spin and angle before and after each bounce. Video clips were transferred to a computer for analysis. The horizontal and vertical distance scales were calibrated simply by measuring the actual and image diameter of each ball.

Friction Force Results

Simultaneous measurements of the vertical force, N, and the horizontal friction force, F, are shown in Figures 40.2-40.5. Figure 40.2 shows the bounce of a new tennis ball on the low friction surface, and Figure 40.3 shows the bounce on the high friction surface. Figure 40.4 shows the bounce of a superball and Figure 40.5 shows the bounce of a basketball, both balls being incident on the low friction surface. In all cases, the ball was incident at low speed, typically about 3 m/s. The properties of each ball are listed in Table 40.1.

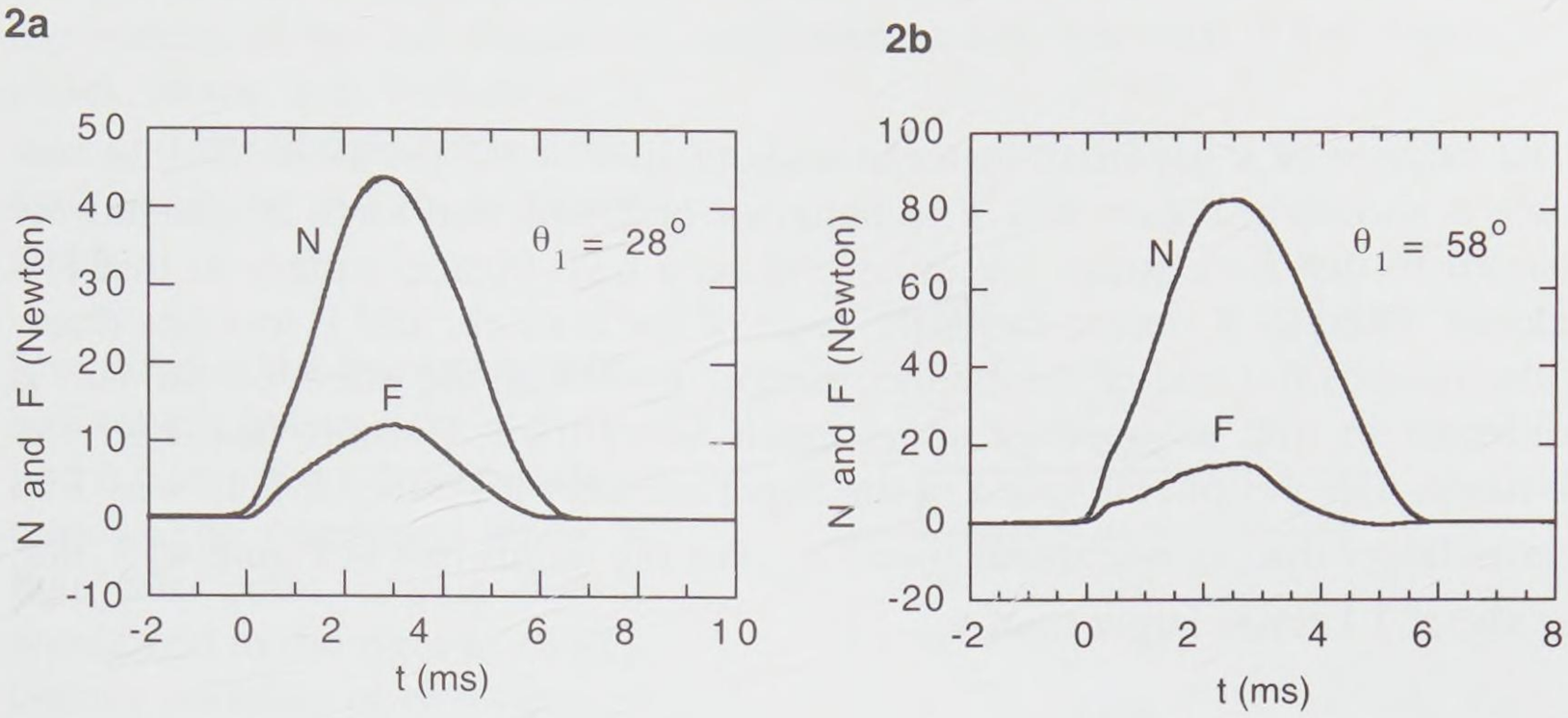
Table 40.1 Ball Properties

Ball	Mass m	Radius R
Tennis	57.4 gm	33.0 mm
Superball	46.4 gm	23.0 mm
Basketball	589 gm	120 mm

The results in Figure 40.2 demonstrate that a tennis ball incident on a low friction surface bounces without significant reversal of the friction force during the bounce. At low angles of incidence the ball slides throughout the bounce and the ratio F/N remains approximately constant during the bounce. In Figure 40.2a, $F/N = 0.27 \pm 0.03$, which gives the coefficient of sliding friction. At higher angles of incidence, the friction force dropped to zero during the bounce, and it dropped to zero earlier as the

Figure 40.2

Results for a tennis ball incident on the smooth, low friction surface.

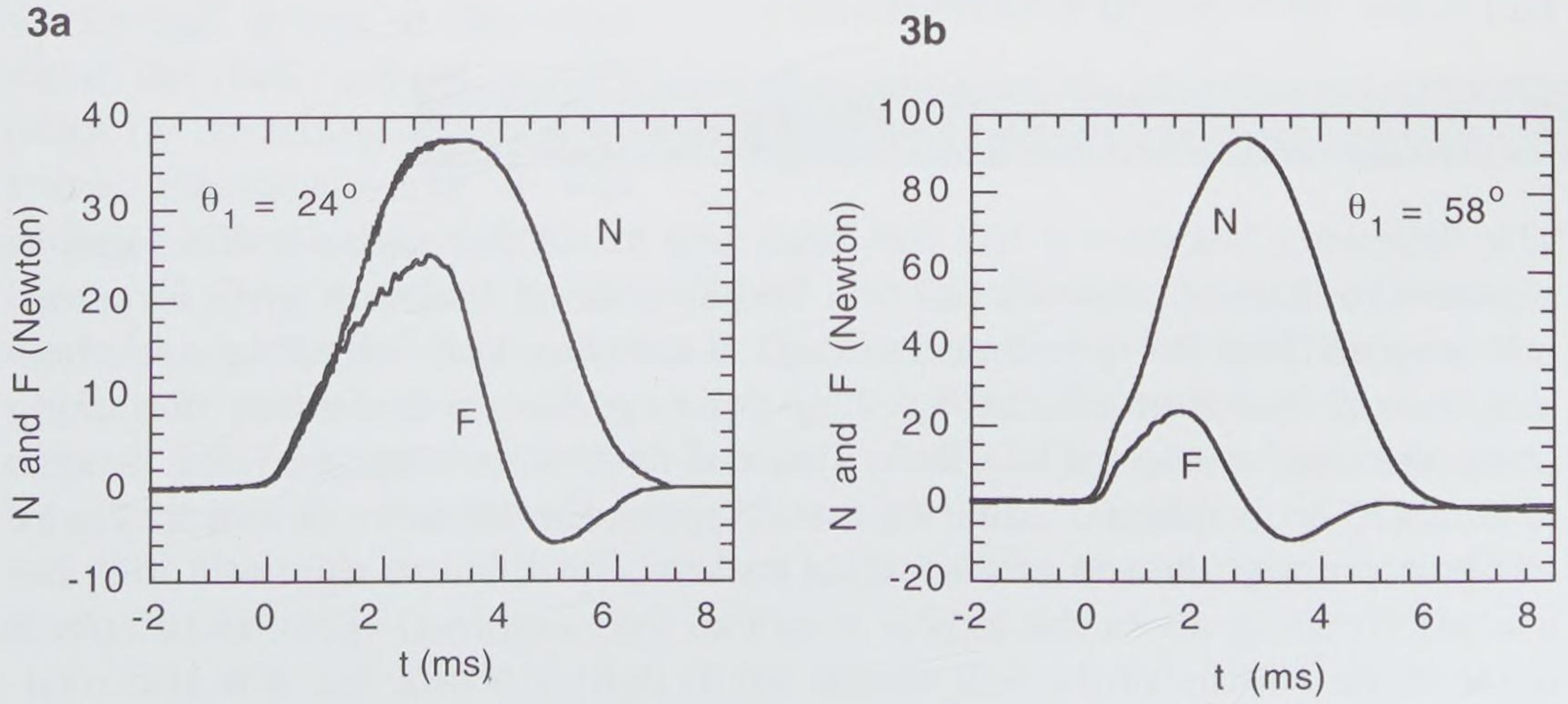


angle of incidence was increased. These results are qualitatively consistent with the bounce model described by Brody, who predicted that the ball would start to roll at a progressively earlier stage as the angle of incidence was increased. However, there is no sudden transition from sliding to rolling during the bounce, and the ball was observed to spin faster than allowed by the rolling condition. The fact that the friction force dropped to zero during the bounce can be interpreted to mean either that the ball rolled or that the friction force was positive in some areas of contact and negative in others. For reasons described in more detail below, it can be inferred that the ball did not roll during the bounce but it first gripped the surface and then commenced to slide backwards.

In practice, the coefficient of sliding friction between a tennis ball and a court surface is usually about 0.5 or larger. On such a surface, the ball is predicted to slide throughout the bounce if the angle of incidence is less than about 16°. At higher angles of incidence, Brody predicted that the ball would enter a rolling mode, and hence, the friction force would drop to zero. Figure 40.3 shows that at these higher angles of incidence the bounce of a tennis ball on a high friction surface is characterized by a significant reversal of F during the

Figure 40.3

Results for a tennis ball incident on the rough, high friction surface.



bounce. The reversal in the direction of F occurs earlier in time as the angle of incidence is increased, allowing two reversals to occur in Figure 40.3b. In Brody's model, the ball rolls at an earlier stage as the angle of incidence is increased, but F does not reverse direction.

The bounce of a superball incident without spin at 36° on a low friction surface is shown in Figure 40.4. The behavior of the friction force is qualitatively similar to that for a tennis ball bouncing on a high friction surface in that F is almost equal to N during the early stage of the bounce, and F reverses direction towards the end of the bounce period. However, the effect on the ball is different in that a superball spins much faster than a tennis ball after the bounce. The peripheral speed of the superball after it bounced, $R\omega_2$, was 1.75 times larger than its horizontal speed v_{x2}. For the tennis ball in Figure 40.3, $R\omega_2$ is about 1.1 times larger than v_{x2}.

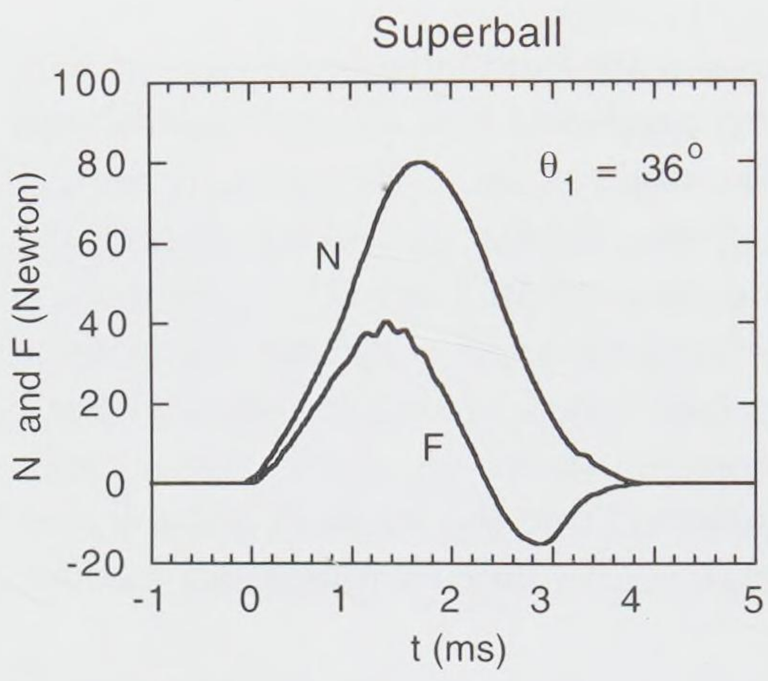


Figure 40.4

Superball on smooth surface.

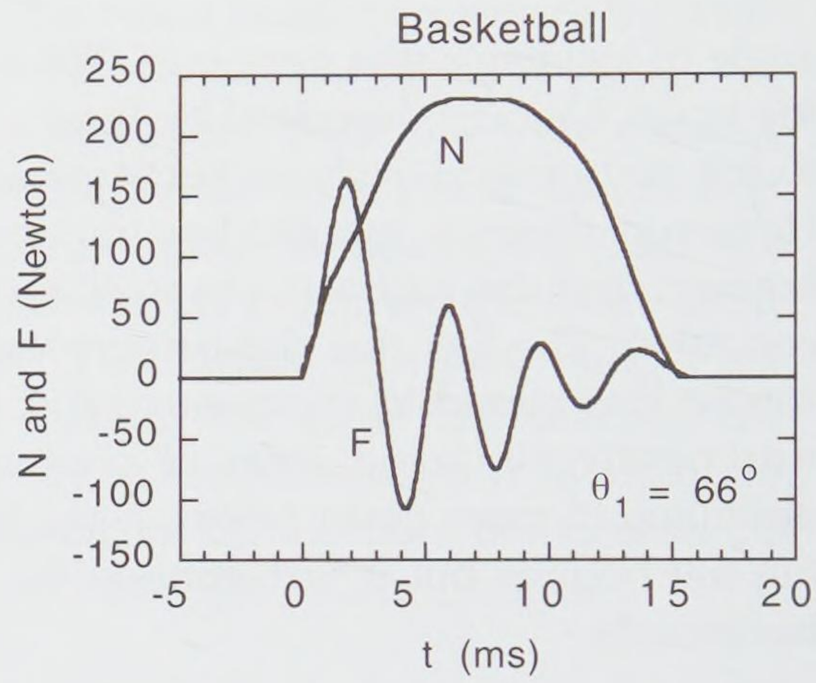


Figure 40.5

Basketball on smooth surface.

Figure 40.5 shows the bounce of a basketball incident with almost zero spin at 66° on the low friction surface. It is especially obvious in this case that the ball vibrates horizontally during the bounce, causing the friction force to reverse direction six times. The half period of oscillation is 15 ms in the vertical direction, and the full period of oscillation in the horizontal direction is about 4 ms. There is no simultaneous 250 Hz oscillation in the N waveform, indicating that this mode of oscillation involves horizontal displacements in the wall of the ball rather than vertical displacements.

Explanation of Friction Force Results

The difference between a ball that bites and a ball that slides is illustrated in Figure 40.6. Part (a) shows a ball that is sliding along a surface while it rotates. It is assumed that every point on the ball is traveling from left to right at a horizontal speed of 8 m/s and that every point on the circumference is rotating about the axis at a speed of 6 m/s. The axis itself is traveling from left to right at 8 m/s. The combined effect of these separate motions is shown in Figure 40.6b. A point at the top of the ball is traveling from left to right at 8 m/s, but it is also rotating about the axis at 6 m/s so the combined speed is 14 m/s. A point at the bottom of the ball travels left to right at 8 m/s, but it is also rotat-

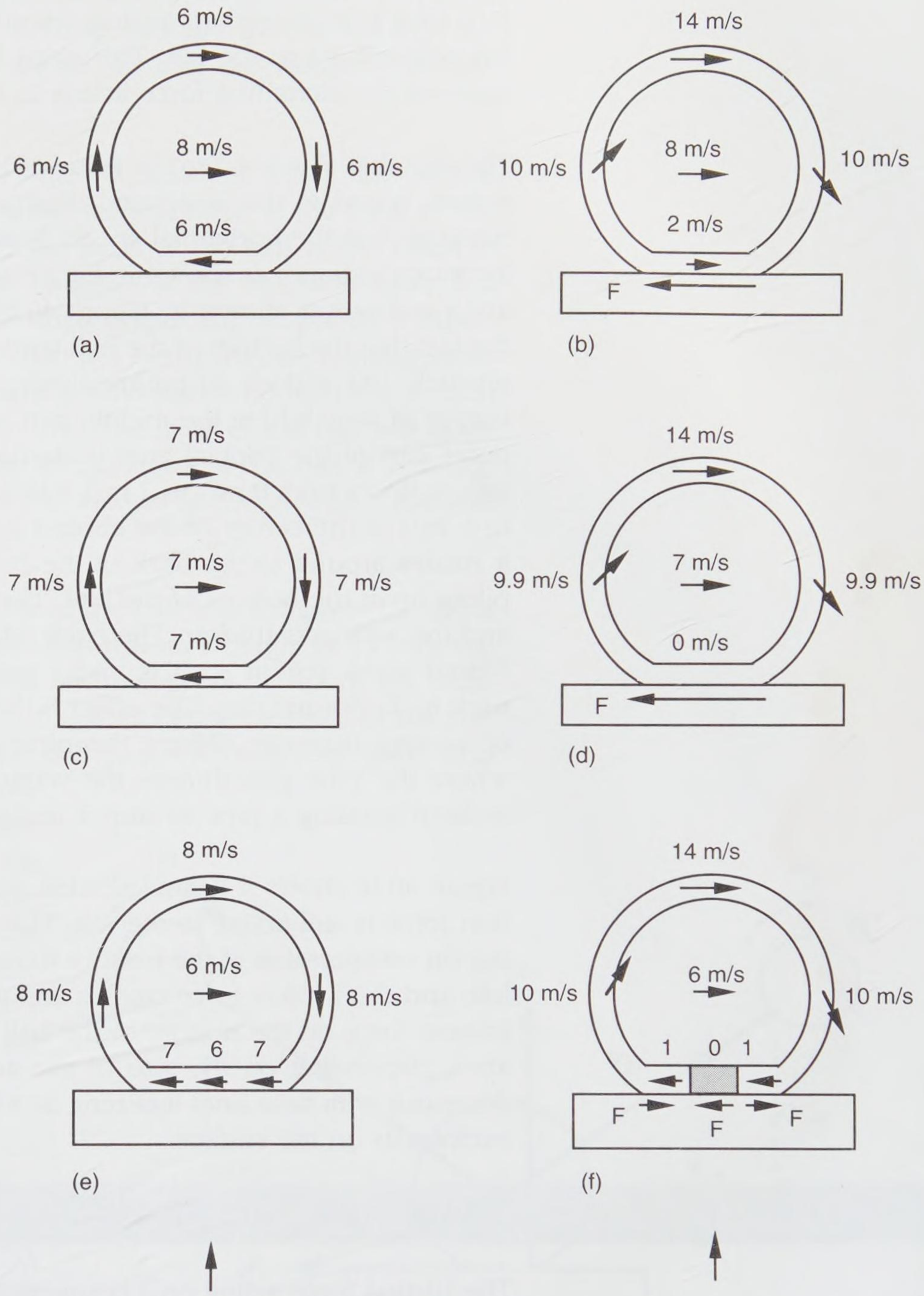
ing right to left at 6 m/s, so the combined effect is motion left to right at 2 m/s. Every point at the bottom of the ball therefore slides along the surface at 2 m/s, and the friction force F acts backwards as shown in Figure 40.6b.

The point at the front end of the ball travels left to right at 8 m/s and it also rotates downwards at 6 m/s. The combined effect is that this point moves downwards and to the right at 10 m/s (vector addition of two velocity components at right angles requires that we add the squares of each component; in this case $10^2 = 8^2 + 6^2$)

The effect of the friction force is to reduce the horizontal speed of the whole ball and to make it rotate faster. Suppose that the horizontal speed decreases to 7 m/s and the rotational speed increases to 7 m/s as shown in Figure 40.6c. The combined effect of these separate motions is shown in Figure 40.6d. In this case, the bottom of the ball comes to rest on the surface, since the horizontal motion forwards is cancelled by backward rotation. When an object is at rest on a surface and remains at rest, or when it is rolling at constant speed, the friction force is zero. In this case the friction force is not zero since the whole ball is not at rest, and since the ball arrived at this point by stretching in the horizontal direction while it was sliding. The ball shown in Figure 40.6b is stretched horizontally by a small amount. It stretches because there is a horizontal force on the bottom acting to the left and because the upper part of the ball is moving to the right. It might be easier to visualize this effect if you imagine that the ball has a top half and a bottom half connected by a spring. If the ball travels from left to right and you grab hold of the bottom half, then the top half will tend to keep moving to the right, causing the spring to stretch.

Figure 40.6

Motion of a ball when it slides (a and b) or bites (c - f).



The upper part of the ball in Figure 40.6d is therefore in a stretched state, and it tends to pull to the right on the bottom of the ball. There is also a large force on the ball in a direction perpendicular to the surface. If the coefficient of static friction is large enough, the bottom of the ball will remain stuck to the surface (like a book pushed against a vertical wall) and the static friction force on the ball will act to the left. This stops the ball sliding along the surface and it opposes the stretching force acting to the right.

The fact that F is not zero in Figure 40.6d means that the ball will continue to reduce speed in the horizontal direction and it will continue to rotate faster. Suppose that the horizontal speed decreases to 6 m/s and the rotational speed increases to 8 m/s as shown in Figure 40.6e. The combined effect of these separate motions is shown in Figure 40.6f. The situation here is complicated by the fact that the bottom of the ball tends to remain stuck. It eventually becomes unstuck, but it does so progressively, not suddenly. Figure 40.6e shows one instant of time where the middle part of the contact area is still stuck, and the outer part of the contact area is starting to slide. As the ball rotates about its axis, it slows from 8 m/s to 7 m/s when it first contacts the surface, and it slows to 6 m/s at the center of the contact area. It then speeds back up to 8 m/s as it rotates around to the back of the ball. This means that bits of the ball start piling up at the bottom of the ball. The front edge of the ball gets compressed and the wall gets thicker. The back edge of the ball is rotating faster than the central stuck region so it is under tension and the wall thickness decreases back to its normal size. The effect is the same as water flowing through a pipe of varying diameter. Where the pipe gets fatter, the water slows down and where the pipe gets thinner the water speeds up. Rubber does this by itself without needing a pipe to stop it leaking out.

Figure 40.6f shows a central shaded area that is still stuck and where the friction force is still acting to the left. The annular region outside this area is sliding (in compression at the front or tension at the back) along the surface to the left, and the friction force on this annulus therefore acts to the right. The total friction force on the bottom of the ball can therefore be positive, zero or negative, depending on the size of the area that is stuck. The size of that area decreases with time until it is zero, at which point the whole contact area slides backwards on the surface.

Slamming on the Brakes

The friction force acting on a bouncing ball slows it down in a horizontal direction and causes it to rotate. It also has another effect that we haven't mentioned so far, and it is essentially the same effect that occurs when you apply the brakes of a vehicle. That is, the weight of the vehicle shifts forwards. Every time you apply the brakes, the front end of the vehicle moves down closer to the road and then it bounces back up again when you come to a complete stop. This is especially noticeable if you slam on the brakes to stop quickly. The friction force on the wheels slows the vehicle in a horizontal direction and causes it to rotate due to the torque acting about the center of mass. As the front end moves down, the vertical force on the front wheels increase. This results in a torque that acts in the opposite direction and prevents the vehicle spinning like a ball.

When a vehicle is at rest or moving at constant speed, the vertical force on the front wheels is roughly the same as the vertical force on the back wheels.

Together, the total vertical force on all four wheels is equal to the weight of the vehicle. When the brakes are applied, the vertical force on the front wheels increases and the vertical force on the rear wheels decreases as if the weight of the vehicle shifted forwards. The engine and the passengers and everything else stays roughly where they were so nothing actually moves towards the front wheels, but the effect is the same so we say that the weight shifts forwards.

A similar thing happens to a tennis ball when it bounces, since the vertical force at the front of the ball is larger than the vertical force at the back of the ball. The distance between the front and the back of the ball is too small to stop it rotating, but there is a shift in the "weight" of the ball, and there is a torque acting backwards on the ball that stops it spinning as fast as it otherwise would. The vertical force on a ball when it is bouncing has nothing to do with its actual weight since its actual weight is tiny compared with the vertical force due to compression of the ball. However, the front of the ball will compress more than the back of the ball due to the fact that the ball is rotating. The amount of compression depends on the vertical speed at which the ball hits the surface.

If you drop a ball from a height of, say, 3 feet, the ball will compress by about 1/4-inch before it bounces back up. If you drop it from a height of 12 feet, the ball will hit the surface at twice the speed and compress by about 1/2 inch. When a ball is spinning, as in Figure 40.6, the front of the ball strikes the surface at a higher speed than the back of the ball so it will compress more. This was not shown in Figure 40.6 since we considered only the combined effects of horizontal motion and rotation. If we add in the vertical motion, then we would get the complete picture, including a shift in the "weight" forwards. The result is that the normal reaction force, N, acts vertically through a point ahead of the center of the ball as shown in Figure 40.7.

There are more symbols than necessary on this diagram, but they are needed for a complete mathematical description of the bounce as described by Cross (2003a, b). The important feature here is that if N is shifted by a distance D, then there is a torque ND that opposes the torque FR, where R is the ball radius. Measurements of ball spin and speed show that D is typically about 4 mm, but it can be as large as 11 mm on a clay court. The large value of D on clay is probably due to an accumulation of clay under and towards the front of the ball, since the ball sweeps clay particles along and leaves a definite depression in the court.

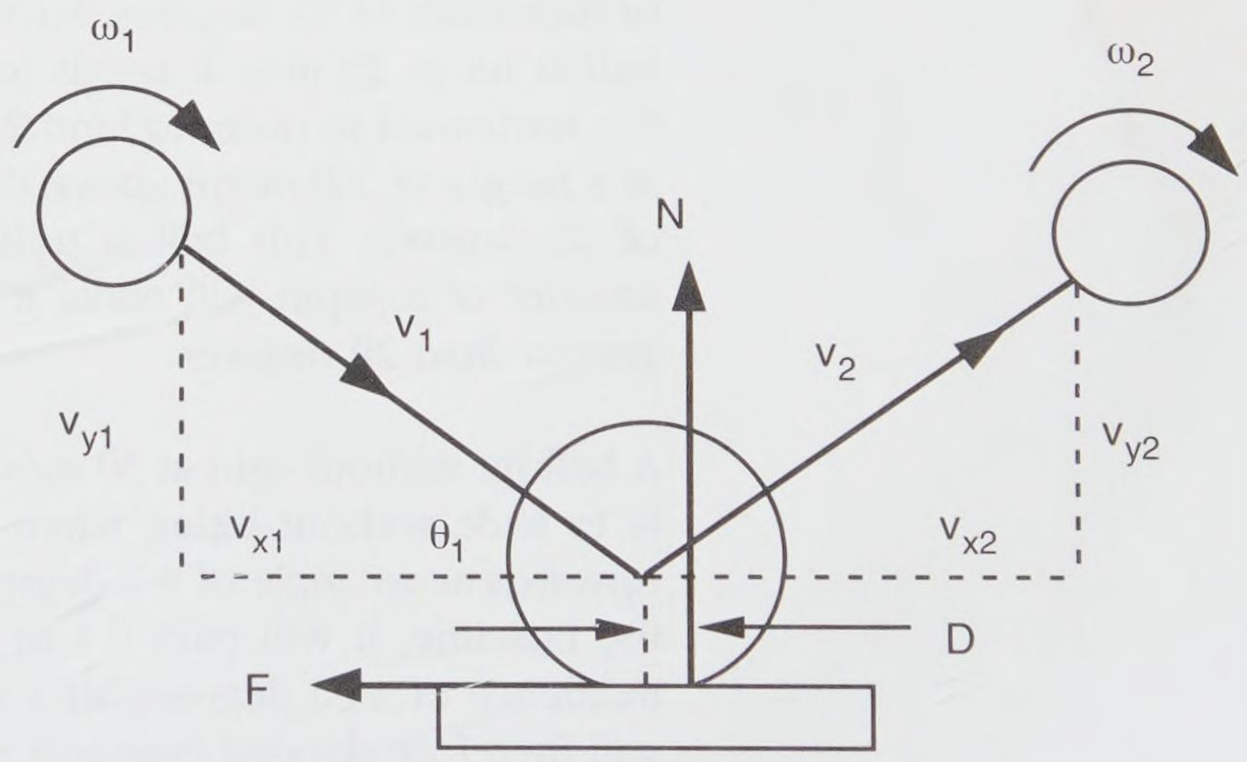


Figure 40.7
Geometry of a bouncing ball.

A consequence of all this is that the speed of a court depends not only on the COF and the COR but also on D. The value of D is not important if the ball slides throughout the bounce but it is important if the ball bites. It is for this reason that, in Table 39.2, the smooth concrete slab was the fastest surface at low angles, but it became the slowest surface at high angles.

Perception of Court Speed

An interesting question is whether players base their estimate of court speed on low angle bounces or high angle bounces or both. I suspect that it is mainly on medium pace, high angle bounces since the ball travels too fast in a fast serve for players to make a reliable estimate of the court speed. Furthermore, the percentage change in ball speed is larger at high angles than at low angles, meaning that differences in court speed are magnified at high angles of incidence, not only in a relative sense but also in absolute terms. A medium pace groundstroke takes about 0.1 seconds longer to cross the baseline on a clay court than on a grass court. A 0.1 sec difference is much easier to pick than say a 0.02 sec difference. A fast serve at 120 mph that lands on a clay court takes only about 0.02 seconds longer to cross the baseline than on a grass court, assuming that it is served at the same angle and the same speed. But how can players judge if the 0.02 s difference was due to a change in court speed rather than a change in serve speed or angle? I suspect they can't. It would be interesting to do an experiment using a radar gun to find out if they can pick the difference between a 120 mph and a 124 mph serve, since the ball would then take 0.02 s longer to cross the baseline for the slower serve.

Calculations of ball trajectories show that the ball can bite the court for a medium pace groundstroke even if the ball is hit without any topspin. For example, suppose that a ball is hit without spin from a height of 1 meter above one baseline and it lands 2 meters inside the other baseline. If it is hit at 20 m/s, it would need to be hit upwards at 20 degrees to the horizontal. It will pass 2.0 meters above the top of the net and it will land 2 meters inside the baseline at an angle of incidence of 31 degrees. Such a ball will bite on essentially all courts. If the ball is hit at 25 m/s, it needs to be hit upwards at an angle of 11 degrees to the horizontal in order to land 2 m inside the baseline. It will pass over the net at a height of 1.0 metre above the net, and it will land at an angle of incidence of 20 degrees. This ball is unlikely to bite if it has no topspin, but a small amount of topspin will make it bite since the angle of incidence will then be greater than 20 degrees.

A ball hit without spin at 30 m/s is relatively fast for a groundstroke and is likely to slide without biting when it bounces. Such a ball would need to be hit upwards at an angle of 6.7 degrees to the horizontal in order to land 2 m inside the baseline, it will pass 0.5 m above the net and it will land at an angle of incidence of 14.6 degrees. At a speed of 30 m/s or more, the speed of a court will therefore depend primarily on the coefficient of sliding friction. At a speed of 25 m/s or less, the speed of a court will depend on the coefficient of friction but it will also depend on D.

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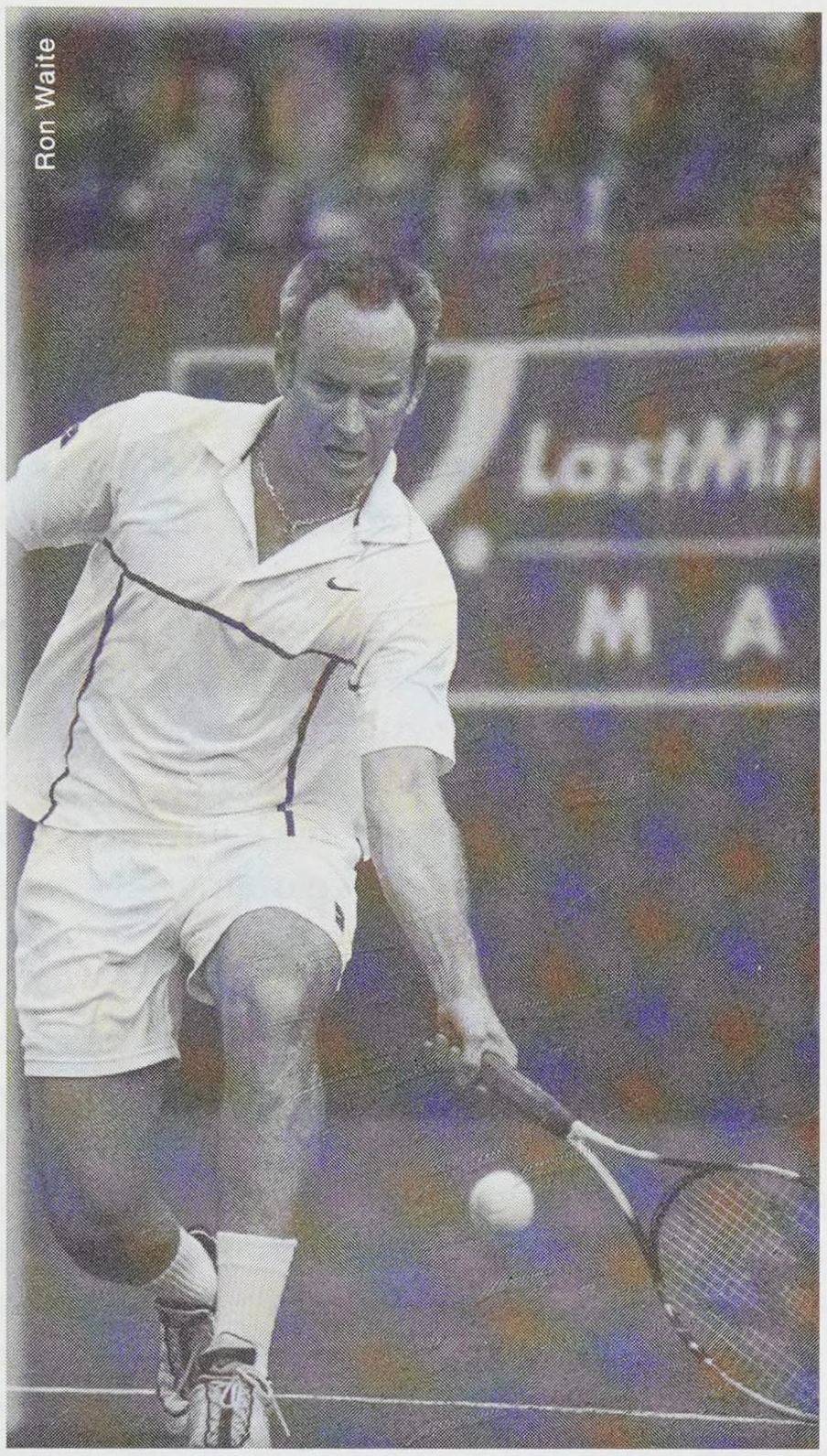


John McEnroe, United States

41

Ball Spin and Bounce Off A Racquet

Spin, Bounce and Strings

By Rod Cross

Introduction

Players these days like to put as much topspin or backspin on the ball as possible, consistent with reasonable ball control. There are many different opinions as to how this is best achieved. The most common are:

- thin strings dig into the ball, allowing players to put more spin on the ball.
- thick strings, widely spaced, are better so that the ball squeezes between the strings
- Strings with a rough texture will get a better grip on the ball.
- Soft strings like natural gut act to cup the ball, which gives a better grip on the ball.
- hit the ball on the rise
- hit the ball when it is dropping
- hit the ball towards one edge of the ball instead of in the middle
- grip the racquet so that the racquet face is inclined forwards
- for a right-handed serve, toss the ball over the left shoulder

Obviously, all of these methods can't be correct, but some of them are. From a physics point of view, there are only three simple rules regarding spin:

Spin Rule 1: The faster you hit the ball the faster it will spin.
Spin Rule 2: The strings don't make much difference.
Spin Rule 3: Maximum spin occurs at an angle of incidence near 40 degrees.

Rule 2 will surprise many players and is explained in more detail below. Rule 3 means that an insect sitting on the strings would see the ball coming in at an angle of about 40 degrees to the string plane. Most tennis players can't hit the ball comfortably at that angle, but a lot of table tennis players do (see Note 1). The least amount of spin occurs if the ball strikes the strings almost at right angles (depending on the spin of the incident ball) or if the ball hits the strings at a very small grazing angle and just brushes the strings lightly. The reason that about 40 degrees gives the maximum spin is that, at that angle, the ball stops sliding and starts biting.

Players tend to use more spin on slow courts such as clay, in part to get the ball to kick up at a steeper angle. On grass courts, the ball slides and tends to stay low after bouncing. Players can take advantage of the low bounce on grass by hitting the ball with less topspin, or with backspin, or by hitting the ball faster and lower over the net, so that the ball does not bounce up into the comfortable waist high hitting zone.

If you watch a ball spinning, it is impossible to tell how fast it is spinning apart from "pretty fast" or "pretty slow". So how do players know if the ball spins faster or not? The easiest way is to observe how much the ball curves through the air. The bigger the spin the more the ball curves. If one is comparing, say, thin strings with thick strings, it might be hard to pick the difference in spin or the curve if the difference is only small. Are the players talking about 20% extra spin with thin strings or is it only 2%? The best way to check this out is to measure the spin, as well as the speed and angle of the racquet head, to make sure that everything else stays the same when the string gauge is varied. Up until about 1996 there were very few measurements of a spinning tennis ball. High speed cameras are now available to capture the spin, but they are expensive. As a result, measurements of a spinning tennis ball can still be counted on one hand.

The amount of spin off a racquet or off the court depends on a whole bunch of factors, some of which are important, some are not so important, and some have never been measured, so it is not actually known for certain if they are important or not. Recently, measurements of spin off the strings were made by Simon Goodwill at the University of Sheffield. The ball was fired at the strings at speeds up to 40 m/s, typical of a high speed groundstroke. These measurements are the first to our knowledge on how the spin varies with string gauge. The main result is that it doesn't. It doesn't vary much with string tension either. Rob Bower obtained a similar result in 1996 for his Master's thesis at Sydney University. Rob fired balls at 45 degrees to the string plane and measured the rebound spin for three different string tensions. The spin increased by only 5% when the tension was increased by 50% (from 18 kg to 27 kg).

Bounce Basics

The basic physics of a ball that bounces obliquely off a surface is described in Chapters 39 and 40. In general, the speed, spin and angle all change when a ball bounces, by amounts that depend on whether the ball slides throughout the bounce or whether it starts to bite during the bounce. We considered only

a bounce off the court in Chapter 39, but a bounce off the strings is essentially the same. The main difference is that the racquet head is pushed away from the ball if the racquet is initially at rest. Alternatively, the racquet head slows down if it collides with a ball. Another difference is that strings are relatively slippery compared with most court surfaces so the angle of incidence at which a ball stops sliding and starts to bite is typically about 40 degrees on the strings, compared with about 20 degrees on a court. Most balls are incident at angles less than 20 degrees on a court so most balls slide throughout the bounce on the court. By contrast, most balls are incident on the strings at angles greater than 40 degrees and they bite the strings (or equivalently, the strings bite the ball).

The difference between sliding and biting is illustrated in Figure 41.1. A ball incident obliquely on a surface without any spin will initially start to slide on the surface. As it does so, it slows down in a direction parallel to the surface and it starts rotating. If it acquires enough spin, then the bottom of the ball can rotate backwards at the same speed that the whole ball moves forwards, in which case the bottom of the ball comes to rest and it bites the surface. The ball can still advance forwards even though some points remain stuck, since points at the back of the ball lift up off the surface as the ball rotates while other points at the front of the ball rotate down onto the surface. This is shown in Figure 41.1. The point on the ball in contact with point B on the surface remains stuck when the ball bites, while the point on the ball that was originally stuck on point A lifts up off the surface.

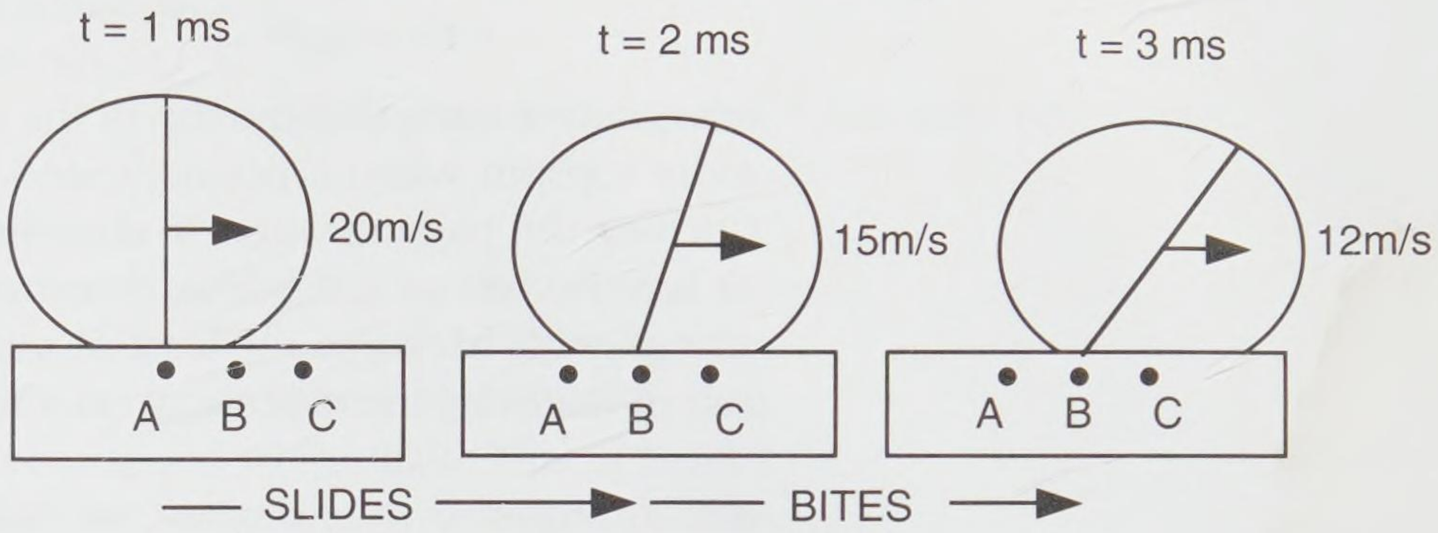


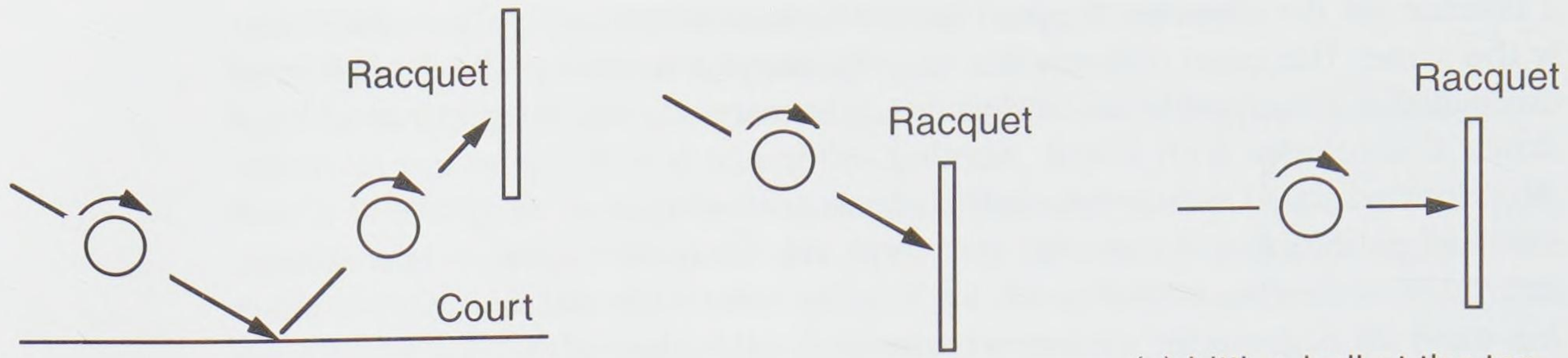
Figure 41.1

Sliding and biting. A point on the bottom of the ball slides from A to B and then remains stuck at point B when the ball bites.

As explained in Chapter 40, biting and rolling are similar for a tennis ball. If a ball rolls during the bounce, then the rebound spin and speed do not depend on the coefficient of friction of the surface, but they do depend on the angle of incidence (see Note 2). In essence, this explains the experimentally observed fact that the ball spin off the strings does not depend on whether the strings are thin or thick, rough or smooth or soft or stiff. These differences would be important if a ball slides along the strings throughout the whole bounce period, but this would require the ball to be incident on the strings at an angle less than 40° to the string plane. More commonly, the ball is incident closer to a right angle to the string plane, in which case the ball will bite before it bounces off the strings.

Topspin or Backspin Into the Strings?

When a ball bounces off the court, it acquires even more topspin than it had when the player at the other end of the court hit the ball. A moderately fast ball arrives at the receiver spinning between 50 and 100 revolutions/sec (314 to 628 radians/sec), but as far as the receiver is concerned, the ball could be incident on the strings either with topspin or with backspin, as shown in Figure 41.2. Figure 41.2a shows a situation where the ball is incident on the court with



(a) hitting ball on the rise

(b) hitting ball as it drops

(c) hitting ball at the top of its bounce

Figure 41.2

Hitting a ball on the rise or as it drops or at the top of its bounce.

topspin (meaning that the top of the ball is spinning forwards), it acquires even more topspin when it bounces, and it is incident on the racquet with topspin (turning the page through 90 degrees will show this more clearly). If the player hits the ball as it is falling downwards, then the ball will be incident on the strings with backspin. At least, it certainly looks that way in Figure 41.2b. In fact, it might be the other way around depending on what the racquet is doing.

If the racquet is at rest when the ball hits the strings, then the ball is incident on the strings with topspin if it is rising, and it is incident with backspin if it is dropping, exactly as shown in Figure 41.2. But racquets are rarely at rest when they hit a ball. If the racquet is moving horizontally at 20 m/s when the ball hits the strings, and if the ball is traveling towards the racquet with a horizontal speed of 20 m/s, then the ball approaches the racquet at a horizontal speed of 40 m/s relative to the racquet. That will change the angle of incidence on the strings (since the relative horizontal speed changes, but the relative vertical speed is unaffected), but it won't change a ball incident with topspin into one incident with backspin (or vice versa).

Now consider a situation where the racquet is rising upwards at around 30 degrees to the court when the racquet hits the ball. If the ball is also rising upwards at 30 degrees to the court, and if the speed of the ball is the same as the speed of the racquet, then the height of the ball above the court will remain the same as the height of the racquet, so the ball is heading at right angles onto the strings. An insect sitting on the strings would see the ball coming in at right angles. In that case, we can't say that the ball is incident with topspin or backspin. It is incident with spin, but it doesn't make any sense to say it is incident with topspin or backspin. Think about Figure 41.2a, b and c. In (a) the ball is incident on the racquet with topspin. In (b) it is incident on the racquet with backspin. In (c) it is incident on the racquet with spin but it is neither topspin nor backspin.

If a spinning ball is incident at right angles to the strings, then friction between the ball and the strings will act to reduce the rate at which the ball spins. In most cases of interest, there will be no reversal in the spin direction, unless the ball happens to be a superball. If you happen to have a superball, then try it. A superball that is dropped vertically onto a hard floor with spin will bounce with a small amount of spin in the opposite direction. Suppose the racquet and the ball are both traveling horizontally towards each other in Figure 41.2c. The ball is incident from left to right with topspin with respect to the court surface, but it is neither topspin nor backspin with respect to the strings. The ball will

still spin in the same direction after it is hit, but it then travels right to left so it has backspin with respect to the court. In order to hit a ball so that it has topspin after it bounces off the strings, the direction of the spin has to be reversed when the racquet hits the ball, as indicated in Figure 41.3. That requires a bit of effort on the part of the player, and it is necessary that the racquet rise upwards faster than the ball. Even greater topspin can be imparted to the ball if the ball is falling rather than rising when it is hit (assuming that the ball is hit with the same speed), since the ball will be incident on the strings at a smaller angle to the strings.

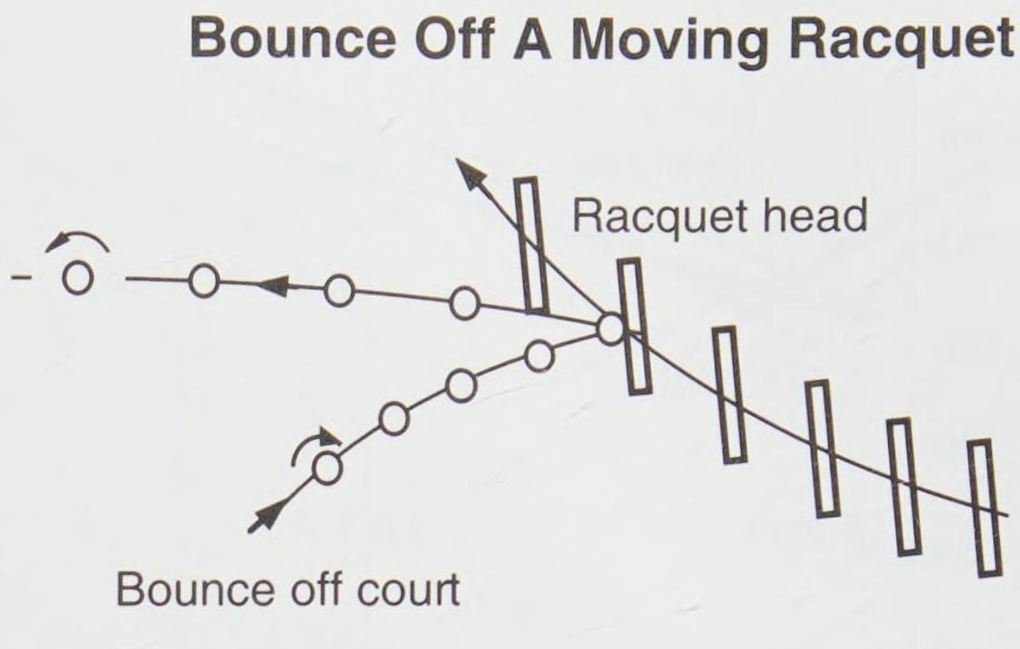


Figure 41.3

Hitting a rising ball so that it rebounds with topspin.

If the ball is rising at 20 degrees and the racquet is rising at 30 degrees, and if the ball and the racquet have the same speed, then the racquet is rising up faster than the ball as if the ball was falling downwards to meet the racquet. In that case the ball would be incident on the strings with backspin, as in Figure 41.2b. But what if the racquet is travelling at a different speed to the ball? That is harder to think through logically but the situation can be analyzed with a bit of simple mathematics as shown at the end of this Chapter.

Ball Speed, Spin and Angle Measurements

The performance of the strings in a racquet can be measured by firing a ball at the strings and measuring the rebound speed and angle of the ball (Knudson, 1997; Bower and Sinclair, 2000). Manufacturers do this routinely, but in most cases, the ball is fired perpendicular to the string plane in order to measure the performance of the racquet. Some manufacturers fire balls at an oblique angle in order to measure the durability of the strings when subject to repeated impacts. It is only very recently that the spin has been measured using a high speed video camera. This was done in 2001 by Simon Goodwill at the University of Sheffield as part of his PhD thesis, by firing balls at a racquet at speeds between 22 m/s and 39 m/s and an angle of incidence of 50 degrees to the string plane – i.e., 40 degrees to the normal (Goodwill and Haake, 2002). The angle of incidence was not varied, but the amount of spin on the incident ball was varied from -600 rad/s (the minus sign meaning backspin) to +100 rad/sec.

The racquet head was firmly clamped to stop it flying away every bounce, but this does not detract from the relevance of the experiment. It simplifies the experiment. It shows directly the difference between the strings and a court, and it is relatively easy to calculate from these results the effects of NOT clamping the head. This is a similar situation to that described in Chapter 14 where the COR can be measured by clamping the racquet head and bouncing a ball at right angles off the strings. The experiment could also have been done by clamping the handle instead of the head, which would be one step closer to the real thing, but the results are still very interesting. Some of these results are shown in Figure 41.4, and the main conclusions are:

(a) the rebound spin of the ball does not depend significantly on string tension, string type, string gauge, or spacing between the strings;

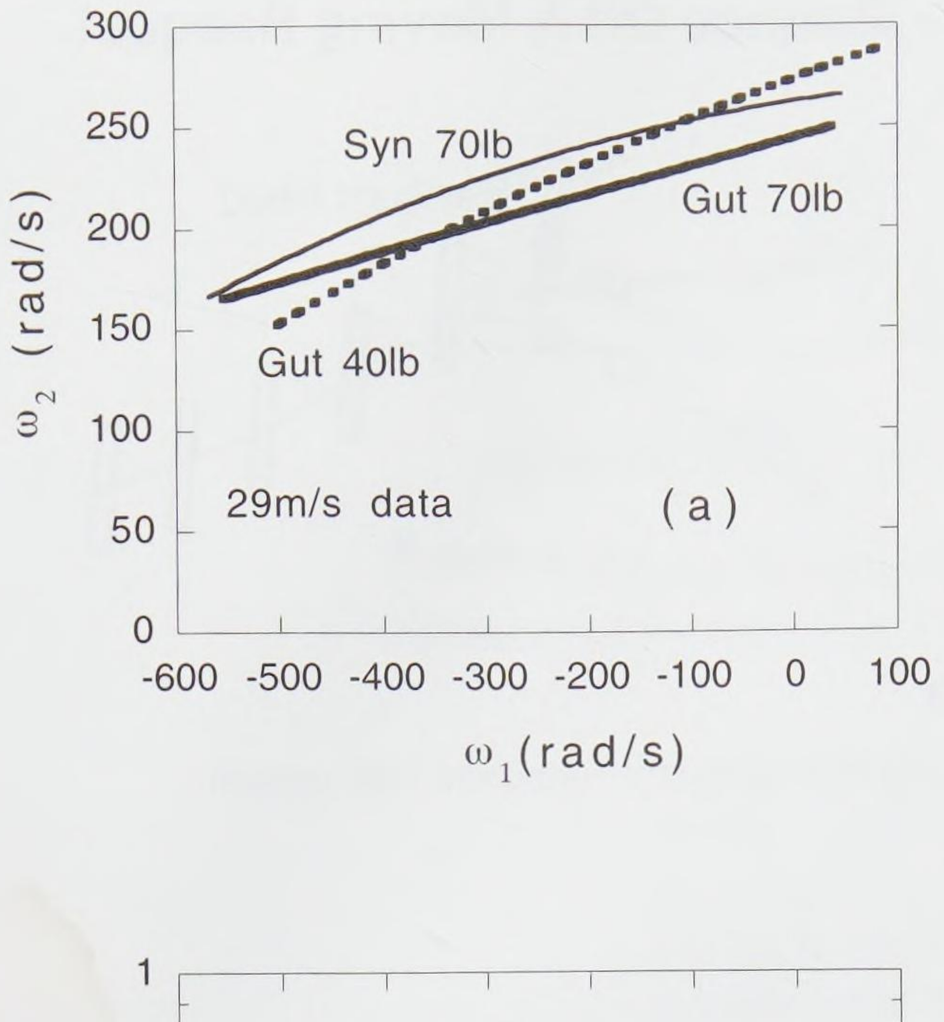


Figure 41.4

Rebound spin, speed and angle for a ball incident at 29 m/s and at 40 degrees to the normal vs incident ball spin. The racquet head was clamped to prevent it rotating. Results are shown for natural gut and nylon (Syn) strung at 40 lb or 70 lb. θ_2 is the rebound angle to the normal.

(b) the rebound speed of the ball increases slightly as the tension is lowered, and

(c) the rebound angle depends on string tension, string type and the spacing between strings. The ball rebounds at an angle closer to the normal as the string tension is reduced.

All three parameters depend on the spin, ω_1, of the incident ball as shown in Figure 41.4. Even though the ball was incident with backspin, it bounced off the strings with topspin in all cases. As one might expect, the spin of the ball is reduced when the incident ball has a lot of backspin, but the effect is not very strong. For example, if ω_1 is decreased from 0 to -600 rad/s, then ω_2 decreases from about 250 rad/s to about 150 rad/s. Brody's (1984) bounce model indicates that if ω_1 is decreased by 600 rad/s, then ω_2 will also decrease by 600 rad/s if the ball slides throughout the bounce. If the ball starts rolling during the bounce, then Brody's model indicates that ω_2 should decrease by 213 rad/s. The actual decrease in ω_2 from 250 to 150 rad/s can be explained by the fact that the ball bites the strings.

Clues to the behavior of the strings can be found in measurements of the v_{x2} / v_{x1} ratio, the COR, and the ratio $S = R\omega_2 / v_{x2}$, as shown in Figure 41.5. On a tennis court, v_{x2} / v_{x1} is typically about 0.4 to 0.5 at high angles of incidence, as described in Chapter 39. In Goodwill's experiment, the ball was incident at an angle of 50° to the string plane, and the v_{x2} / v_{x1} ratio was about 0.42 on all strings tested when the ball was incident without spin. This ratio dropped to much lower values when the ball was incident with backspin, since it took longer for the ball to stop sliding before it started to bite. The results in Figure 41.5c show that the ball gripped all strings during the bounce, since $R\omega_2 / v_{x2}$ was larger than 1.0 in all cases. When the ball first hit the strings, it started sliding on the strings, but the friction force caused it to slow down, reverse its direction of spin and then bounce with topspin.

The ball slowed down most on natural gut at low tension and least on the nylon strings at high tension, as if the ball dug deeper into the softer strings than the stiffer strings. Alternatively, the softer strings might stretch further in a direction parallel to the string plane, acting in a manner similar to the spaghetti strings described in Chapter 39. The latter effect appears to be the most significant one. Measurements of ball spin and speed indicate that the normal reaction force acts ahead of the center of the ball by a distance of about 3 or 4 mm on each of the strings tested and at all values of the

incident spin. This is similar to the behavior on a court surface. Even if the ball does dig into the strings, it does not shift the normal reaction force any more than it does on a court. On the other hand, the results in Figure 41.5c show that $R\omega_2 / v_{x2}$ is largest for gut at low tension, particularly when the ball is incident with a lot of backspin. Values of $R\omega_2 / v_{x2}$ greater than about 1.4 are normally observed only with a superball or with spaghetti strings.

Superballs spin much faster than other balls when they bounce, and so do tennis balls when they bounce on spaghetti strings. In both of these cases, there is a highly elastic deformation in a direction parallel to the surface, resulting in extra spin and a larger than usual slowing down of the ball in a direction parallel to the surface. Figure 41.4a shows that there is no significant effect on spin, but Figure 41.5a shows that there is a big effect in slowing the ball in a direction parallel to the surface. Part of the reason for the low values of v_{x2} / v_{x1} with large backspin is that the ball takes longer to stop sliding so the friction force acts for a longer time, but the ball is especially slow at low string tensions. This indicates that loose strings move further in the string plane and take more sideways energy out of the ball, but the strings do not spring back, so there is not the same increase in ball spin as there is with spaghetti strings. In other words, the large values of $R\omega_2 / v_{x2}$ shown in Fig. 41.5c are not due to excessive spin, but to a big slowing down of the ball on the looser strings.

An additional effect on soft strings is that the COR is enhanced at all values of the incident spin (when compared with stiff strings). The combined effect of the increase in the COR and the decrease in v_{x2} / v_{x1} on soft strings is that the ball bounces off the strings at a steeper angle (closer to a line perpendicular to the string plane). If you examine the diagrams below, you will see that this will have the effect of sending the ball higher over the net. When players break a string during a groundstroke, the ball usually flies out rather than landing in the net. However, a more serious problem would arise if string movement at low tension was more variable than string movement at high tension, as one might expect. In that case the player would lose some control of the ball, especially if the player spins the racquet around between shots and hits from both sides of the racquet. Some players (especially Lleyton Hewitt) like to push the strings back into position between each point. It seems to work for him!

In order to determine the behavior of a ball bouncing off a hand-held racquet, the data shown in Figure 41.4 needs to be incorporated into a model that takes into account

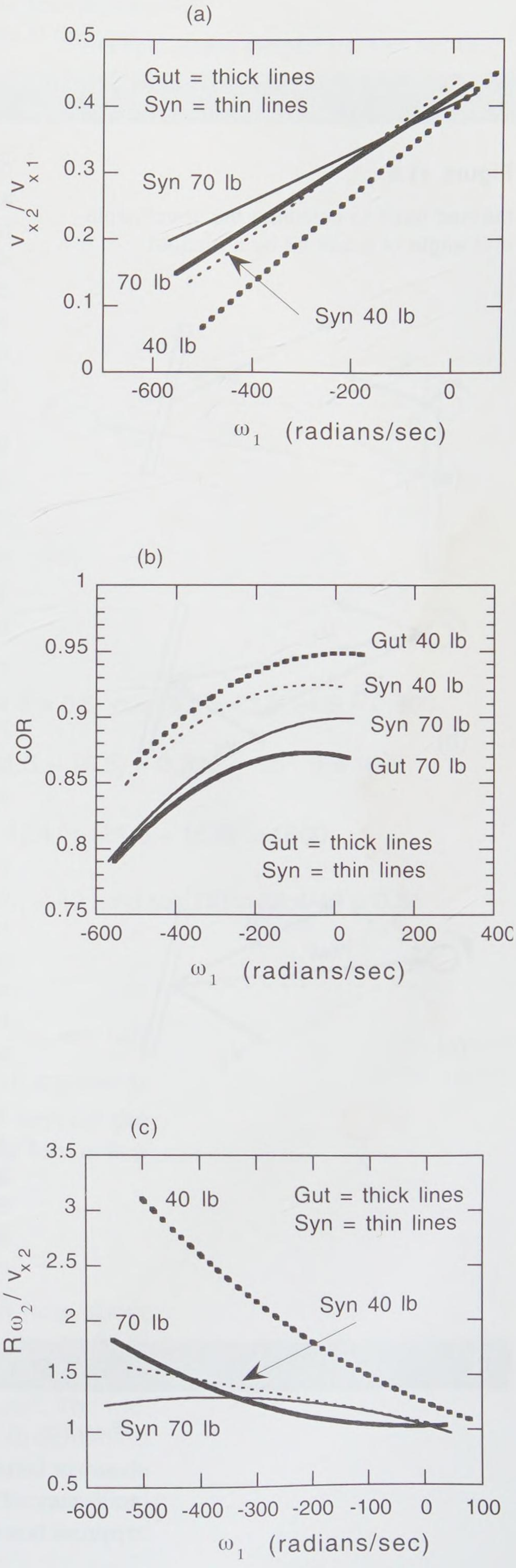


Figure 41.5

the speed and direction of the racquet, as well as the speed and direction of the ball (Cross, 2000).

How To Calculate the Bounce Off a Moving Racquet

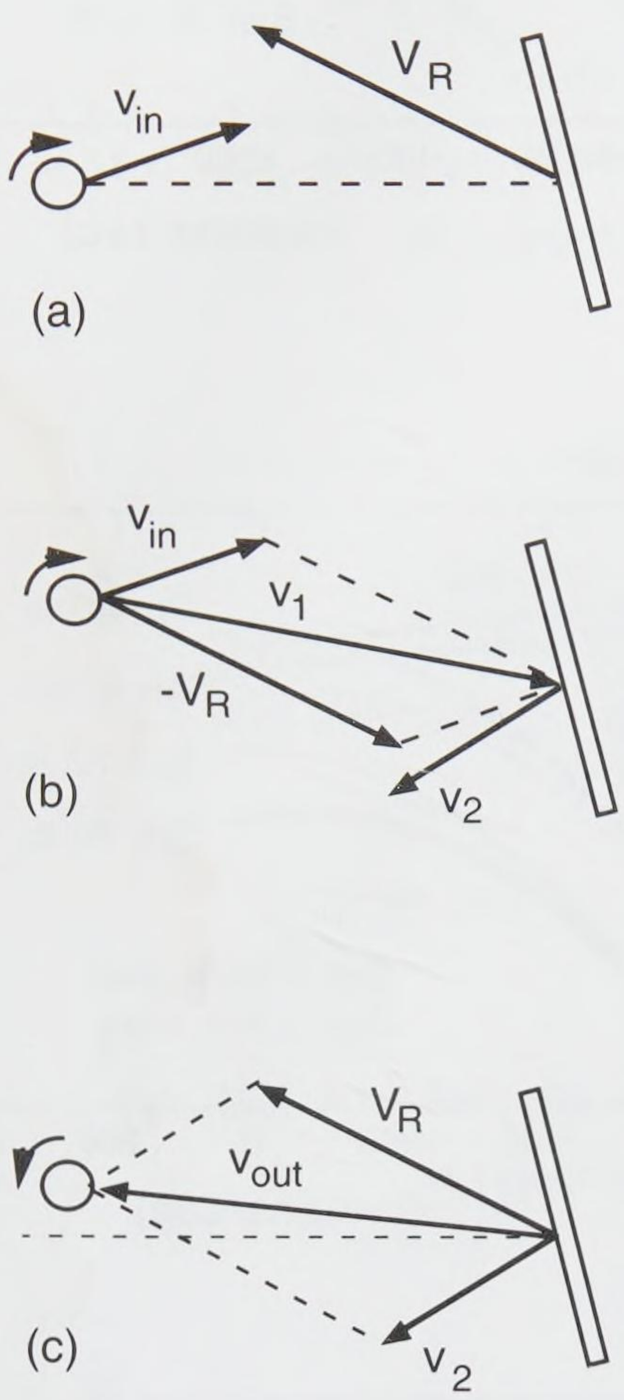


Figure 41.6

Method used to calculate the speed, spin and angle of a ball hit by a racquet.

Suppose that the ball has topspin off the court and is rising upwards as in Figure 41.6a. Suppose also that the racquet head is tilted fowards and is rising upwards to meet the ball. The speed and direction of the ball and racquet are represented by the length and direction of the two arrows. It is easiest to think about this situation if we first imagine that the racquet is at rest. To do that, we need to change to the racquet frame of reference and then change back to the court frame of reference, as follows:

If we subtract V_R from the speed of both the racquet and the ball, then the racquet is at rest, and the ball approaches the racquet at speed v_1 and with backspin, as shown in Figure 41.4b. It bounces off with speed v_2 and with topspin (if the ball is hit hard enough). The v_1 arrow is obtained by drawing the parallelogram (dashed lines), and v_2 needs to be calculated or measured separately. The racquet will also change speed when it is struck by the ball.

Now add V_R back to the speed of the ball (by drawing the parallelogram) as shown in Figure 41.6c. Then the ball bounces off with speed v_{out} and with topspin.

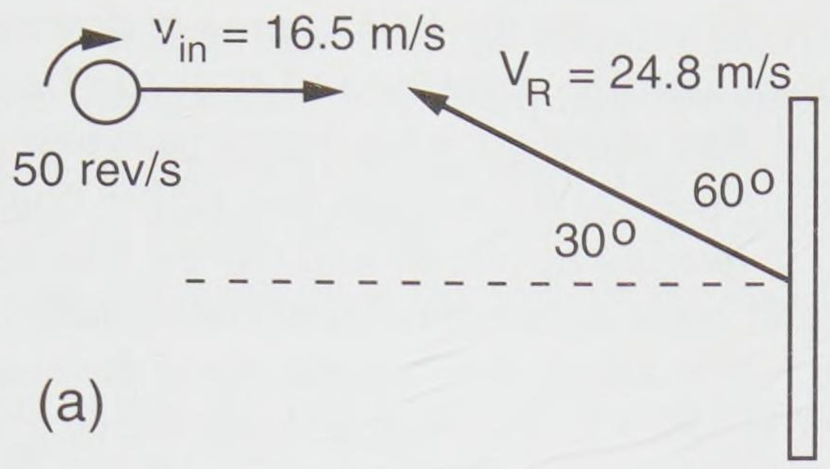
Note that changing the frame of reference has no effect on the spin of the ball. It just changes the speed and angle of incidence or reflection of the ball onto or off the racquet. This is an important effect that is sometimes overlooked. If a ball is fired at a stationary racquet to measure the angles of incidence and reflection, then those angles will be completely different when the racquet is actually swung towards the ball.

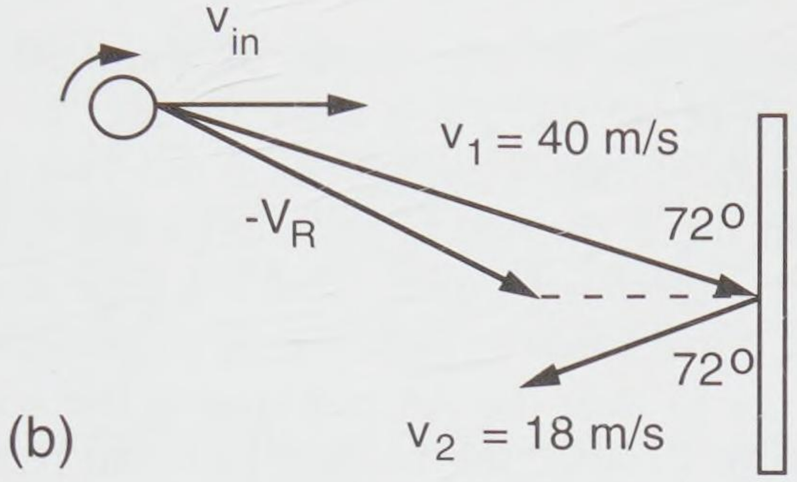
The situation shown in Figure 41.6 is typical of a topspin forehand. A ball hit on the rise is usually incident with backspin on the strings. It is also incident with backspin if the ball is hit at the top of its bounce or if it is hit as it falls towards the court, but the angle of incidence changes in each case. This is shown in the following example where the ball is traveling horizontally before it hits the strings.

Example of Bounce Calculation

The data in Figure 41.7a are typical of a topspin forehand where the racquet head is vertical and rising upwards at 30° to the meet the ball at the top of its trajectory. From the geometry (see Figure 41.7d), we find that the ball approaches the racquet at speed v_1 = 40 m/s with backspin and at an angle of 72° to the string plane.

Calculations show that the ball bounces at an angle of 72°, with speed v_2 = 18 m/s and with topspin at 15 rev/sec, as shown in Figure 41.7b.





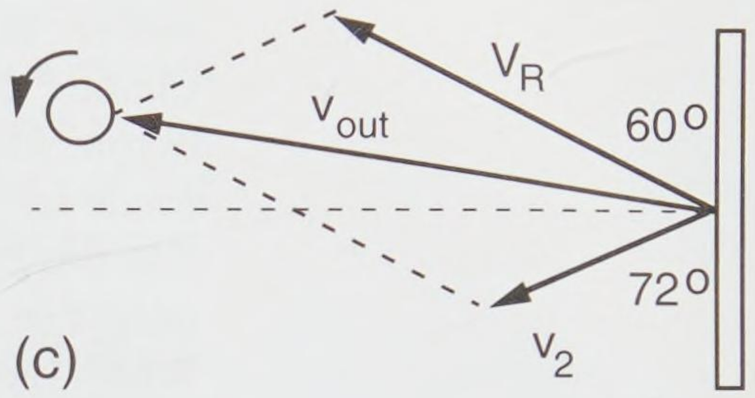
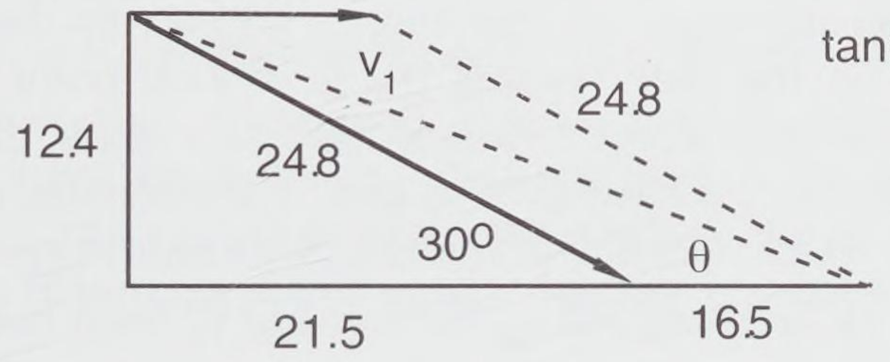


Figure 41.7

Example where racquet rises at 30° to meet the ball.

(d) Incident ball geometry



$\sin(30) = 12.4/24.8 = 0.5 \quad \cos(30) = 21.5/24.8 = 0.866$

$\tan(\theta) = 12.4/(21.5 + 16.5) = 0.326 \quad$ so $\quad \theta = 18^o$

$v_1^2 = 12.4^2 + (21.5 + 16.5)^2 = 1600$

so $\quad v_1 = 40$ and $\sin(18) = 12.4/40 = 0.31$

Adding V_R back to the speed of the ball (Figure 41.7c) shows that the ball bounces with speed v_{out} = 39 m/s, at 10° to the horizontal and with topspin at 15 rev/sec. Hit from the baseline, the ball would actually land beyond the opposite baseline. A solution is not to hit so hard, or tilt the head forwards a fraction.

Serving with Topspin

We have seen how a groundstroke can be hit with topspin, but how about serving with topspin? Does the same method work here? In order to hit a groundstroke with topspin, the racquet head needs to rise upwards to meet the ball. That way, the strings brush upwards against the back of the ball. The friction force between the ball and the strings then acts vertically upwards and causes the ball to rotate with topspin. The ball is ejected forwards and upwards out of the strings in order to pass over the net. That's because the racquet head is moving forwards and upwards when it contacts the ball, and hence the force on the ball is forwards and upwards.

A serve is different since it is hit from a point well above the player's head. That presents two problems. First, how can you get the racquet head to rise

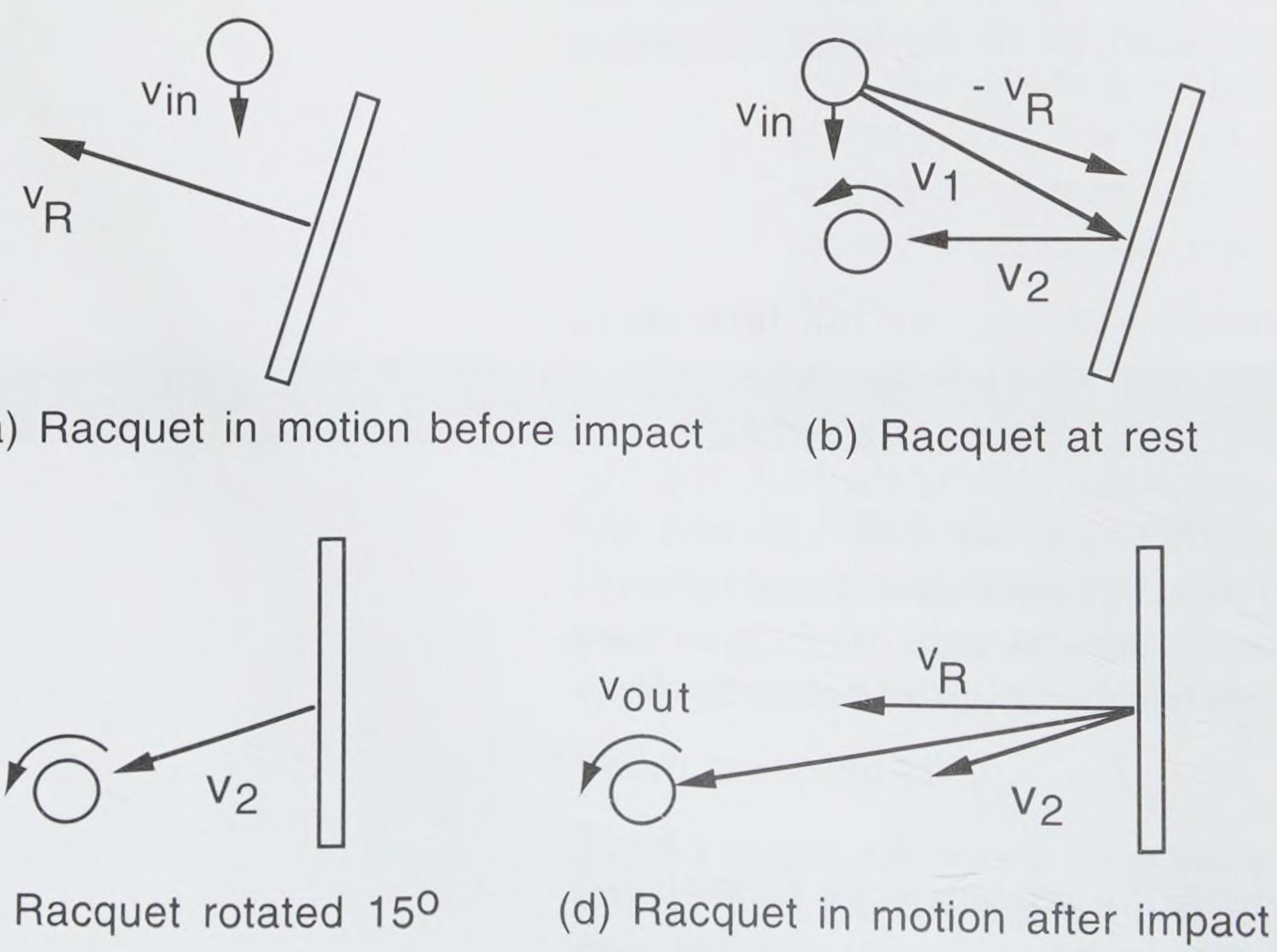
upwards as it contacts the ball when the racquet head is almost as high as you can reach? Even if you can, how do you get the ball to project downwards so it lands in the service court rather than fly upwards and over the back fence? I have not seen any high speed film showing what happens when a ball is served with topspin, but it is likely that (a) the racquet first makes contact with the ball while the racquet head is moving upwards, and (b) the ball leaves the strings 5 ms later when the racquet head is moving almost horizontally. In addition, a high ball toss causes the ball to move downwards along the strings during the serve. That is equivalent in effect to the racquet head moving upwards along the ball even if the racquet head is actually moving horizontally.

In a moderately high speed serve, the racquet rotates at about 80 rad/s just before impact and slows to about 27 rad/s just after impact, as shown in Chapter 20. At this speed, the racquet rotates 23 degrees in the 5 ms period just before impact and 8 degrees in the 5 ms period just after impact, so it must rotate by about 15 degrees during the impact itself. That is a large angle, given that the ball must be projected downwards to pass through a narrow window only 2 or 3 degrees wide in order to clear the net and land in the service box. When hitting a groundstroke with topspin, the racquet face can remain vertical immediately before, during and immediately after impact. That is not the case when serving the ball.

The position of the racquet face during a serve is shown in Figure 41.8. The racquet face is rising upwards at the instant of contact, it is moving forwards, and it may also be moving sideways, i.e., moving up and across the ball from say the 8 o'clock point on the ball towards the 2 o'clock point. Sideways motion will cause the ball to spin about a vertical axis. As a result, the ball will swerve sideways like a hook or a slice shot in golf. The situation just before impact is shown in Figure 41.8a. The ball is falling downwards at speed v_{in} and the racquet is rotating in an arc at speed v_R. Subtracting v_R from both the racquet and the ball (Figure 41.8b) shows the situation in a reference frame where the racquet is rotating about an axis that remains at rest. In that frame, the ball is incident at speed v_1, and it bounces off the strings at speed v_2. At least, that's what would happen if the racquet was originally traveling in a straight line. In fact, the racquet rotates by about 15 degrees during the impact.

If we rotate the racquet and the v_2 arrow by 15 degrees, then we have the situation shown in Figure 41.8c. This is not a strictly valid procedure but it is roughly correct. The force acting on the ball remains essentially perpendicular to the string plane throughout the impact, meaning that it varies in direction by up to 15 degrees during the impact. It varies by the full 15 degrees only at the very end of the impact. The effect on the ball is that its trajectory will change by something less than 15 degrees, and probably by about half of that.

Figure 41.8
Topspin serve



If we assume that the situation in Figure 41.8c is roughly correct, then the rotation axis is still at rest but the racquet has rotated through 15 degrees. To determine the situation from the player's reference frame we need to add v_R to the speeds in Figure 41.8c, as shown in Figure 41.8d. Note that we have also rotated the v_R arrow through 15 degrees since the player sees that the racquet is travelling horizontally at the instant the ball leaves the strings.

The result is that the ball is served with topspin and it is projected downwards at a small angle to the horizontal. From the player's point of view, the racquet is traveling upwards (and forwards) immediately before impact, and this imparts topspin. Since the ball is dropping vertically just before and at impact, this acts to deflect the ball downwards. During the impact the racquet swings around to a more horizontal path, but since the ball is traveling downwards along the string plane, the racquet is still rising upwards with respect to the ball. The friction force on the ball therefore imparts topspin throughout the impact, even if the racquet is travelling horizontally near the end of the impact.

A rough estimate of the amount of topspin imparted to the ball is as follows. Suppose that v_{in} = 3.7 m/s and v_R = 35 m/s. Then the ball is incident in Figure 41.8b at v_1 = 35.2 m/s and at an angle of 84 degrees to the string plane. If the ACOR is 0.4 then the ball will bounce in Figure 41.8b at v_2 = 14 m/s and at about 84 degrees to the string plane. The bounce speed in a direction parallel to the string plane is v_{x2} = 14 x cos(84) = 1.46 m/s. Assuming that the ball rolls, it will spin at $v_{x2} = R\omega$ and hence $\omega = v_{x2}/R$ = 44 rad/s = 7 rev/s. This is much smaller than the subsequent spin when the ball bounces off the court. However it is enough to alter the trajectory of the ball significantly, as shown in Figures 30.14 and 42.3. High speed video film of a topspin serve would be needed to determine if this rough estimate is reliable.

Greater topspin can be imparted to the ball during a serve if the racquet head first makes contact with the ball at an earlier stage during the swing. The head will then be rising faster at impact, and the ball will be incident on the strings (in the racquet frame of reference) at an angle that is not as close to the normal as shown in Figure 41.8b. The player might perceive that he or she is then hitting the ball at a significant angle upwards from the horizontal, but the ball will be (and needs to be) ejected from the strings at a small angle downwards from the horizontal if it is served at moderately high speed.

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Notes

1. It is easy to make a ball spin by dropping the ball vertically onto the strings while the string plane is horizontal, and by "chopping" the bottom of the ball with a sharp, horizontal swing. If the string plane is inclined at a few degrees to the horizontal, the ball can be hit vertically upwards so it spins rapidly, and one can repeat this several times so the ball spins faster and faster after each chop. It might appear that the ball is incident vertically on the strings (since the ball is dropping vertically and the string plane is moving horizontally), but an insect sitting on the strings would see the ball coming in at an angle of only about 40 degrees to the string plane, depending on the speed of the racquet. We say that "in the racquet frame of reference" where the racquet is at rest, the ball is incident at about 40 degrees and it bounces with topspin. A player serving a ball with heavy spin uses a similar chopping or brushing action, but in the racquet frame of reference, the ball is incident typically at around 50 degrees. The object here is not to get maximum possible spin but to get as much spin as possible consistent with a reasonably high serve speed. One has to compromise by trading off spin against speed. Slice, kick and topspin serves all involve spin where the ball rotation axis varies from about 45 to about zero degrees to the horizontal and where the server brushes the strings upwards and across the back of the ball in the direction that the server wants the ball to spin.

2. The simplest way to describe a bouncing ball is to assume that it is rigid and that the surface on which it bounces is also rigid. The ball will start to roll during the bounce at high enough angles of incidence. The horizontal speed and the spin after the bounce are then related by $v_{x2} = R\omega_2$, where R is the radius of the ball and where v_{x2}/v_{x1} is 0.645 regardless of the coefficient of friction and the angle of incidence and the initial ball spin. But for a given incident ball speed, v_{x1} decreases as the angle of incidence increases so v_{x2} and ω_2 also decrease, all the way to zero when the ball is incident at right angles to the surface. A similar situation arises if the ball or the surface is flexible but the ball bites the surface instead of rolling.

Nathalie Tauziat, France

42

Ball Trajectories

Factors Influencing the Flight of the Ball

By Rod Cross

Introduction

A good tennis player knows instinctively how hard to hit a ball and at what angle to get the ball over the net and into the court so that the ball lands at just the right spot. This comes from years of practice. Players might be surprised to know just what they are doing in terms of the actual ball trajectories. For example suppose you hit a forehand at the baseline so that the ball lands on your opponent's baseline 78 feet away. If you hit the same ball but 1% faster it will land 18 inches beyond the baseline. If you hit the ball one degree higher it will land about 6 feet beyond the baseline, depending on the initial speed and angle. If you hit the ball one degree further to the left it will land 16 inches further to the left.

When serving a ball at moderate to high speed, the ball must be served a few degrees down from the horizontal. Too far down and the ball will hit the net. Not enough and the ball will be long. The range of possible angles is only about two degrees, and it gets smaller if the ball is served faster or by a shorter player (Brody, 1987). The range of angles increases if the ball is served with topspin. Typical ball trajectories are shown below, together with an explanation of the trajectories.

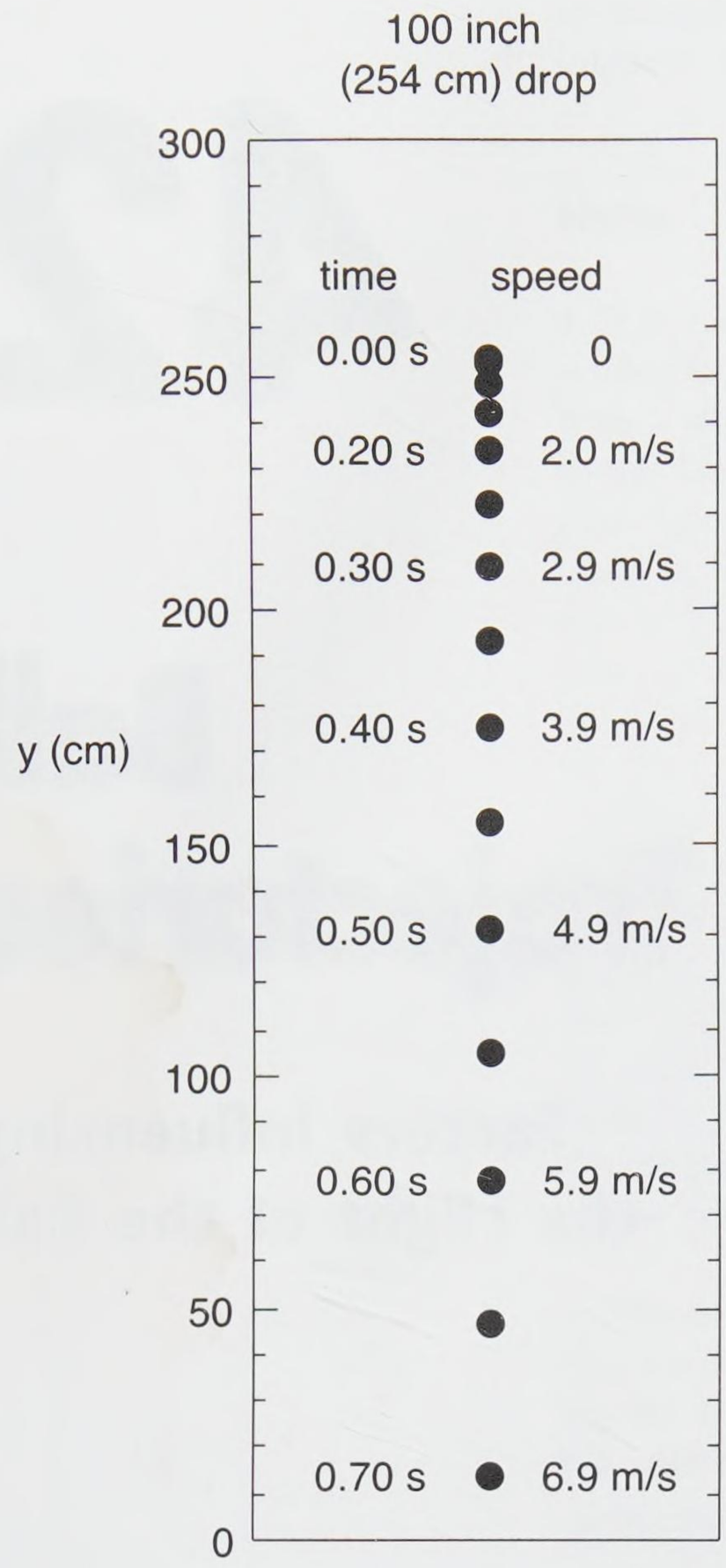


Figure 42.1

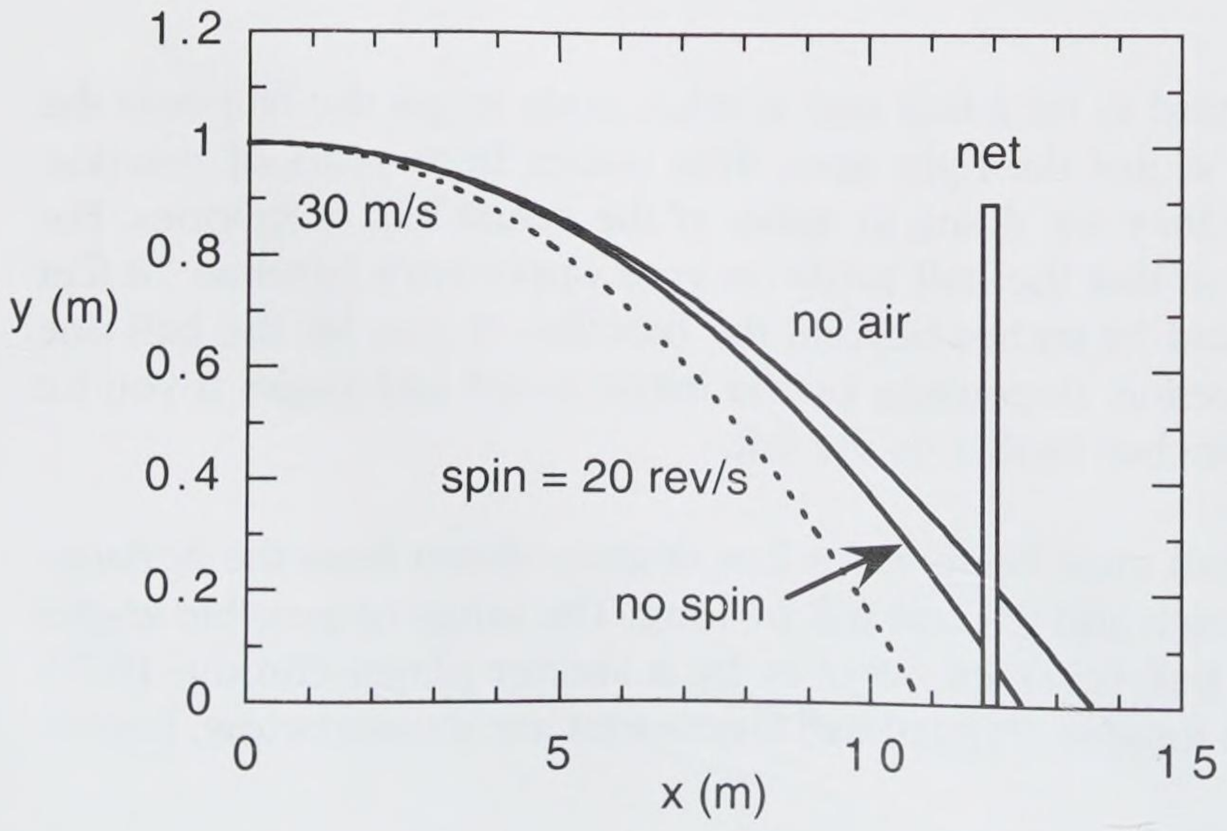


Figure 42.2

Examples of Ball Trajectories

(a) Dropping a Ball (Free Fall)

Suppose that a ball is dropped from a height of 1.0 m (about waist height) and is allowed to fall to the ground. Provided that it is not hit or thrown then it starts out with zero speed. It will accelerate downwards due to the force of gravity. If we ignore air resistance, then the vertical distance (y) travelled in time t is given by $y = gt^2/2$ where $g = 9.8$ m/s^2 is the acceleration due to gravity. After = 0.1 s, y = 0.049 m and after 0.2 s, y = 0.196 m. The ball will hit the ground at t = 0.452 s. On the way down, the speed increases and is given by v = gt. At t = 0, v = 0. At t = 0.1, v = 0.98 m/s. Just before it hits the ground, at t = 0.452 s, v = 4.43 m/s. The corresponding results for a ball dropped from a height of 100 inches (2.54 m) is shown in Figure 42.1. Air resistance is very small in this case since the ball speed is small. If allowance is made for air resistance, a ball dropped from a height of 100 in would hit the ground at 0.725 s instead of 0.720 s.

(b) Horizontal hit with free fall

Now suppose that the ball is hit at 30 m/s (67 mph) from a height of 1.0 m so that it starts moving in a horizontal direction, parallel to the ground, as shown in Figure 42.2. It is easy to calculate where the ball will land provided we ignore the force on the ball due to air resistance. Then the only force acting on the ball is the gravitational force pulling the ball vertically towards the ground. The ball will accelerate downwards and change its vertical speed, but there is no change in the horizontal speed since there is no force in the horizontal direction. In other words, the ball will keep moving horizontally at 30 m/s until it hits the ground. It starts out with zero speed in the vertical direction, and moves downwards a vertical distance $y = gt^2/2$, as in our first example. It therefore takes 0.452 s to hit the ground. During that time, it travels a horizontal distance x = 30 x 0.452 = 13.56 m.

In this case air resistance does make a difference since the ball is moving fairly fast and since the air resistance increases as the ball speed increases (Table 42.1).

If the ball has no spin then it will actually hit the court after 0.470 s after traveling a horizontal distance of 12.46 m. The average horizontal speed is 12.46/0.47 = 26.5 m/s. The horizontal speed drops slightly as the ball moves through the air, and so does the final vertical speed since air resistance exerts a force backwards and upwards

along the path of the ball. If the ball has topspin, there is an additional force that acts at right angles to the path of the ball, in a downwards and backwards direction in this case. This force is called the Magnus force. For example, if the ball is spinning at 20 revolutions/sec, the ball will hit the court after 0.410 s after traveling a horizontal distance of 10.85 m.

Table 42.1. Trajectories With Same Launch Speed and Angle Under Various Conditions

Conditions	Velocity	Time to Bounce	Travel Distance
No air	30 m/s	0.452 secs	13.56 m
Air no spin	30 m/s	0.470 secs	12.46 m
Air w/ topspin (20rev/s)	30 m/s	0.410 secs	10.85 m

The distance from the baseline to the net is 39 feet (11.887 m) and the height of the net in the middle is exactly 3 feet (0.9144 m). A ball hit horizontally at 30 m/s will not clear the net even if it starts at a point 1.0 m off the ground. To get the ball over the net it has to be hit upwards at a certain angle to the horizontal. It can't be hit too steeply or it will land past the baseline at the other end of the court. There is a certain range of angles that will get the ball over the net and inside the baseline. The range of angles is relatively small.

(c) Hitting at Various Launch Angles

Figure 42.3 shows the trajectories of a ball hit from a height of 1.0 m, at 30 m/s and at various angles to the horizontal, including the effects of air resistance. In order to just clear the net, a ball hit without spin must be hit upwards at 4.0° to the horizontal. If the ball has topspin at 20 revolutions/sec, it must be hit upwards at 5.5° to the horizontal. If a ball is hit without spin at 8.1° to the horizontal, it lands on the far baseline. If the ball has topspin at 20 revolutions/sec, it must be hit upwards at 11.9° to land on the baseline. The range of available angles is 4.1° without spin or 6.4° with topspin.

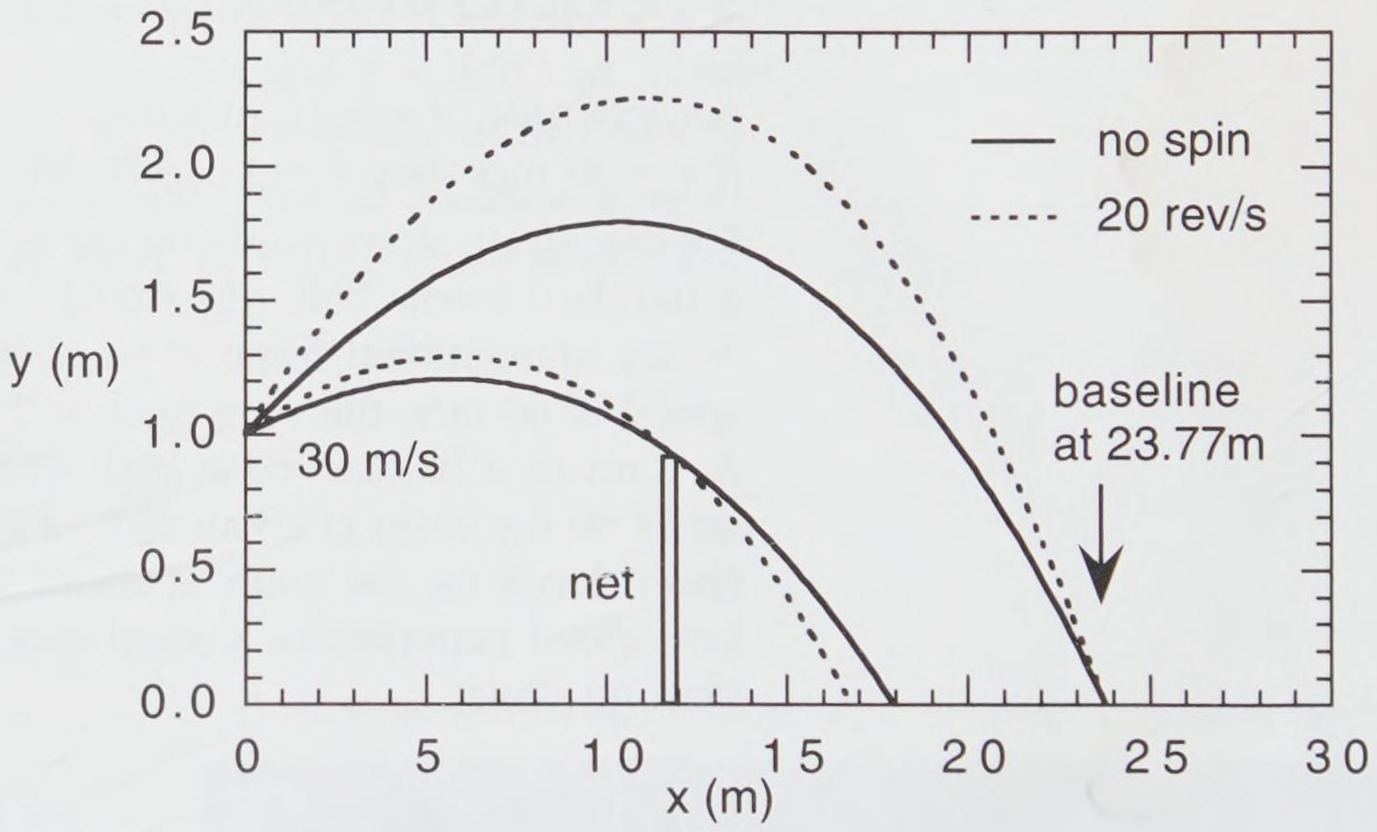


Figure 42.3

Drag Force

(a) Drag force pushes ball backwards and slows it down.

A ball traveling through the air experiences a backwards force due to the fact that the air pressure on the front of the ball is larger than the force at the back of the ball. This force is called the drag force. The force is large because air is heavy. Not as heavy as water, but a room full of air contains about 90 kg or 200 lb of air. When a tennis ball is at rest, air pressure exerts a force of 150 lb on the front of the ball and 150 lb on the back of the ball. The ball doesn't collapse since the air inside the ball exerts a similar force outwards. When a ball is traveling through the air, the force on the rear side of the ball is typically about 0.2 lb lower than the force on the front side, depending on the ball speed.

(b) Formula for drag force.

The flow of air around a tennis ball is turbulent at all ball speeds of interest in tennis, due to the relatively rough surface of the ball. This actually simplifies the analysis of ball motion since the drag coefficient remains constant for all speeds of interest. Other balls used in sport are generally smoother, but this complicates the analysis since the drag coefficient usually decreases suddenly at a sufficiently high ball speed (an effect known as the drag crisis). The drag force is proportional to the ball speed squared and is given by the formula

$$F = C_d A d v^2/2 \qquad (42.1)$$

where $d = 1.21$ kg/m^3 is the density of air, $A = \pi R^2$ is the cross-sectional area of the ball, R is the radius of the ball, v is the ball speed and C_d is the drag coefficient. The only factor here that can be influenced by the player is the ball velocity. If the ball was a flat, circular disk, the drag coefficient would be 1.0. But a ball has a rounded nose which streamlines the air flow and reduces the drag. For a relatively new tennis ball, C_d is about 0.55. A used ball experiences a slightly lower drag force since it has a smoother surface. A standard new ball of radius 3.3 cm therefore experiences a backwards force

$$F = 0.55 \times 0.00342 \times 1.21 \times v^2/2 = 0.00114 \times v^2 \quad \text{Newton (N).}$$

For example, if v = 10 m/s then F = 0.114 N. If v = 20 m/s then F = 0.456 N. If v = 30 m/s then F = 1.026 N. We can compare this with the force of gravity F = mg acting downwards on the ball. Since g = 9.8 m/s^2 and m = 0.057 kg for a standard tennis ball, mg = 0.057 x 9.8 = 0.559 N. The drag force is as large as the gravitational force even at moderate ball speeds. At a very fast serve speed of 60 m/s, the drag force is 7.3 times bigger than the gravitational force. As a result, a ball served at high speed slows down rapidly through the air and lands on the court at about 75% of the serve speed. In fact, a ball served at any speed lands on the court at about 75% of the serve speed. A ball served at a low speed experiences a small drag force, but it spends a relatively long time slowing down.

Magnus Force

(a) Force due to spin

An additional force arises if the ball is spinning. A spinning ball sets the air around it in motion in a thin layer near the surface of the ball. The flow of air around the ball is altered in such a way that the air pressure on top of a spinning ball is decreased if the ball has topspin and is increased if the ball has backspin. As a result, a ball that travels horizontally experiences a force downwards if it has topspin or upwards if it has backspin. If the ball spins about a vertical axis then the force causes the ball to swerve sideways. The force due to spin is called called the Magnus force and it always acts at right angles to the drag force and to the spin axis. (see Figure 42.4). Consequently, if a ball with topspin is rising upwards at an angle to the court, the Magnus force tends to push the ball down onto the court and it pushes it forwards in a direction paralliel to the surface. If a ball with topspin is falling towards the court surface, the Magnus force pushes the ball downwards and backwards. A ball with topspin therefore falls onto the court at a steeper angle than a ball without top-

spin. Conversely, a ball with backspin tends to float through the air and falls to the court at a shallow angle.

(b) Formula for the Magnus force.

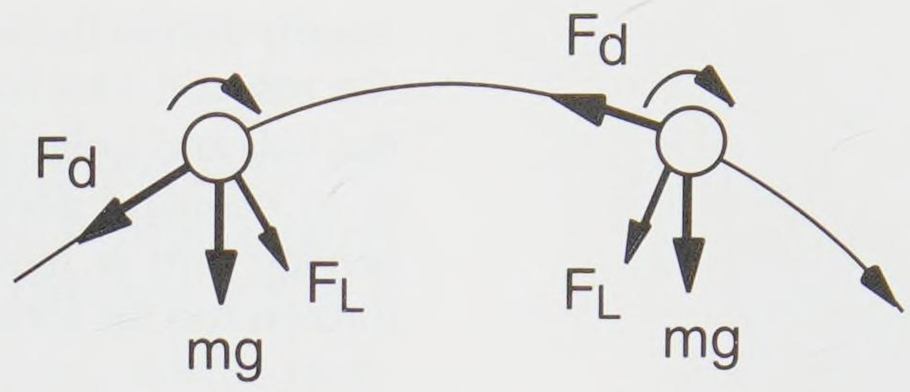
The magnitude of the Magnus force on a tennis ball is given by

$$F = C_L A d v^2/2 \qquad (42.2)$$

where C_L is called the lift coefficient since the ball experiences a vertical lift force if it has backspin and is given by

$$C_L = 1/[2 + (v/v_{spin})] \qquad (42.3)$$

where $v_{spin} = R\omega$ is the peripheral speed of the ball, R is its radius and ω is the angular speed about a horizontal axis perpendicular to the path of the ball. The angular speed varies typically from about 100 to about 500 radians/sec which translates to a spin between 16 and 80 revolutions per second (1 rev/sec = 6.28 radians/sec). Suppose that v = 30 m/s, ω = 300 rad/s and R = 0.033 m. Then v_{spin} = 9.9 m/s, v/v_{spin} = 3.03 and C_L = 0.20. The lift coefficient in this case is smaller than the drag coefficient, but not a lot smaller. The lift coefficient never gets to be as large as the drag coefficient. At most, C_L = 0.5, which is the limit when v_{spin} is much larger than v.



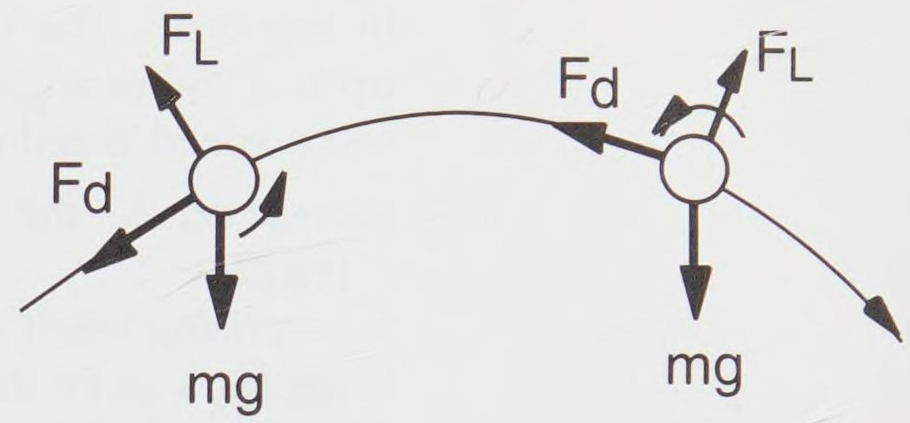


Figure 42.4

Ball spin also affects the drag coefficient (Stepanek, 1985). A good fit to the experimental data is given by

$$C_d = 0.55 + 1/[22.5 + 4.2(v/v_{spin})^{2.5}]^{0.4}$$

which indicates that C_d increases from 0.55 to 0.84 as v_{spin} increases from zero to a value much larger than v.

Serve Trajectories

Armed with the relevant values of the lift and drag coefficients, one can calculate the trajectory of a tennis ball through the air for any initial set of conditions. The equations describing the trajectory are given in the Appendix to this chapter. Some typical serve trajectories are shown in Figure 42.5 for a ball served at 110 mph (177 kph) from a height of 2.8 m (9 feet 2 inches). A person of height h serves the ball from a height typically about 1.5h. For example, a person of height 6 feet

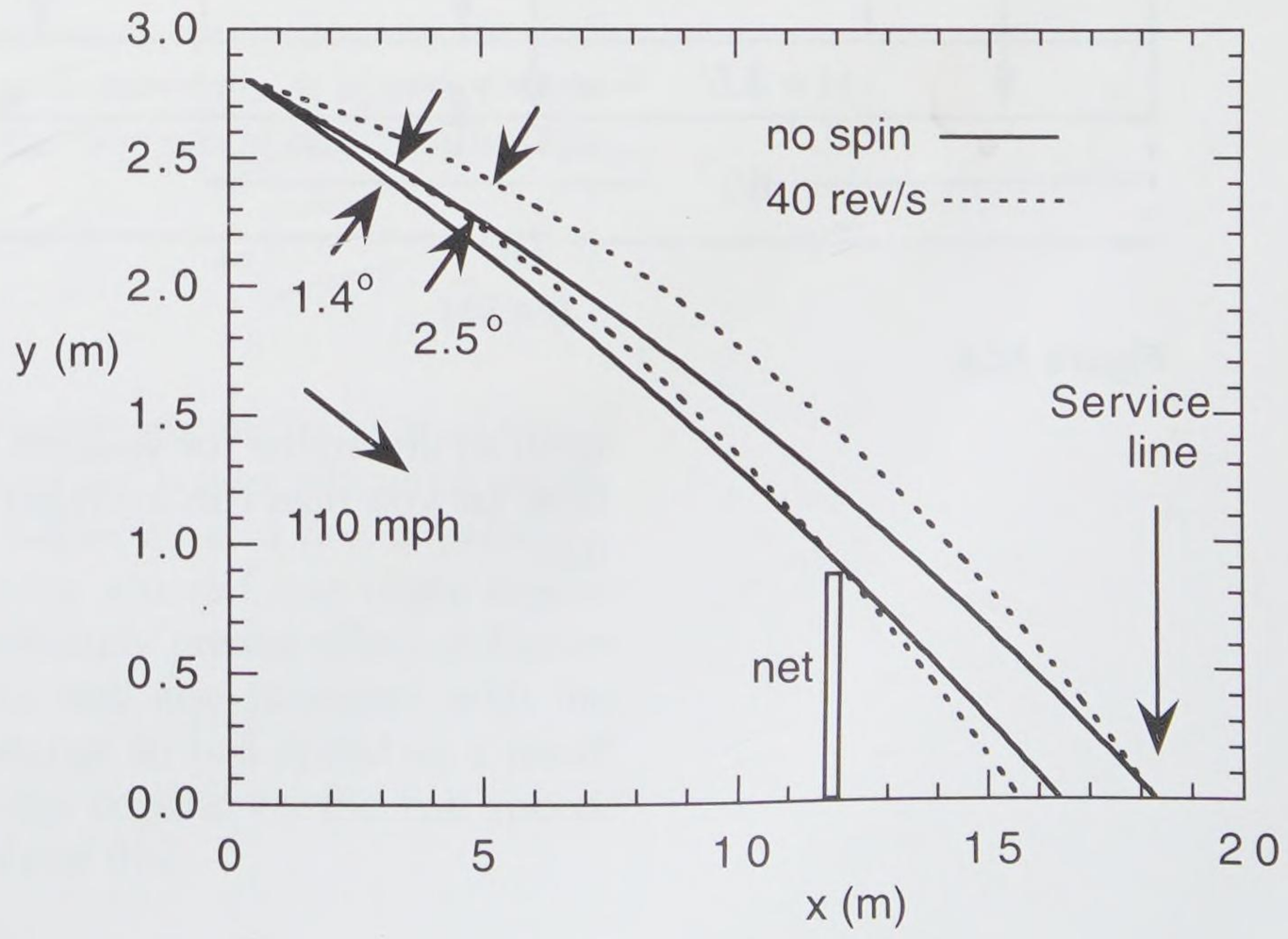


Figure 42.5

usually serves the ball from a point about 9 feet above the court, even if he or she tosses it a lot higher. A high ball toss allows the server enough time to sight the ball and judge its position accurately. A high ball toss also allows the server to maintain good balance while the tossing arm is lifted upwards and the serving arm is moved backwards. If the toss is too high, the ball might be thrown too far forwards or backwards.

The trajectories in Figure 42.5 were calculated for a serve down the center line so that the ball just cleared the net or so that it landed on the service line (point B in Figure 42.6). Results are given for a perfectly flat serve with no spin and for a serve with topspin at 40 rev/sec. One of the advantages of serving with topspin is that the available range of serve angles is increased (from 1.4° to 2.5° in this case). The ball also approaches the court at a steeper angle and kicks up at a higher angle. The results in Figure 42.5 are typical of a first serve. If the serve speed is reduced to 85 mph and the spin increased to say 50 rev/s for a second serve, the range of available angles increases to 3.8°.

A surprising result is that the range of available angles for a serve is almost the same for a serve in the far corner of the service court (point D in Figure 42.6). Even though the net is 3.2 inches (81 mm) higher along that path, the distance from the server to the far corner is 18 inches (457 mm) longer. As a result, the range of available angles is reduced by only about 0.2°. It is therefore almost as easy to serve into the far corner as to serve down the center line. The hard part is to serve even wider than the far corner. There is, however, one advantage of serving towards the far corner. If you aim for the center line and miss in the horizontal direction, then you have only a 50-50 chance that the ball will be in. If you aim towards the far corner and miss in the horizontal direction then the ball can still land in, provided you serve the ball short.

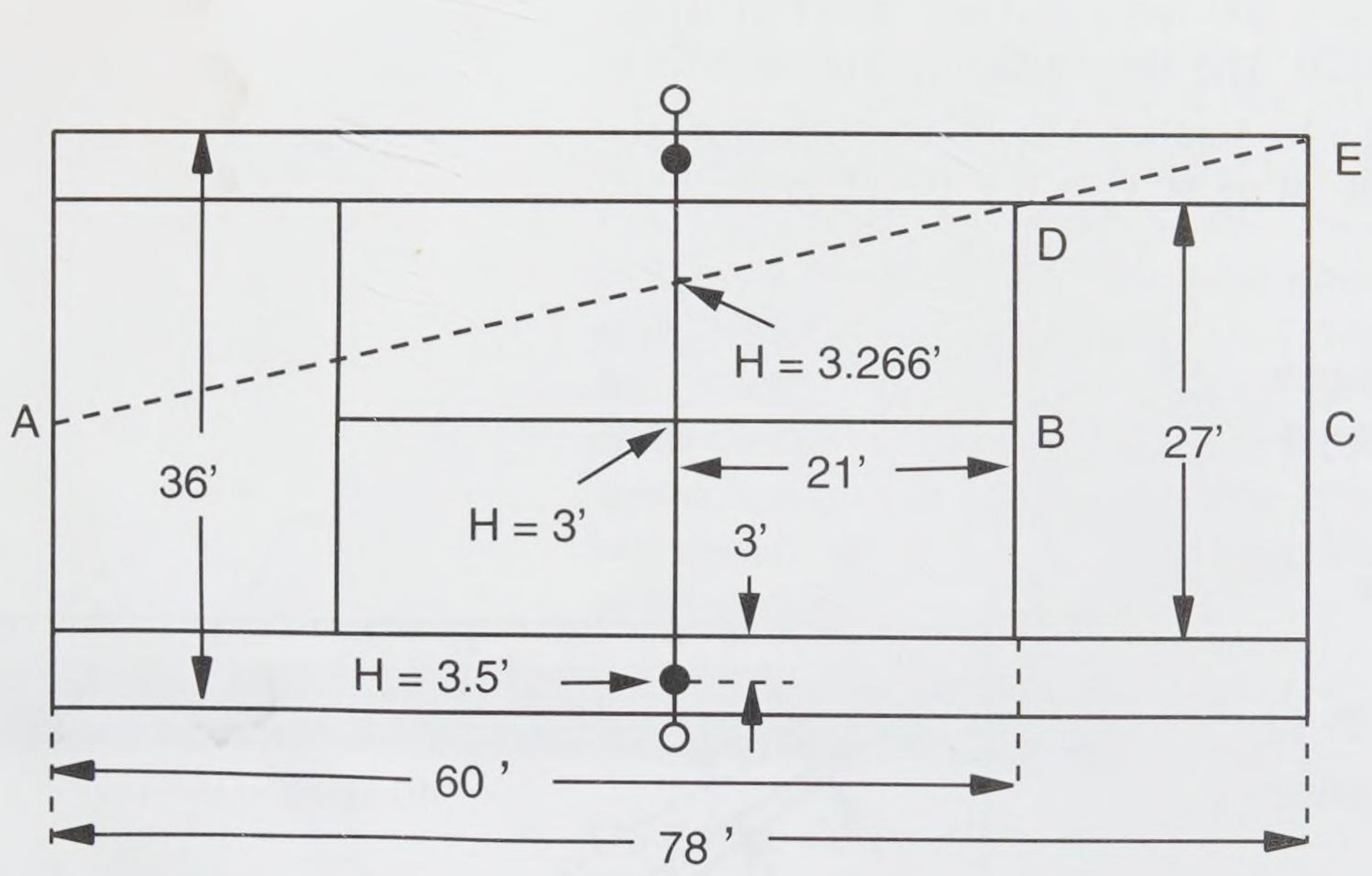


Figure 42.6

To compensate for possible horizontal direction errors, you should never aim exactly down the center line, but a bit towards the receiver. Just how far is something that you need to determine for yourself. Try serving down the line a few times and see how far you miss (on average). That's how far you should aim away from the line.

Appendix 1: Trajectory Equations

Suppose that a tennis ball of mass m and radius R is traveling at speed v and at an angle θ upwards from the horizontal. The velocity in the horizontal direction is then $v_x = v \cos(\theta)$ and the velocity in the vertical direction is $v_y = v \sin(\theta)$. The forces acting on the ball are mg downwards, $F_d = C_d\, A d v^2/2$ backwards and $F_L = C_L\, A d v^2/2$ upwards and at right angles to the path of the ball (assuming the ball has backspin). The equations of motion are then

$$m dv_x/dt = -F_d \cos(\theta) - F_L \sin(\theta)$$

$$m dv_y/dt = F_L \cos(\theta) - F_d \sin(\theta) - mg$$

which can written as

$$dv_x/dt = -kv\,(C_d\, v_x + C_L\, v_y) \qquad (43.a1)$$

$$dv_y/dt = kv\,(C_L\, v_x - C_d\, v_y) - g \qquad (43.a2)$$

where $k = d.\pi.R^2/(2m)$. If the ball has topspin, the sign in front of C_L must be changed in each of these equations. These equations can be solved numerically, but care is needed to avoid numerical errors. A good check is to solve for a purely vertical drop without spin. In that case the equation of motion is

$$dv/dt = -kC_d\, v^2 - g \qquad (42.a3)$$

which can be solved analytically to check the numerical result. For example, if m = 57 gm, g = 9.8, d = 1.21 kg/m^3, diameter = 2R = 65 mm, C_d = 0.50, initial speed = 15 m/s downwards starting at y = 30 m, then the ball hits the ground at y = 0 at speed v = 21.006 m/s after 1.6199 sec.

The essential features regarding the horizontal motion of a tennis ball can be described analytically if one ignores the small vertical component of the ball speed. Consider a case where a ball is traveling horizontally at speed v without spin, and where the equation of motion in the horizontal direction has the form

$$dv/dt = -k\, C_d\, v^2 \qquad (42.a4)$$

The ball will subsequently develop a velocity component in the vertical direction due to the gravitational force, but this component is typically much smaller than the horizontal component and can be neglected as a first approximation. Since the drag force increases with the velocity squared, one might expect that the drag force on a ball would have a significantly greater effect at higher ball speeds. However, the kinetic energy of the ball also increases with the velocity squared. As a result, the percentage change in ball speed as a result of the drag force, over a given distance, does not depend on the ball speed. Equation (42.a4) can be integrated directly to show that

$$v/v_o = 1/(1 + k\,C_d\,s) \qquad (42.a5)$$

where v_o is the initial speed and $s = v_o t$ is the distance the ball would travel at speed v_o in a time t. For a standard 66 mm diameter, 57 gm tennis ball, $k\,C_d = 0.020\ m^{-1}$ when $C_d = 0.55$. Over the distance s = 17.888 m from a point 40 cm in front of the baseline to the opposite service court line, the ball speed drops to $v = 0.737 v_o$, regardless of the initial speed. For example, if v_o = 160 kph, then v = 118 kph, which agrees well with the numerical solution (113.6 kph) . The numerical solution takes into account both the horizontal and vertical motion of the ball.

Equation (42.a5) can also be integrated directly to show that the time taken to travel a horizontal distance x is given by

$$t = [\exp(k\,C_d\,x) - 1]/(k\,C_d\,v_o) \qquad (42.a6)$$

For a 66 mm diameter tennis ball, the time taken to travel from a point 40 cm in front of the baseline to the opposite service line is therefore $t = 21.5/v_o$. If v_o = 44.44 ms (160 kph), then t = 0.484 s, which also agrees closely with the numerical solution (t = 0.487 s).

Appendix 2: Simple Ball Speed Measurement

Suppose you hit a forehand or launch a ball from the middle of the baseline so it passes over the middle of the net and lands on the service line tee. The ball speed off the racquet or out of the ball launcher can be determined with the aid of Figure 42.7. This shows the height at which the ball passes over the top of the net when it is hit without spin from a height of either 0.7 m or 1.0 m above the baseline. All you need to do is measure the height above the net and then read the ball speed off the graph. For example, if the ball passes 2.0 m above the top of the net, it started out at 16.2 m/s, and it lands on the service line at an angle of incidence of 36°. If it passes 0.2 m over the net, it started out at about 26 m/s and it lands on the service line at an angle of incidence of about 15°.

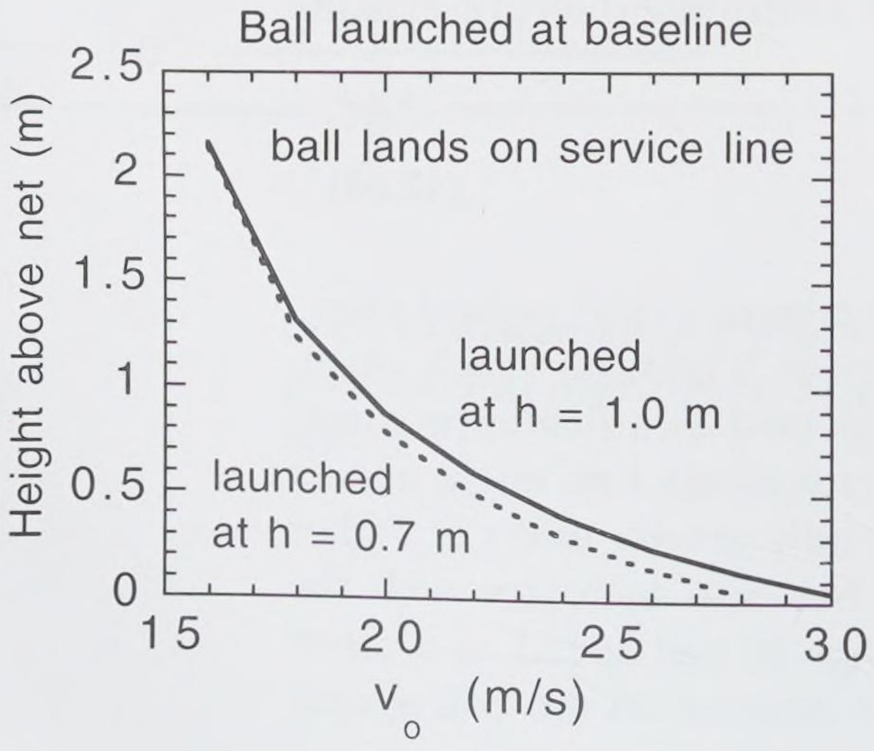


Figure 42.7

Height over net vs ball speed

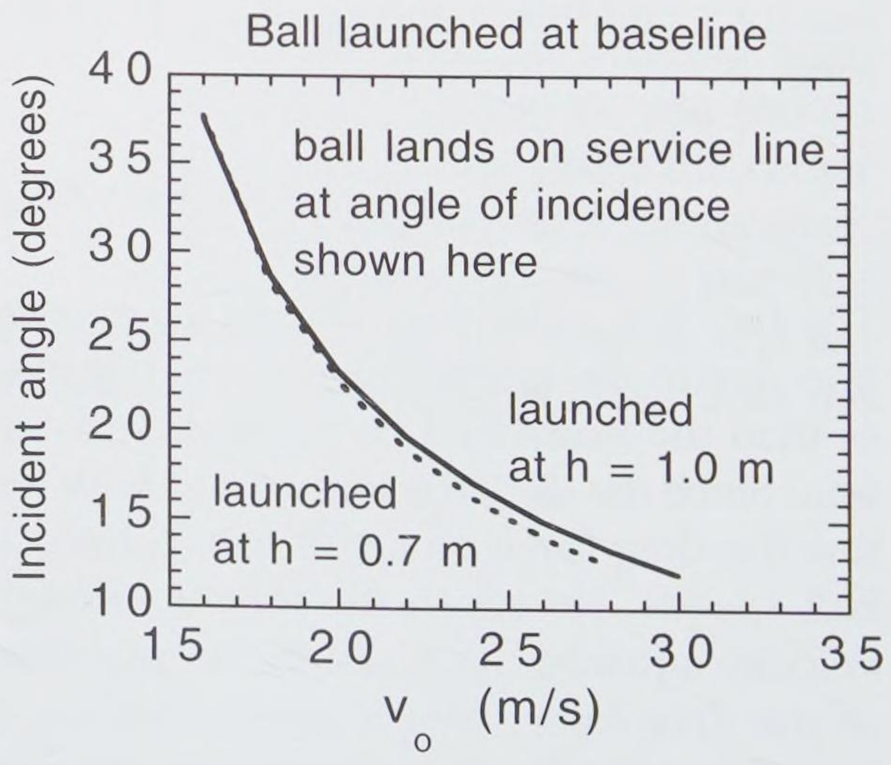


Figure 42.8

Angle of incidence on the court

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A

Appendix A
Glosssary

Acceleration

The rate of change in velocity. It is measured in ıneters per second per second (m/s^2): $a = \Delta v/t$. If a vehicle accelerates from 20 m/s to 30 m/s over 5 seconds, $a = 10/5 = 2$ m/s^2. If the speed stays constant at 20 m/s, then $a = 0$.

Acceleration due to gravity (g)

The rate at which velocity increases for a freely falling body. Near the earth's surface g = 9.8 meters per second per second (m/s^2).

Air resistance

Friction of the air upon a ball in flight. It generates a drag force acting backwards on the ball.

Acrylic

An acrylic paint is a spreadable plastic compound mixed with pigment of a particular color used to coat tennis courts. It can be thinned with water and dries much faster than oil-based paints. It can also be mixed with sand to form a slightly rough surface as used on the Rebound Ace and DecoTurf courts at the Australian and US Open. The amount of sand controls the speed of the court to a small extent. Some areas of a court can wear more than others, typically leaving smooth patches near each baseline.

Amplitude

The maximum displacement on either side of the equilibrium position of a racquet that is vibrating.

Angular momentum

Angular momentum is the momentum of a rotating object, but it is defined in a different way to linear moment, since different parts of a rotating object travel at different speeds. If a racquet of mass M is moving in a straight line at speed v, then its linear momentum is Mv. If the racquet is rotating at angular speed ω, then its angular momentum is $I\omega$, where I is the moment of inertia about the axis of rotation. In a collision between a racquet and a ball, the total linear and the total angular momentum (of the racquet plus the ball) are both conserved.

Anisotropic

A material that exhibits different properties when tested in different directions. Graphite is an example. It is stronger, for example, along its length than perpendicular to it. Titanium, on the other hand has the same strength in all directions and is called isotropic.

Atom

Smallest unit of an element. It consists of a positively charged nucleus surrounded by electrons with a total negative charge that is equal and opposite the positive charge of the nucleus.

Axis of rotation

When a racquet or a ball rotates, it rotates about a particular axis in such a way that points furthest from the axis rotate at the greatest speed, and points on the axis rotate at zero speed. Each point on the racquet or the ball follows an arc (or possibly a full circle), the center of which coincides with the axis of rotation. The axis of rotation of a racquet, just before the racquet collides with the ball, is typically about 6 inches beyond the butt end of the handle.

Balance point

The balance point of a racquet is near the middle of the racquet, directly below its center of mass. It is the point where a racquet will balance on one finger or a straight edge. It is usually quoted as the distance in cm from the butt end of the handle to the balance point and is typically between 32 and 39 cm. Also called center of mass.

Bite

Players and commentators often say that the ball "bites" the court or that the strings "bite" the ball. They are correct in that the bottom of the ball can grip the surface and come to a complete stop in a direction parallel to the surface. Even though the bottom of the ball stops, the top and middle of the ball do not come to a stop. The result is that the ball bounces off the court at lower speed in a direction parallel to the surface than when it slides across the surface, so it kicks up at a steeper angle off the court.

Bumper guard

The plastic strip around the tip of the frame used to prevent the frame and strings getting scratched or damaged.

Center of mass

In order to describe the motion of a racquet or a ball, it helps to imagine that the whole mass of the racquet or the ball is located at a particular point called the center of mass (CM). The CM of a ball is inside the ball, in the middle, despite the fact that there is nothing there apart from some air. The CM of a racquet may also be located in the air space around the racquet, and is typically somewhere near the throat. A racquet can be balanced on a narrow edge if the edge is directly below the center of mass. If you toss a racquet in the air, the racquet will tumble and turn, but the center of mass traces out a perfectly smooth parabolic path.

Center of percussion

Located near the center of the strings, but a few cm closer to the throat. If a ball is struck at the center of percussion, there is no sudden movement of the handle towards or away from the net, and hence, there is no sudden shock felt by the hand or arm.

Clamps

Used by stringers to hold each string in position while the next one is being installed. A good clamp properly installed should stay in place and not move very far when the string is tensioned, but all clamps move a small amount.

Clay

Clay courts are common in Europe, but not elsewhere. The clay is made from finely crushed bricks or tiles and has the texture of a fine powder at the French Open. It is called clay since bricks and tiles are made by baking clay, but clay itself is soft, like plasticene. The court is relatively slow compared with most other courts, although players can slide into and out of shots with ease since the fine particles make the surface slightly slippery under foot.

Closed face

Describes racquet head orientation at time of ball contact. A closed face has the string plane tilted forward and downward when the ball contact is made.

Coefficient of friction (COF)

When an object slides on a surface there is resistance to its motion caused by friction between the object and the surface. The friction force F acts in a direction parallel to the surface, opposite the direction of motion of the object, and is proportional to the force N on the

object acting perpendicular to the surface. The COF is defined as COF = F/N and is a number typically between 0.1 and 0.9. The COF between a ball and grass is about 0.6. Between the ball and a clay court it is about 0.8. Between the ball and the strings of a racquet it is about 0.4. If the ball rolls along a surface, then COF decreases to about 0.05.

Coefficient of restitution (COR)

When a ball is dropped vertically onto the ground or onto a head-clamped racquet, the COR is defined as the ratio of the rebound speed to the incident speed. The COR is a number between 0 and 1 that describes how well the ball bounces. A plasticene ball does not bounce at all off a hard surface and the COR in that case is zero. A more general definition of the COR is used to describe the impact of a ball on a hand-held racquet. If the ball impacts and rebounds at right angles to the string plane, then the COR is defined as the ratio of the relative speed of the ball and the impact point on the racquet after the impact to the relative speed before the impact. For example, if the ball is incident at 10 m/s and the racquet approaches the ball at 20 m/s, then the relative speed is 30 m/s. The ratio of the ball speed after the impact to the ball speed before the impact is called the ACOR (apparent coefficient of restitution), and it is usually measured under conditions where the racquet is initially at rest. If the ball is incident obliquely on the court or on a racquet, then the COR is defined in terms of the perpendicular components of the velocity of the ball and the racquet.

Composite

A combination of two or more materials into a structure. Usually one material contributes most to the strength (reinforcement) and the other holds the material together (the matrix). Racquets are usually a "graphite reinforced matrix" and string a "nylon (or other) reinforced matrix" where the matrix is a resin.

Conservation of energy

In a closed system, the amount of energy remains constant. It may be converted to other forms, but the amount is always the same. When we say kinetic energy is lost in a collision of a racquet and ball, we mean it has been converted to another form that cannot be used in propelling the ball off the racquet.

Conservation of momentum

The total momentum before a racquet and ball collide is equal to the total momentum after they collide. The sum of the mass times velocity of the racquet and of the ball before the collision equals their sums after the collision: $(mv)_{before} = (mv)_{after}$.

Court speed

The ratio of the horizontal speed after the bounce to the horizontal speed before the bounce doesn't have a special name because it depends on the angle of incidence. In practice, the ratio is usually between 0.40 and 0.75 on all courts and at all angles of incidence of practical interest. In theory the ratio could be 0.9 or more at very low angles of incidence, but a ball never hits the court at angles less than about 10 degrees. The ratio is sufficiently important that it ought to have a special name. In this book, we will call it "Court Speed," since that is what it describes, but it does vary with the angle of incidence. The International Tennis Federation has a different definition to describe the speed of a court – Pace (see Pace).

Creep

An object that is subject to a tension force will stretch by an amount that is roughly proportional to the force. Even though the force may be held constant, the object may keep stretching at a slow rate for hours or years. This effect is called creep and it is especially noticeable in tennis strings and other plastic materials. Alternatively, if a string is under tension, and it is clamped at each end so its length stays constant, then the tension will slowly drop. Both effects are due to the gradual breakage and slippage of bonds in a material that is subject to a tension force.

Constant pull stringing machine

When a string is pulled to a given tension, whereupon the pulling is stopped, it will immediately begin to lose tension. A constant pull stringing machine recognizes the decline in tension and pulls back up to the desired tension. This is a "continuous" or "constant" process until the stringer clamps the string. It usually takes 2-5 seconds from pulling to clamping, so a continuous pull machine will stretch the string for a longer time than will a lockout machine which stops pulling as soon as the string reaches the desired (reference) tension. As a result, a racquet strung on a continuous pull machine will have a stingbed that is about 5-10% stiffer than one strung on a lockout machine. Many stringers know this and compensate accordingly.

Copolymer

Copolymers involve making long chain molecules (polymers) by using more than one type of basic repeating unit (mer). See polymer.

Covalent bonding

Bonding between atoms by sharing electrons in the outer shell of each atom. This is how atoms bond to one another in polymer chains that compose strings. These bonds are very strong. The bonds between adjacent chains within strings are not so strong and are called secondary or polar bonds. These are more bonds of attraction rather than interaction and sharing, and as such, are weaker and are the reason string creeps – secondary bonds between adjacent chains can "break" and the chains slide or slip relative to each other.

Customization

This typically refers to altering the weight, balance, swingweight, stability and sweetspot of a racquet by adding lead tape to strategic areas of the racquet.

Cyclops

A device called a Cyclops machine is used on some courts to determine if a serve is in or out. Several parallel infrared beams pass over and beside the service line about 6 mm above the court. A ball that intercepts one or more of the beams and not another can be judged by the machine to be in or out. The machine can't be used on the baseline since a player's foot might interrupt the beam.

DA, RA and RDC units

The stiffness or "flex" of a racquet frame can be measured by locating it between two bars and pushing on a third point to measure how far the racquet bends. It's like bending it across your knee, but with a dial or readout to give you a number. There are several racquet diagnostic machines available to do this and to measure stringbed stiffness as well. Unfortunately, there is no industry standard and the different machines don't use the same measurement units. The USRSA uses a Babolat machine calibrated in RDC (Racquet Diagnostic Center) or RA units. Other machines such as the Pacific diagnostic machine use DA units which are similar, but not exactly the same. Readings in both units vary typically from 55 (soft frame) to 85 (very stiff frame).

The stiffness of a racquet frame decreases as soon as it is strung since the strings act to bend the frame further when the frame is deliberately bent to measure its stiffness. Ideally, one should measure frame stiffness without the strings since the higher the string tension the lower is the frame stiffness. Stiffness readings are also affected slightly by the relative softness of different grips.

Deadspot

If a ball is dropped on a stationary, hand-held racquet with the strings in a horizontal plane, then the ball will bounce best near the throat and worst near the tip. There is usually a spot near the tip, especially with light racquets, where the bounce height is zero. That is called the dead spot. If the racquet is swung to serve a ball at the dead spot, then the speed of the ball off the strings is equal to the speed of the dead spot.

DecoTurf

The court surface used for the US Open. Players need to wear thick-soled shoes on this court to get a good grip, which is why shoes squeek more on this surface. Players at Wimbledon wear shoes with much thinner soles. The court is classified as a hardcourt and is built up from an asphalt base using several layers of acrylic mixed with rubber and sand to form a hard surface cushioned slightly by the rubber content.

Density

Mass per unit volume of a substance.

Drag force

A ball served at 100 mph slows down through the air and lands on the court at about 75 mph. The force exerted by the air on the ball is called the drag force. It acts backwards on the ball. The drag force is small at low ball speeds and increases at high ball speeds. The drag force increases with the ball diameter and increases if the cloth fluffs up.

Drawing

The process of stretching a string under heat during its manufacture to orient all the polymer chains composing it in the same direction. The more unidirectional the polymer chains, the stronger and stiffer the string.

Dwell time

The amount of time that the collision between the ball and the strings takes. Typically this is about 0.005 seconds (5 milliseconds). It can be increased a millisecond or two by lowering string tension and decreased by increasing tension.

Effective mass

When a ball strikes a stationary racquet anywhere on the strings, the racquet recoils and it also rotates. The combined recoil and rotation speed at the point of impact is greater than that for an impact at the racquet's center of mass. As a result, the racquet behaves

at the impact point as if its mass is less than its actual mass. The effective mass of a racquet near the tip is about the same as the mass of a tennis ball. In the center of the strings, the effective racquet mass is about half the actual mass.

Elastic collision

A collision where all the kinetic energy is conserved. So, the sum of the ball's and racquet's kinetic energy before the collision would equal that after the collsion. Of course, this does not occur in a racquet and ball collision. Some of the kinetic energy is transformed into vibrational energy in the racquet, ball and strings, as well as into heat and sound. In this case, we say the collision was inelastic.

Elasticity

If a certain material is elastic, it can be stretched and it will then return to its original shape when the stretching force is removed. The latter property means that the material is also resiliant. The words elastic and resiliant mean the same thing scientifically. In common usage, something that is very elastic can be stretched a long way, like a rubber band. Plasticene can also be stretched a long way, but it is not elastic since it doesn't resume its original shape after stretching. Steel is elastic provided it is not stretched too far since it will return to its original length when the stretching force is removed. A tennis string is elastic if the stretching force is removed soon enough, otherwise it will not return to its original length when the stretching force drops to zero.

The amount of stretch under a certain tension force is described by the stiffness or softness of the material. A steel wire is a lot stiffer than a tennis string. It is also less flexible meaning that it is harder to bend. If a steel wire is stretched too far, it can be taken past its elastic limit and then it remains permanently stretched when the tension force is removed. Some materials stretch easily, but they don't return to their original length after stretching due to breakage of bonds in the material. Such materials are said to be visco-elastic. Tennis strings stretch or bend easily compared with say steel wire of the same diameter so they are softer or less stiff and more flexible. Tennis strings are also visco-elastic. If you take a given length of string, stretch it for a while and release the tension, the string will be a bit longer than its original length. The longer it is under tension the more it stretches and the longer it will be when the tension force is completely removed. A string in a racquet stays the same length for many years since it remains under tension and since it is anchored at both ends at a fixed length. However, the tension drops slowly over time and every time a ball is hit.

Elastic limit

The amount a material can be stretched, bent, etc., and still return to its original form. Beyond that point, the material remains permanently deformed.

Elastomer

Rubber materials that can stretch several times their length and return to the original length when the force is released. Elastomers are used in vibration damping in racquets. The elastomer stretches or compresses and stores energy elastically. However, upon release of the force, it returns the energy more slowly than it received it. Because the energy is transferred over a longer period of time, the peak force experienced by the hand will be less.

Flex

The word flex means bend. When describing a racquet, it refers to the DA or RA reading of racquet stiffness.

Force

A force is a push or pull which can cause an object to accelerate or decelerate or change its shape by stretching or compressing. A force is always exerted by something on something else. The earth pulls downwards on an object of mass M with a force F = Mg where g is the acceleration due to gravity. In the SI system of units M is measured in kg, g = 9.8 m/s^2 and F is measured in Newton. The earth pulls down on a 1 kg object with a force F = 9.8 N. The weight of an object (as opposed to its mass) is the force exerted by the earth on the object. The weight of an object of mass 1 kg is therefore 9.8 N. In common usage it is often stated that the weight of a 1 kg mass is 1 kg or that it weighs 1 kg. In the USA it is more common to say that the earth pulls down on an object of mass 1 lb with a force of 1 lb and it weighs 1 lb. This avoids the problem of multiplying by g, but a more careful distinction between mass and weight or force must be made when doing calculations. An object of mass M subject to a force F will have an acceleration a given by F = Ma or a = F/M. If a string is stretched by applying a force F at each end, it will not accelerate since the total force on the string is zero (since equal and opposite forces are applied at each end), but it will stretch by an amount x given by F = kx or x = F/k, where k is the stiffness of the string. In this case, the force F is more commonly called the string tension.

Frame of reference

In order to describe the speed and location of an object that is moving, we need a coordinate system. That system is our frame of reference. The frame need not be at rest. It might itself be moving, but we usually prefer to use a reference frame that does remain at rest. Your reference frame might then be different to someone else's reference frame. If you are sitting in a train and throw a ball vertically into the air, the ball will rise straight up and fall straight back into your hand, exactly as if you were on a tennis court. People outside the train will see the train and the ball coming towards them at high speed. We can describe any particular event in whatever reference frame we like, but the event will appear to be different in different frames of reference, even though Newton's laws of tennis do not depend on which reference frame we choose. At least, that is the case when two reference frames move at constant speed with respect to each other. Rotating frames or accelerating frames or frames moving near the speed of light need to be treated with more caution since Newton's laws don't apply in those reference frames.

Frequency

If any part of a racquet (or a guitar string) vibrates back and forth 100 times every second we say that the frequency is 100 Hz. Frequency is measured in Hertz (Hz), which is the same as cycles per second. If the frequency is 200 Hz, that actually means your hand gets hit 400 times per second, since it is surrounding the handle on both the "back" side and "forth" side of the vibration.

Gauge

Tennis string diameters vary from about 1.20 mm to about 1.40 mm and are specified by a gauge number. The nominal diameters are approximately as follows: 18G = 1.20 mm, 17G = 1.25 mm, 16G = 1.30 mm, 15L = 1.35 mm and 15G = 1.40 mm. The actual gauges vary from one manufacturer to the next and they vary along the string (especially on textured or rough surfaces) and they will vary slightly from one batch to the next. Some strings are elliptical in cross section so the diameter is different depending on which diameter is measured. If you are concerned about actual string gauge, then you will need to measure it with a micrometer. Thin gauge strings usually wear out faster. They are not necessarily softer than thicker gauges. Thin strings stretch further when they are installed, but they tend to stretch less than thick strings when they are used to hit a ball since they get stiffer quicker at high operating tensions. Thin strings tend to hold tension better than thick strings. Thin strings do not impart more spin or less spin to the ball than thick strings.

Graphite

This is one of the materials used to construct almost all modern tennis racquets. It is one of the two main forms of pure carbon (diamond being the other) and is used to make pencils. It can also be manufactured as a long and very strong filament which can be woven as a cloth. Many layers of cloth are glued together with epoxy resin to manufacture a tennis racquet, often with other layers of kevlar or fiberglass. The result is a very light and very strong composite material, especially when it is constructed as a hollow tube. Graphite tubes are also used to construct modern arrows, kites, fishing rods, etc. Spider webs are actually stronger than graphite filaments (for any given diameter), but no one has figured out how to make a tennis racquet from spider webs.

Grommet

Plastic grommets are inserted through the holes in the frame to protect the string from the sharp edges of the holes in the frame.

Gut

Natural gut is manufactured from the intestines of beef cattle. It is the preferred string of many professionals, but it is relatively expensive. Intestines are designed to stretch and shrink to accommodate a large meal and have excellent elastic properties. To manufacture a string, the intestines are cut into several long ribbons. It takes two or three animals to produce one string. The ribbons are first washed and then twisted together to form a structure like a thin rope since it is impossible to join the ribbons end-to-end. The rope is dried, mechanically polished to form a smooth string and then coated to make it more water and abrasion resistant and hence more durable. Natural gut has the property that it does not get significantly stiffer at tensions above the normal strung tension, whereas synthetics do. As a result, natural gut is generally softer. However, it can play as a stiff string if strung at high tension, since it holds its tension better than most synthetics.

Hardcourt

A court with a hard surface. It may be solid concrete or asphalt or have an upper layer of rubber and acrylic mixed with sand to make the surface slightly rough.

Head-heavy

If the balance point of a racquet is midway along its length, the racquet is neutrally balanced. If the balance point is closer to the tip of the racquet, the racquet is head heavy. If the balance point is closer to the butt end of the handle, the racquet is head light or handle heavy.

Headsize

Modern racquets have a much larger head than old wood racquets. Head sizes are classified as midsize (< 95 $in.^2$ / 613 cm^2), midplus (95-104 $in.^2$ / 613-671 cm^2.), oversize (105-117 $in.^2$ / 677-755 cm^2), or superoversize (> 117 $in.^2$ / 755 cm^2).

Horizontal velocity

When a ball flies through the air, it has a speed in both the horizontal and vertical directions. When it bounces, the friction force changes the horizontal velocity. The relationship between the horizontal and vertical speeds after the bounce, as well as the spin, is what characterizes the speed and angle of the bounce.

Impulse

When a racquet hits a ball, we say that the ball was acted upon by an impulsive (or sudden) force. The impulse is defined as the force F multiplied by the duration of the force t, and is equal to the change in the momentum of the ball. That is,

$$\text{Impulse} = Ft = mv_2 - mv_1 = \text{change in momentum.}$$

Inertia

This is the property of racquets and balls to resist any change in their motion. An object's mass is a measure of this inertia.

Inelastic

An object that does not return to its original form when it is stretched by a force and then released.

Inelastic collision

See elastic collision.

Isotropic

The properties of the material are the same in all directions. Metals, like titanium, are isotropic. Graphite is not. Because graphite is not isotropic, its stiffness and strength depend on the direction of the long axis of the material compared to the stressing force. That is why a racquet is made of layers of graphite arranged at different angles in different parts of the racquet, depending on the expected directions of forces in that area.

Kevlar

A very stiff and strong synthetic material developed by Dupont used in some racquet frames and to make tennis strings.

Kinetic energy

The kinetic energy E (in Joules) of an object of mass M (kg) is the energy due to its motion and is given by $E = Mv^2/2$ where v is the speed of its center of mass (in m/s). A tennis ball of mass 0.057 kg served at a speed of 50 m/s has kinetic energy E = 71.25 Joules. An 80 kg person jogging at 5 m/s has kinetic energy E = 1000 Joules. If the object is rotating about a fixed axis, then $E = I\omega^2/2$ where I is the moment of inertia about that axis, and ω is the angular velocity due to rotation (measured in radians/sec). One revolution/sec is equal to 6.28 radians/sec.

Layup

By-hand process of placing different layers of unidirectional graphite at different angles to each other in a racquet. A racquet is composed of 7-12 layers, with various layers lying between 0 and 90 degrees to each other. This is important to the strength and stiffness of the racquet because these depend on the direction of the graphite fibers at any given location.

Lockout

See constant pull stringing machine.

Magnus force

When a ball is spinning, the flow of air around the ball is altered in such a way that the air pressure is reduced at the bottom of a ball with topspin or at the top of a ball with backspin. As a result, a downwards force is exerted by the air on a ball with topspin and an upwards force is exerted on a ball with backspin. This force is called the Magnus force. A ball hit with topspin is forced downwards onto the court with a higher vertical speed than a ball without spin, so it bounces higher (assuming that both balls are hit with the same speed).

Mass

The mass of an object is usually measured in kg or gm or oz or lb. It represents the resistance of the object to

acceleration. A heavy object is more difficult to accelerate than a light object and will require a larger force to accelerate it to the same speed in the same time. If a light and a heavy object are subject to the same force acting for the same length of time, the light object will accelerate to a higher speed. The relation between these quantities is given by F = Ma where F = force, M = mass and a = acceleration.

Mer

The smallest unit that is repeated over and over (thousands of times) to make a polymer, which is also called a long chain molecule. Sometimes called a "monomer."

Midsize

Generally speaking, racquets with a headsize < 95 $in.^2$ / 613 cm^2. Midplus racquets have a headsize between 95-104 $in.^2$ / 613-671 cm^2.

Modulus

Describes the relationship of stress to strain on an object when a pull of a given force is applied along its length: Modulus = stress/strain. Modulus is a measure of a material's stiffness.

Moment of Inertia

The moment of inertia of an object represents its resistance to change in rotation and is commonly known as the swingweight when referring to a racquet. If the racquet has mass M (kg) and length L (cm), then the swingweight or moment of inertia about an axis through the butt end of the handle is approximately $ML^2/3$ ($kg \cdot cm^2$). For example, if M = 0.3 kg and L = 70 cm, then the swingweight is approximately 490 $kg \cdot cm^2$. The swingweight about an axis 4 inches from the butt end is smaller than this by about 180 $kg \cdot cm^2$. The value of the swingweight will be larger if the racquet is head heavy, and smaller if the racquet is headlight.

Momentum

The momentum of an object is its mass times its velocity. When two objects have a head-on collision, the total momentum of the two objects after the collision is the same as the total momentum before the collision even if some of the kinetic energy is lost.

Node

A point on a vibrating object that does not move. In a racquet this occurs in two locations, one about 15 cm from the tip and the other about 15 cm from the butt end of the handle. If the ball hits at the node, the racquet will not vibrate. If it hits anywhere else, the racquet will vibrate everywhere but the nodes.

Open Face

Describes racquet head orientation at time of ball contact. An open face has the top of the string plane tilted backward and upward when the ball contact is made.

Open stance

Players these days often stand facing the net when returning the ball, rather than side-on. Open stance means facing the net, in which case the racquet is swung around in front of the body rather than in a direction towards the net.

Overgrip

A thin grip that is wrapped on top of an existing grip. Cheaper than replacing the whole grip with a replacement grip. You can choose between soft, tacky, sticky, smooth, perforated, embossed, ribbed, firm or cushioned and various combinations of these properties. A smooth, firm grip allows you to feel everything that's happening to the handle, a bit like a manual rather than an automatic.

Oversize

Generally speaking, racquets with a headsize between 105-117 $in.^2$ / 677-755 cm^2.

Pace

The International Tennis Federation (ITF) definition to describe the speed of a court is called Pace. The formula is slightly complicated, but if the ball slides throughout the bounce, then Pace = 100(1 - COF) where COF is the coefficient of sliding friction. For a grass court, COF is about 0.6 and hence Pace = 100 x 0.4 = 40. For a clay court COF is about 0.8 and hence Pace will be about 100 x 0.2 = 20.

Peak force

The maximum force obtained during a collision. When the ball hits the strings, for example, the force starts at 0, climbs to a peak just as the strings achieve maximum deflection as the ball comes to a rest, and then goes back to 0 as the ball leaves the strings.

Peak tension

Upon impact, the strings stretch and therefore increase in tension. At maximum deflection, they will achieve a peak tension, whereupon the tension will decrease as the ball is ejected. The string will not return to its before-collision tension. It will be slightly less after each impact.

Player Rating

In the USA, players are rated on a scale from 1 to 7. A 7.0 player is one of the top 100 players in the world. A 1 or 2 is a beginner.

Period

The time it takes a vibrating object to complete one back-and-forth cycle. Typically about 0.007 seconds for a vibrating racquet.

Piezoelectric

A material that is piezoelectric generates an electric charge when a force deforms it. The charge produced is proportional to the force. The opposite is also true. A piezo material is deformed when it is subjected to an electric charge, and the deformation is proportional to the amount of charge.

Polymer

Long chain molecules made from the combinations of thousands of repetitions of a basic molecular unit called a mer. "Poly" means many and "mer" means unit. Most string materials are polymers as are the resins in racquets.

Potential energy

When a ball or any other object of mass M is lifted off the ground to a height h, the work done to lift it is given by the force Mg times the distance h. This is stored as potential energy Mgh. When the ball is released, the stored potential energy is converted to kinetic energy and the potential energy drops to zero at ground level. The ball hits the ground with a speed v given by $Mv^2/2 = Mgh$. The kinetic energy of the ball is converted to elastic potential energy in the ball as the ball squashes. The ball comes to rest momentarily and loses all its kinetic energy in doing so. If the ball has a stiffness k and is squashed by an amount x, the stored elastic energy is $kx^2/2 = Mv^2/2$. Some of this energy is lost in heating up the ball, but the rest is converted back to kinetic energy as the ball bounces off the ground. Similarly, when the strings impact a ball the strings store elastic potential energy by stretching and give almost all that energy back to the ball.

Power

There is no such thing as a 100 Watt or a 0.2 Horsepower racquet or string. Racquets and strings don't generate any power by themselves. All the power comes from the player, but the racquet and strings allow some of that power to be transferred to the ball with an efficiency that depends on the particular racquet and strings. Power is one of those terms used in tennis, like feel or playability, that can't easily be defined, but everyone has a good idea what it means. Essentially it means that if you can hit a ball faster with one racquet or string than another, then the racquet or string is more powerful or more efficient in transferring energy from the arm to the ball. However, a precise definition of racquet power is complicated by the fact that it depends on the impact point and it depends on how the racquet is swung. A ball hit from the tip won't travel at the same speed as a ball struck in the middle of the strings since the tip and the middle travel at different speeds. The difference will depend on how much wrist action the player uses.

Pressureless ball

A ball that is sold in a cardboard box instead of a pressurised container since there is no excess air inside the ball. The rubber wall is slightly thicker and stiffer than in a pressurized ball. It bounces the same as a pressurized ball and lasts a lot longer, but it sounds slightly different, since it tends to vibrate a bit more. The ball never goes flat, but the cloth cover will wear.

Pressurized ball

A ball that is pressurized with air to increase its stiffness so that it bounces to the correct height. The air gradually leaks out over time or after a few sets.

RA

See DA units.

Radar gun

The speed of a ball can be measured remotely using a radar gun. The gun emits a beam of microwaves which bounces off both the racquet and the ball and it returns to a microwave detector in the gun. The frequency of the microwave beam is shifted by any moving object by an amount that depends on the speed of the object. If the detector is set to detect the maximum frequency shift, it will record the speed of the ball rather than the speed of the racquet. The gun actually records $v \cos(\theta)$

where v is the speed of the ball and θ is the angle between the path of the ball and the path of the microwave beam. If the ball is headed say 5 degrees away from the gun, then the recorded speed is 0.996v. If θ = 10 degrees, then the gun records 0.985v.

Rebound Ace

This is the court surface used for the Australian Open. It consists of a layer of rubber about 6 mm thick glued to a concrete or asphalt base with a 1 mm thick and slightly rough acrylic upper surface. The speed of the court can be adjusted slightly by repainting the upper surface with fine sand mixed in with the paint.

Recoilweight

Not an accepted physics term, this is the tennis player's version of moment of inertia about the center of mass. It describes the racquet's resistance to rotational acceleration about the center of mass as the axis of rotation. This is usually a little less than half the swingweight. Whenever the ball hits any place besides the center of mass, it tends to rotate the racquet backwards. The higher the recoilweight, the less the rotational acceleration for a given impact velocity.

Resilience

If an object is stretched or squashed, its length or diameter will change. If the force on the object is then reduced to zero, the object will return to its original shape if it is perfectly resiliant. Materials such as plasticene are not resilient since they do not return to their original shape but remain plastically deformed. A tennis string under tension for some time does not return to its original length if the tension is reduced to zero. However, a tennis string is perfectly resiliant if it remains under tension in a racquet since it always returns to the same length after stretching due to an impact with a ball. The main strings often move sideways and get stuck at a slightly longer length, but they are still perfectly resiliant in a direction perpendicular to the string plane. Some materials take time to recover to their original shape and are therefore not very resiliant. They make good vibration dampeners.

Shock

When a ball impacts a racquet, the racquet translates backwards, it rotates, and it bends creating a bending wave. All three of these motions arrive at the hand at the same time about 2 milliseconds after impact. Together, they jolt the hand and this is what a player feels as shock. The hand stops the rotation and translation, but the vibration continues for about 25-50 ms, depending on the racquet and where the impact was.

Short side

A racquet stringer starts with a length of string about 40 feet (12 m) long. About 1/3 of the string (the short side or the short end) is fed through one of the grommet holes so that the other 2/3 (the long side) remains outside the racquet. The stringer can string the short end first without having to pull the whole string through each hole.

Spaghetti strings

These originated in Germany around 1975, but they were quickly banned by the International Tennis Federation because they allowed a player to put excessive spin on the ball. The original version was strung with only about 5 cross strings and used spaghetti tubing to cover the main strings to allow them to slide more easily over the cross strings. The main strings were then tied together to allow the mains to move freely as a single unit across the cross strings. As a result, the mains stretched elastically in a direction parallel to the string plane (as well as in the direction perpendicular to the string plane like other strings) and imparted extra spin as they returned to their original position.

Speed

How fast an object like a ball travels from one location to another. It does not take into account the direction of motion, just the distance traveled per unit of time: speed = distance/time.

Speed of a court

The speed of a court refers to the change in the horizontal speed of the ball when it bounces. There is a relatively large change on a slow court like clay and a relatively small change on a fast court such as grass. The coefficient of sliding friction (between the ball and the surface) is about 0.8 for clay and about 0.6 for grass. However, the speed difference between clay and grass is not as large as one might expect. A ball served on clay takes only about 0.02 seconds longer to cross the baseline than on grass, assuming it is served at around 100 mph or more.

A bigger time difference will be noticed if the ball is hit at a slower speed, partly because the ball is slower, but also because the ball will be incident on the court at a higher angle. The ball slows down by a larger fraction when it is incident at a larger angle.

Standing wave

A standing wave is one that does not propagate like a normal wave, but "stands still". A standing wave is generated when a propagating wave reflects backwards off the ends of the medium supporting the wave and meets the incoming wave. The two waves interfere constructively at some points and destructively at points in between, with the result that the medium supporting the wave (racquet, guitar string, tennis string, etc.) vibrates back and forth rapidly. Points that vibrate the most (tip and butt end of the handle) are described as antinodes, and points that don't vibrate at all are called nodes.

Stiffness

If a tennis string is anchored at one end and pulled by a force F at the other end, the string will stretch by an amount x. The stiffness k of the string is defined by the relation F = kx (or k = F/x) and is measured in lb/in. or N/m. If k is very large, then the string is very stiff. If k is small, then the string is softer or more elastic. Since a long or a small diameter string will stretch further than a short or fat string, k depends on the length of the string and its diameter as well as the type of string. A stretched string also has a certain stiffness when it is pushed sideways. If a sideways force F at the middle of the string causes the string to move sideways by a distance y, then the transverse stiffness is given by k_p = F/y. In this case, k_p depends mainly on the string tension and the string length, but at large values of F or y it also depends on the string stiffness for a longitudinal stretch. The transverse stiffness of the whole string plane depends on all these factors, as well as the total number of strings and on the diameter of the object used to push on the strings.

The longitudinal stiffness of a string may remain constant or it may change as x gets bigger. If it changes, then it is useful to define the dynamic stiffness by k = DF/Dx, where DF is the increase in the force (starting at a certain specified non-zero value), and Dx is the increase in the length of the string.

The stiffness of a ball or a racquet can be defined in the same way. If a ball or racquet squashes or bends by a distance x when subject to a force F, then the stiffness k = F/x. However, racquet stiffness and stringbed stiffness is more commonly measured in DA or RA units. A simple way to judge the stiffness of a racquet is to tap the tip with your finger and listen to the handle vibrate (holding the frame lightly about 6 in. from the butt end with the handle close to your ear). If it vibrates at around 200 Hz, it is very stiff (and/or very light). Most racquets vibrate at around 120 to 150 Hz. Stiff racquets are like tight guitar strings. The higher the tension or the stiffer the racquet the higher the vibration frequency.

Strain

Strain is related to stress and is given by the extension or compression of an object divided by the length of the object. If a 10 inch length of tennis string stretches by 1 inch, then the strain = 1/10 = 0.1. If the stress increases, then the strain will increase. The ratio of the stress to the strain is a constant for any given material, independent of the diameter or the length of the material.

Stress

Stress is an engineering term describing the tension or compression or shear force acting per unit area on an object. For a tennis string, the stress is given by the string tension divided by the cross-sectional area of the string, and is measured in pounds per square inch or Newton per square metre.

String dampener

A small object located on the strings close to the frame whose function is to dampen vibrations of the strings so that the impact sounds like a thud instead of a ping. Andre Agassi uses a rubber band. About half the professionals use them, and the other half don't.

Stringbed

The stringbed is simply the bed of strings used to hit the ball.

String saver

The main strings move across the cross strings every time a ball is hit, eventually wearing a groove in the main strings. The main strings usually break first, at one of the grooves. String savers are small circular plastic inserts that lie between the mains and the crosses to reduce wear. A drop of superglue would probably do the same job by preventing sideways movement.

Sweetspot

The impact point on the strings that feels best when you hit a ball. Points nearby feel almost as good, which enables manufacturers to claim that their sweet spot is bigger than a single point. The sweetspot is located near the middle of the strings in a modern racquet.

Swingweight

When a racquet is swung it may feel light or heavy. Part of the heaviness depends on its weight, but one racquet can feel heavier than another even if both racquets have the same weight. A racquet with a heavy head and a light handle will feel heavier when it is swung than one with a light head and a heavy handle. The heaviness depends on how the weight is distributed along the racquet and it is measured in terms of the swingweight. The technical name for swingweight is the moment of inertia. The actual value of the swingweight depends on the location of the axis of rotation. Measurements of swingweight are normally done by the swinging the racquet about an axis in the handle located 4 inches from the butt end.

Synthetic gut

A synthetic string, usually nylon, which first appeared in the 1960's as a more durable replacement for natural gut.

Tension

A tennis string is relatively soft and floppy when it is not under tension. If the string is tied down at one end and pulled at the other end, then the string will stretch and it will be under tension. The tension force acts in a direction along the string. A string under tension will resist any attempt to push it sideways.

Throat

The middle section of a racquet near the top end of the handle.

Titanium

Titanium is used mostly as titanium oxide to make white paint. As a metal or an alloy it is very strong and light and withstands high temperatures better than most other metals so it is suitable in aircraft engines. It's use in tennis racquets, balls or strings is less directly associated with performance, but it seems to work well as a selling point.

Topspin

A ball can spin about a horizontal axis at right angles to its direction of motion in one of two directions. If the ball has topspin, then the top of the ball rotates in the same direction as the direction of motion. The bottom of the ball rotates in the opposite direction to the direction of motion. A ball rolling forwards on the ground has topspin. A ball that bounces off the court almost always bounces with topspin. If it bounces off the court with topspin, then it is usually incident on the strings of a racquet with backspin. The ball doesn't change its spin direction just before it hits the strings, but the point on the ball that contacts the strings does so in the same manner as a ball hit with backspin contacts the court surface. If a ball is thrown on the floor and lands under a table, then the ball bounces off the floor with topspin, but it approaches the underneath side of the table with backspin.

Torque

Defined as the force times the distance of the lever arm to the axis of rotation. A torque tends to rotationally accelerate an object. For instance, when a ball bounces, the friction force of the court surface exerts a torque equal to the frictional force times the radius of the ball. This torque causes the ball to change its rate of spin.

Twistweight

More correctly known as the polar moment of inertia, twistweight describes the racquet's resistance to rotational motion around the longitudinal (length-wise) axis. It is the same as swingweight, but around a different axis. The twistweights for most racquets are between 13 and 24 $kg \cdot cm^2$. Swingweights, on the other hand, typically lie between 250 and 350 $kg \cdot cm^2$, indicating that it is much more difficult to rotationally accelerate a racquet around an axis 4 inches from the butt than it is to do the same about the long axis.

Vector quantity

Quantity that has both a magnitude and direction. Examples are force, velocity, acceleration, torque. Vector quantities are depicted by arrows. The arrow indicates the direction and the length indicates the magnitude.

Velocity

The speed and direction of an object's motion. If a ball and racquet approach each other with the same speed but from opposite directions, then one has a positive velocity and the other a negative velocity, even though their speeds are the same.

Vertical speed

See horizontal speed.

Vibration

A racquet will vibrate back and forth about 140 times per second when it strikes a ball. The vibration fre-

quency is said to be 140 Hz. Very stiff racquets vibrate faster, up to about 200 Hz. Old wood racquets vibrate at about 90 Hz. The strings themselves vibrate at about 500 Hz.

Wave

A wave is a disturbance that propagates from one point to another. Sound waves travel through the air at a speed of 340 m/s. Sound travels through water at 1450 m/s and through solids at about 6000 m/s. Light and radio waves are electromagnetic waves that travel through air or a vacuum at 300,000,000 m/s. Waves at the beach travel on the surface of the water at a speed of a few m/s. An impact of a ball and a racquet sends a transverse bending wave along the racquet at a speed of around 200 m/s. The racquet bends locally around the impact point, displacing adjacent bits of the racquet from each other in a direction transverse to the direction of propagation of the wave. The adjacent bits in turn pull on neighboring bits, but it takes time for each bit to react to a bending force since each bit takes time to accelerate up to maximum speed. It is this delayed reaction time that causes the bending wave to travel along the racquet at relatively low speed (compared with sound or light). The stiffer and lighter the racquet the faster is the bending wave and the faster the racquet vibrates. If the racquet vibrates at, say, 140 Hz, the associated bending wave travels up and down the racquet 140 times each second.

Weight

When you weigh yourself on a set of bathroom scales you do so to measure your weight in lb or kg. At least that's what most people do. Scientists and engineers have stolen this common word to mean something else, in order to distinguish mass from weight. In scientific language, weight is the force Mg (measured in Newton) acting on a mass M (measured in kg) due to the pull of the earth. A mass of 1 kg weighs 9.8 Newton. If bathroom scales gave a reading in Newton, then scientistists and engineers would agree that they are used to measure weight. But bathroom scales give a reading in lb or kg, so people actually mass themselves rather than weigh themselves. In the USA, most people get around the problem by measuring mass in lb and weight or force in lb, but that is not scientifically correct. This book is scientifically incorrect whenever it quotes string tension in kg or lb since tension is a force, not a mass.

Widebody

The rules of tennis specify that the strung surface shall not exceed 15.5 inches (39.37cm.) in overall length, and 11.5 inches (29.21cm.) in overall width. It says nothing about the thickness of the racquet head in a direction perpendicular to the string plane. The thickness is typically about 20 mm, but anything over about 22 mm is regarded as a widebody racquet. An increase in frame thickness adds to the stiffness of the head.

Work

In general useage, the term "work" implies that some effort is required. In physics, work has a specific meaning. When a force F is applied to move an object through a distance d in the direction of the applied force, then the work done is defined to be $W = Fd$. A number of different things can happen to the object as a result of this work, depending on the circumstances. If an object of mass M is accelerated in a horizontal direction from rest to a speed v, then the kinetic energy of the object will be equal to the work done, i.e., $W = Fd = \frac{1}{2}Mv^2$. If the object is lifted through a height h by a force Mg without any increase in its speed, then the potential energy of the object increases by an amount $W = Fd = Mgh$. The work done by a player in swinging a racquet results in an increase in the speed and kinetic energy of the racquet.

B

Appendix B Units and Conversions

To convert from USCS to SI, multiply the USCS units by the conversion factor. To covert from SI to USCS, divided the SI units by the conversion factor.

U.S. Customary System (USCS)		Conversion factor	International System (SI)	
Acceleration				
foot per second squared	ft/s^2	0.305	meter per second. squared	m/s^2
Energy				
foot-pound	ft-lb	1.356	joule	J
Force				
pound	lb	4.448	newton	N
Force per Length				
pound per inch	lb/in.	175.127	newton per meter	N/m
Length				
inch	in.	2.540	centimeters	cm
foot	ft	0.305	meter	m
mile	mi	1.609	kilometer	km

U.S. Customary System (USCS)		Conversion factor	International System (SI)	
Mass				
pound	lb	2.2	kilogram	kg
Torque				
pound-foot	lb-ft	1.356	newton meter	N•m
pound-inch	lb-in.	0.113	newton meter	N•m
Stress				
pound per square inch	psi	6894.760	pascal	Pa
Velocity				
foot per second	ft/s	0.305	meter per second	m/s
inch per second	in./s	0.025	meter per second	m/s
mile per hour	mph	0.447	meter per second	m/s
mile per hour	mph	1.609	kilometer per hour	km/hr

Other Useful Data

Acceleration due to Earth's gravity = 9.8 m/s^2 = 32.13 ft/s^2

Angles

1 rad = 57.30°
360° = 2π rad
180° = π rad

Area

1 square inch = 6.452 square cm
1 sq cm = 0.1550 sq in.
90 sq in. = 580.6 sq cm
100 sq in. = 645.2 sq cm
110 sq in. = 709.7 sq cm

Energy

1 Joule = 0.7376 ft-lb
1 ft-lb = 1.356 Joules

Force

1 Newton = 0.2248 lb
1 lb = 4.448 Newton (lb here is a unit of force)

In the USA one pound can be a unit of mass or force in everyday use.

Length

1 inch = 25.4 mm
12 inch = 1 foot = 304.8 mm
1 cm = 10 mm
1 metre = 100 cm =1000 mm = 39.370 inches = 3.281 feet
1 km = 1000 m = 0.6214 mile
1 mile = 5280 feet = 1.609 km
Length of typical racquet = 27 inch = 685.8 mm
3 feet = 0.9144 m = net height in middle

3.5 feet = 1.067 m = net height at posts
4.5 feet =1.372 m = distance between tramlines
21 feet = 6.401 m = distance from net to serve line
21 feet = 6.401 m = distance of net posts from centre of net.
27 feet = 8.230 m = width of singles court
36 feet = 10.973 m = width of doubles court
39 feet = 11.887 m = distance of baseline to net
78 feet = 23.774 m = length of court

Mass

1 kg = 1000 g = 2.205 lb = 35.27 oz
1 lb = 16 oz = 0.4536 kg
1 oz = 28.35 gm

Speed

1 m/s = 3.6 km/hr = 3.281 ft/sec = 2.237 mph
100 mph = 160.9 km/hr = 44.70 m/s
140 mph = 225.3 km/hr = 62.58 m/s = fastest serve speed up to 1996.
100 km/hr = 62.15 mph = 27.778 m/s
50 m/s = 111.85 mph

Spin

1 revolution/sec = 60 rpm = 6.28 radian/sec
1000 rpm = 16.67 rev/sec = 104.7 radian/sec
3000 rpm = 50 rev/sec = 314.16 radian/sec

String Gauge

18G = 1.20 mm
17G = 1.25 mm
16G = 1.30 mm
15L = 1.35 mm
15G = 1.40 mm

String Tension

50 lb = 22.676 kg
60 lb = 27.211 kg
70 lb = 31.746 kg
20 kg = 44.100 lb
25 kg = 55.125 lb
30 kg = 66.150 lb

Time

1 ms = .001 s

C

Appendix C Equipment Rules

The following are the equipment 2002 rules for balls (rule 3) and racquets (rule 4), as specified by the International Tennis Federation.

3. THE BALL

Balls that are approved for play under the Rules of Tennis must comply with the following specifications:

a. The ball shall have a uniform outer surface consisting of a fabric cover and shall be white or yellow in colour. If there are any seams they shall be stitchless.

b. The ball shall conform to the requirements specified in Appendix I (Regulations for making tests specified in Rule 3) Section iv for size and be more than 1.975 ounces (56.0 grams) and less than 2.095 ounces (59.4 grams) in weight.

c. More than one type of ball is specified. Each ball shall have a bound of more than 53 inches (134.62cm) and less than 58 inches (147.32cm) when dropped 100 inches (254.00cm) upon a flat, rigid surface e.g. concrete. Ball Type 1 (fast speed) shall have a forward deformation of more than .195 inches (.495 cm) and less than .235 inches (.597 cm) and return deformation of more than .295 inches (.749 cm) and less than .380 inches (.965 cm) at 18 lb. (8.165kg) load. Ball Types 2 (medium speed) and 3 (slow speed) shall have a forward deformation of more than .220 inches (.559cm) and less than .290 inches (.737cm) and return deformation of more than .315 inches (.800cm) and less than .425 inches (1.080cm) at 18 lb. (8.165kg) load. The two deformation figures shall be the averages of three individual readings along three axes of the ball and no two individual readings shall differ by more than .030 inches (.076cm) in each case.

d. For play above 4,000 feet (1219m) in altitude above sea level, two additional types of ball may be used.

i. The first type is identical to Ball Type 2 (medium speed) as defined above except that the ball shall have a bound of more than 48 inches (121.92cm) and less than 53 inches (134.62cm) and shall have an internal pressure that is greater than the external pressure. This type of tennis ball is commonly known as a pressurised ball.

ii. The second type is identical to Ball Type 2 (medium speed) as defined above except that the ball shall have a bound of more than 53 inches (134.62cm) and less than 58 inches (147.32cm) and shall have an internal pressure that is approximately equal to the external pressure and have been acclimatised for 60 days or more at the altitude of the specific tournament. This type of tennis ball is commonly known as a zero-pressure or non-pressurised ball.

iii. The third type of ball which is recommended for use for play on any court surface type above 4,000 feet (1219 m) in altitude is the Ball Type 3 (slow speed), as defined above.

e. All tests for bound, size and deformation shall be made in accordance with the regulations in Appendix I.

f. The International Tennis Federation shall rule on the question of whether any ball or prototype complies with the above specifications or is otherwise approved, for play. Such ruling may be taken on its own initiative, or upon application by any party with a bona fide interest therein, including any player, equipment manufacturer or National Association or members thereof. Such rulings and applications shall be made in accordance with the applicable Review and Hearing Procedures of the International Tennis Federation (see Appendix III).

Note 1: *Any ball to be used in a tournament which is played under the Rules of Tennis, must be named on the official ITF list of approved balls issued by the International Tennis Federation.*

Case 1: Which ball type should be used on which court surface?

Decision. 3 different types of ball are approved for play under the Rules of Tennis, however:

a. Ball Type 1 (fast speed) is intended for play on slow pace court surfaces (see Appendix I).

b. Ball Type 2 (medium speed) is intended for play on medium/medium-fast pace court surfaces (see Appendix I).

c. Ball Type 3 (slow speed) is intended for play on fast pace court surfaces (see Appendix I).

4. THE RACKET

Rackets failing to comply with the following specifications are not approved for play under the Rules of Tennis:

a. The hitting surface of the racket shall be flat and consist of a pattern of crossed strings connected to a frame and alternately interlaced or bonded where they cross; and the stringing pattern shall be generally uniform, and in particular not less dense in the centre than in any other area. The racket shall be designed and strung such that the playing characteristics are identical on both faces.

The strings shall be free of attached objects and protrusions other than those utilised solely and specifically to limit or prevent wear and tear orvibration, and which are reasonable in size and placement for such purposes.

b. The frame of the racket shall not exceed 29 inches (73.66cm) in overall length, including the handle. The frame of the racket shall not exceed 121/2 inches (31.75cm) in overall width. The hitting surface shall not exceed 151/2 inches (39.37cm) in overall length, and 111/2 inches (29.21cm) in overall width.

c. The frame, including the handle, shall be free of attached objects and devices other than those utilised solely and specifically to limit or prevent wear and tear or vibration, or to distribute weight. Any objects and devices must be reasonable in size and placement for such purposes.

d. The frame, including the handle, and the strings, shall be free of any device which makes it possible to change materially the shape of the racket, or to change the weight distribution in the direction of the longitudinal axis of the racket which would alter the swing moment of inertia, or to deliberately change any physical property which may affect the performance of the racket during the playing of a point. No energy source that in any way changes or affects the playing characteristics of a racket may be built into or attached to a racket.

The International Tennis Federation shall rule on the question of whether any racket or prototype complies with the above specifications or is otherwise approved, or not approved, for play. Such ruling may be undertaken on its own initiative, or upon application by any party with a bona fide interest therein, including any player, equipment manufacturer or National Association or members thereof. Such rulings and applications shall be made in accordance with the applicable Review and Hearing Procedures of the International Tennis Federation (see Appendix III).

Case 1. Can there be more than one set of strings on the hitting surface of racket?

Decision. No. The rule clearly mentions a pattern, and not patterns, of crossed strings.

Case 2. Is the stringing pattern of a racket considered to be generally uniform and flat if the strings are on more than one plane?

Decision. No.

Case 3. Can vibration dampening devices be placed on the strings of a racket and if so, where can they be placed?

Decision. Yes; but such devices may be placed only outside the pattern of the crossed strings.

Case 4. In the course of play, a player accidentally breaks the strings of his racket. Can he continue to play with the racket in this condition?

Decision. Yes.

Case 5. Can a battery that affects playing characteristics be incorporated into a racket?

Decision. No. A battery is prohibited because it is an energy source, as are solar cells, and other similar devices.

D

Appendix D
The Mathematics of Ball Bounce

Brody's Bounce Model

The simplest model of a bouncing ball is the one described by Howard Brody in his 1984 paper "That's how the ball bounces." In that model, Howard assumed that the ball and the surface were both rigid so that there was no deformation of the ball or the surface. In that case, the horizontal friction force acting backwards on the ball is given by

$$F = -m dv_x/dt$$

the vertical or normal reaction force is given by

$$N = m dv_y/dt$$

and the rotation speed is given by

$$FR = I d\omega/dt$$

where x and y are the horizontal and vertical coordinates of the center of mass of the ball, ω is the angular velocity and $I = \alpha m R^2$ is the moment of inertia of the ball. For a thin spherical shell, $\alpha = 2/3$, but for a tennis ball with a 6 mm thick wall, $\alpha = 0.55$. Suppose that the ball spends a time τ in contact with the surface before it bounces. During that time the average values of F and N are respectively F_a and N_a. We can integrate the above

equations over the whole contact period, giving

$$F_a\tau = -m(v_{x2} - v_{x1}) \quad (D.1)$$

$$N_a\tau = m(v_{y2} - v_{y1}) \quad (D.2)$$

$$F_aR\tau = I(\omega_2 - \omega_1) \quad (D.3)$$

where subscripts 1 and 2 denote conditions before and after the bounce respectively.

Case Where Ball Slides Throughout the Bounce

Provided that the ball slides throughout the bounce, then $F = \mu N$ at all times and $F_a = \mu N_a$ where μ is the coefficient of sliding friction, so

$$(v_{x1} - v_{x2}) = \mu\,(v_{y2} - v_{y1}) = \alpha R(\omega_2 - \omega_1)$$

The ratio $e = -v_{y2}/v_{y1}$ is called the coefficient of restitution. It is a positive number with a value of about 0.75 (given that v_{y1} is negative if the ball is incident downwards onto the surface). Hence $v_{y2} - v_{y1} = -(1 + e)v_{y1}$ so

$$v_{x2} = v_{x1} + \mu\,(1 + e)\,v_{y1}$$

and

$$\omega_2 = \omega_1 - \mu\,(1 + e)v_{y1}/(\alpha R)$$

It is easier to think about these results if we change the sign of v_{y1} so that v_{y1} is a positive number instead of a negative number, then $e = v_{y2}/v_{y1}$ and

$$v_{x2}/v_{x1} = 1 - \mu\,(1 + e)\tan(\theta_1) \quad (D.4)$$

where θ_1 is the angle of incidence, $\tan(\theta_1) = v_{y1}/v_{x1}$

and

$$\omega_2 = \omega_1 + \mu\,(1 + e)v_{y1}/(\alpha R) \quad (D.5)$$

Case Where Ball Rolls

While the ball is sliding, the horizontal speed decreases and the rotation speed increases. The ball will start to roll at the end of the bounce period if $v_{x2} = R\omega_2$. In that case (v_{y1} now being a positive number)

$$v_{x1} - \mu\,(1 + e)\,v_{y1} = R[\omega_1 + \mu\,(1 + e)v_{y1}/(\alpha R)] = R\omega_1 + \mu\,(1 + e)v_{y1}/\alpha$$

so $v_{x1} - R\omega_1 = \mu\,(1 + e)(1 + \alpha)v_{y1}/\alpha$

or

$$\mu = \alpha(v_{x1} - R\omega_1)/[(1 + e)(1 + \alpha)v_{y1}] \qquad (D.6)$$

This gives the value of the coefficient of friction that will cause the ball to start rolling just at the end of the bounce. If the coefficient of friction is larger than this, then the ball will start rolling before the end of the bounce period. The friction force will suddenly drop to zero when the ball starts to roll and there will be no further change in the horizontal speed or rotation speed of the ball.

Suppose that the ball starts to roll at a time t. Suppose also that the horizontal speed at that time is v_x . Since the friction force drops to zero at this time, the horizontal speed when the ball bounces is also v_x , so we can let $v_{x2} = v_x$ where v_{x2} is the horizontal speed of the ball after it bounces. If we replace τ in equations D.1 - D.3 by t, then we will arrive at exactly the same equations as D.4 and D.5, except that μ is given by Equation D.6. If μ is bigger than the value given by D.6, then the ball will start rolling at an earlier time, but there is no change in v_{x2}, since the condition for the ball to roll is given by $v_{x2} = R\omega_2$ which leads directly to Equation D.6.

Hence v_{x2} will be given by $v_{x2} = R\omega_2$, exactly as if the ball started rolling at the end of the bounce period. The relevant values of v_{x2} and ω_2 can be found using Equation D.6, which can be expressed as

$$\mu\ (1 + e)v_{y1} = \alpha(v_{x1} - R\omega_1)/(1 + \alpha)$$

Hence

$$v_{x2}\ /v_{x1} = 1 - \mu\ (1 + e)\ v_{y1}\ /v_{x1} = 1/(1 + \alpha) + \alpha R\omega_1/[(1 + \alpha)v_{x1}]$$

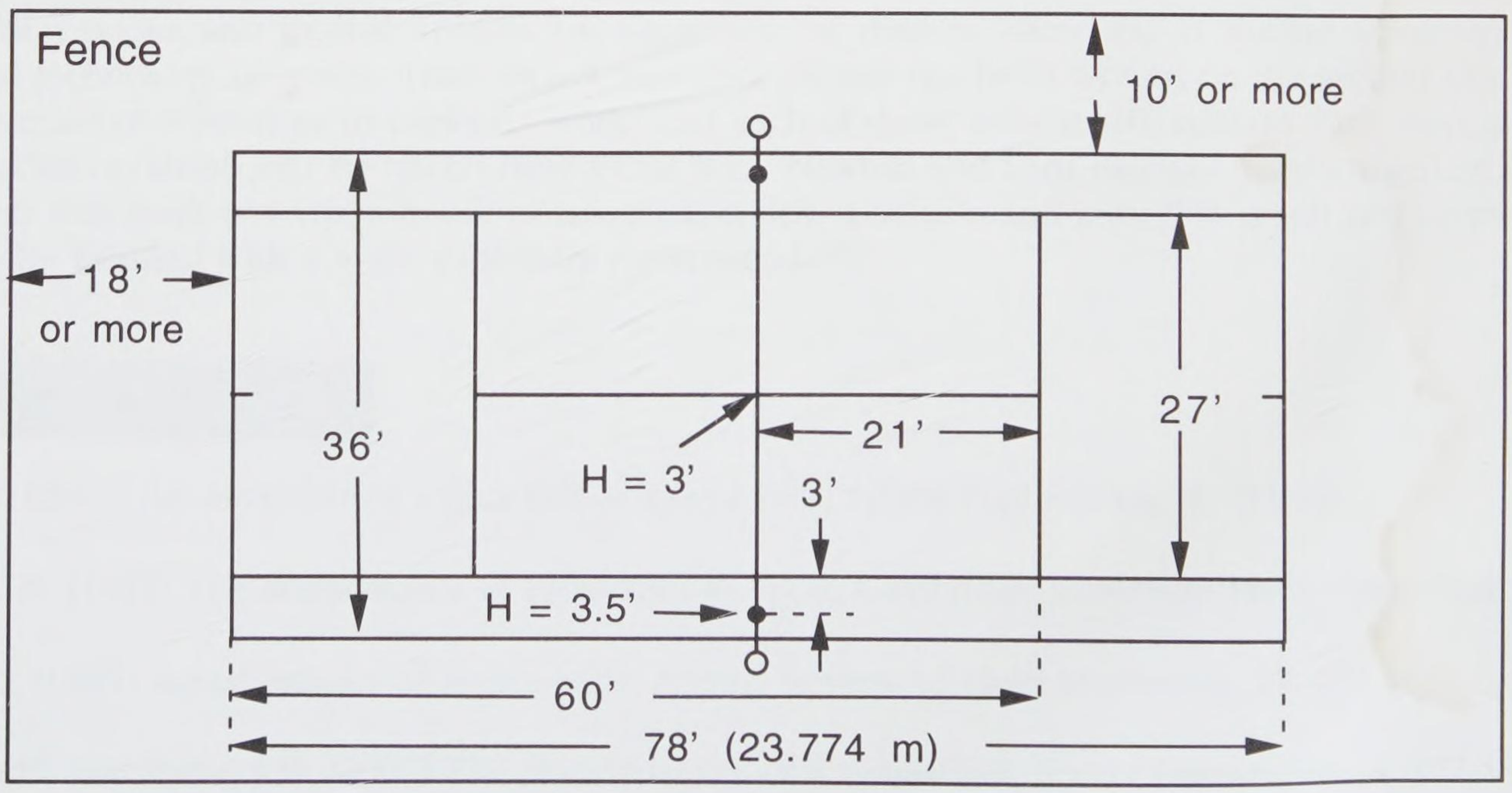
and

$$\omega_2 = \omega_1 + (v_{x1} - R\omega_1)/[(1 + \alpha)R]$$

For example, if $\omega_1 = 0$ and $\alpha = 0.55$, then $v_{x2}\ /v_{x1} = 0.645$ and $\omega_2 = v_{x1}/(1.55R)$ so $R\omega_2 = v_{x1}/1.55 = v_{x2}$.

The above model works fine if the ball slides throughout the bounce and it is roughly correct when the ball bites. However, experimental results show that the ball slides over a slightly larger range of incident angles than predicted above since the normal reaction force acts ahead of the center of the ball when the ball is squashed flat on the bottom. In addition, $v_{x2}\ /v_{x1}$ drops below 0.645 when $\omega_1 = 0$ and when the ball bites.

Appendix E
Tennis Court Dimensions



All measurements are made to the outside of the lines

F

Appendix F References

The following books and journal articles are suggested for readers interested in further information on the physics and technology of tennis. They do not represent all that has been written on the subject but each contains its own list of references to previous work (and each of those in turn will contain their own list of references). Articles on tennis can be traced back to Sir Isaac Newton and Lord Raleigh. Taken together, the reader will discover that there is a vast amount of information now available and a lot that is still not properly understood. Articles labelled with a • are especially recommended.

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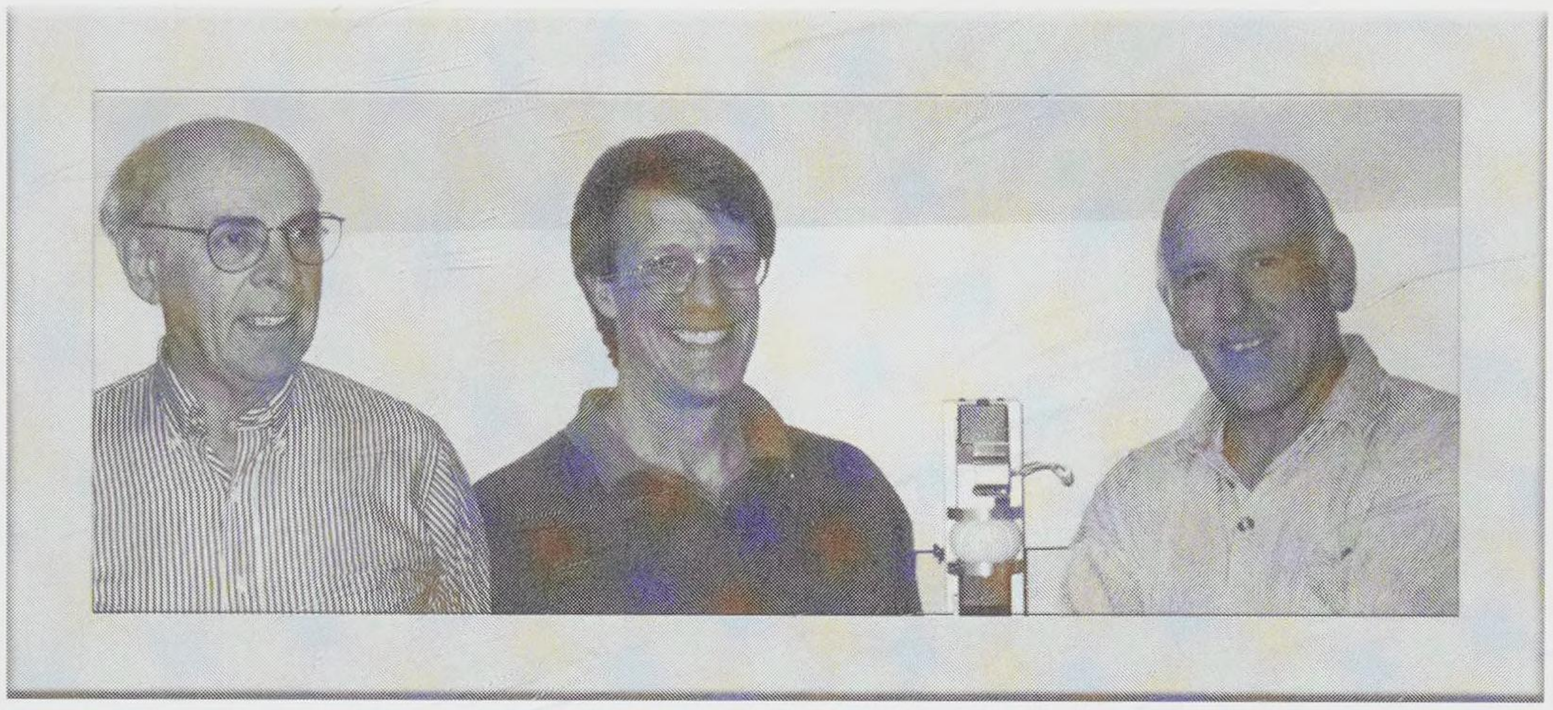


Howard Brody, Crawford Lindsey, Rod Cross

Dr. Howard Brody is an emeritus professor of physics at the University of Pennsylvania, where he was interim varsity tennis coach for part of the 1991 season. He played varsity tennis and earned his bachelor's degree at MIT and his master's and doctoral degrees at Cal Tech. For many years his primary field of research was elementary particle physics and high energy nuclear physics. Professor Brody is a member of the International Tennis Federation Technical Commission, the USTA Sports Science Committee, science advisor to the Professional Tennis Registry, technical advisor to the United States Racquet Stringers Association, on the Editorial Board of the Journal of Sports Engineering, and on the technical advisory panel of Tennis Magazine. His book Tennis Science for Tennis Players was published by the University of Pennsylvania Press in 1987. He and Vic Braden are featured in a video "The Science and Myths of Tennis." He has received the USPTR Plagenhoef award for sports science in 1996 and the International Tennis Hall of Fame Educational Merit Award for the year 2000.

Rod Cross is Associate Professor in Physics at the University of Sydney, Australia. Up until 1995 Rod's main research interest was the physics of Alfven waves in plasmas at temperatures above one million degrees. He switched to the physics of sport in 1996 when government research funding became scarce. His paper in the American Journal of Physics on the sweetspots of a baseball bat was voted as one of the top 75 papers in physics in the USA in 1998. Rod's main research interest now is the physics of tennis, which he has played competitively since 1950. He has published about 25 papers on this subject in physics and engineering journals and has written extensively for USRSA's Racquet Tech on the physical properties of tennis strings and tennis racquets.

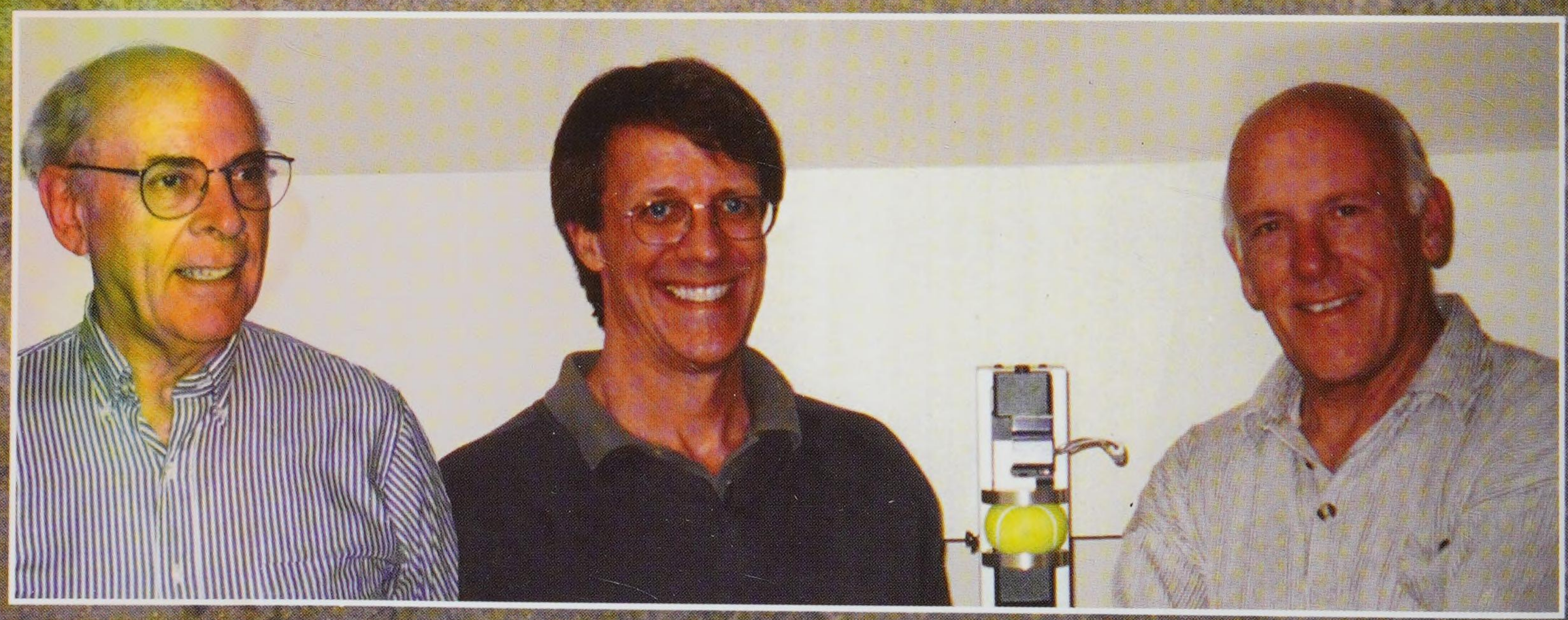
Crawford Lindsey is the publisher and managing editor of Racquet Tech magazine and The Stringer's Digest (publications of the USRSA). He is also the web developer/master of www.racquettech.com, for which he has developed extensive software programs dedicated to racquet selection, customization, optimization and stringing. His bachelor's degree is from Princeton University and he has done further studies in publishing, education and business. He was a product manager for squash and racquetball racquets as well as tennis and ski accessories at Head Sports. His book, The Book of Squash, was published in 1987, and he has authored two other books on unrelated subjects. He has written extensively on the science and technology of tennis in Racquet Tech magazine

The Physics and Technology of Tennis

The life's purpose of a tennis racquet is to change the speed and direction of a tennis ball. This momentous event takes place in approximately five-one-thousandths of a second. During that time, the ball and the racquet launch into a frenzy of technology-enhanced physics:

- The strings stretch, deflect, vibrate, store and return energy.
- The ball squashes, slides, bites, reverses direction, ejects, and flies away like vibrating jello.
- The racquet decelerates, rotates, bends, twists and vibrates.
- Bending waves race up and down the racquet, slamming the handle into the hand.
- Pain, frustration, anger, joy, or elation explode forth from the collision.

Players paying upwards of $300 per racquet and $50 for restringings can spend hundreds of dollars every year trying to influence any of these events in their favor. This book explains the what, how and why of those fateful collisions on each side of the net, as well as the flight and the bounce of the ball in-between.



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